1 INTRODUCTION

Sampling the considerable literature\textsuperscript{1,2,3,4,5,6} about loudspeaker arrays, it's noticed that the focus is often narrowed down to one specific technology. In particular, beamforming and Wave Field Synthesis (WFS) are covered in separate studies and seem to be unrelated, or at best only remotely related, topics. Both will be introduced briefly. Beamforming is a spatial filtering technique used to aim sound in a specific direction. There are many benefits, which include enhanced power efficiency, improved uniformity of coverage, increased direct-to-reverberant ratio, and reduced sound spill. Beamforming techniques can be broadly divided into two categories; mechanical and electronic beamforming, used in curved line arrays and steered column speakers, respectively.

WFS is a spatial audio rendering method, capable of delivering a physically correct reproduction of the auditory scene. Depending on the practical implementation, the loudspeakers are distributed along a horizontal line or across a (vertical) plane. Admittedly, at first glance these technologies seem to have very little in common, but from a wave perspective, it's obvious that they are closely intertwined. More importantly, both technologies can benefit from each other. Principles and features which are considered to be unique for one technology can be incorporated into the other, and vice versa. Using Matrix Arrays, 3D sound fields can be reproduced and controlled precisely in every direction. This opens up new ways for creating and controlling sound in functional as well as creative applications.

2 ARRAY PRINCIPLES

2.1 Wave interference

The main underlying physical principle of any array is wave interference. Sound waves emitted by multiple sources interact with each other. Constructive interference occurs when the individual sound waves reinforce each other, building a wave of greater amplitude. Destructive interference occurs when the waves oppose each other, resulting in lesser amplitude. For waves that are neither fully in-phase nor out-of-phase, partial reinforcement or cancellation occurs. By combining multiple loudspeaker drivers into an array, sound can be reinforced in a specific direction, while being attenuated in other directions. Similar to frequency filters boosting or attenuating certain parts of a signal spectrally, arrays allow spatial filtering of the radiated sound field.

2.2 Beamforming

The traditional way of controlling directivity is by physically pointing a horn-loaded loudspeaker to the audience. Alternatively, loudspeaker arrays (usually in the form of passive column speakers) can be used. Compared to horns, arrays typically have a narrower beam (at least in one dimension) and offer more precise control, especially at mid and high frequencies. Early realizations of loudspeaker arrays suffered though from a frequency-dependent beam width, yielding an uneven coverage and frequency response.

Nowadays, the physics of loudspeaker arrays and their design principles are well understood and most artifacts can be avoided, or at least mitigated. The most important design criteria are the array size, driver density and driver directivity. At low frequencies a large array is needed to obtain sufficient directivity control. At high frequencies a small array will do, but a dense driver distribution is required in order to avoid spatial aliasing up to a certain frequency limit. These criteria can be
quantified as follows. The beamwidth $B_{-6\text{dB}}$, (i.e., opening angle of the beam defined by the -6dB points\(^5\)), is proportional to the ratio between the wavelength $\lambda$ and array length $L$

\[
B_{-6\text{dB}} = K \frac{\lambda}{L} \quad \text{with } K = 69^\circ \quad (1)
\]

To avoid spatial aliasing, the distance $d$ between the drivers should satisfy the spatial Nyquist criterion

\[
d \leq \frac{\lambda_{\text{min}}}{2} \quad (2)
\]

Using $\lambda = c/f$, with frequency $f$ and the speed of sound $c$, the spatial Nyquist frequency can be expressed as:

\[
f_{\text{Nyq}} = \frac{c}{2d} \quad (3)
\]

For frequencies below $f_{\text{Nyq}}$ no spatial aliasing occurs. In case of narrow beams (with beamwidth $B$) a less strict ‘control frequency’ criterion can be derived\(^4\)

\[
f_{\text{ctrl}} = \frac{c}{2d \sin(\pi/2)} \quad (4)
\]

Note that $f_{\text{ctrl}} \geq f_{\text{Nyq}}$. Between $f_{\text{Nyq}}$ and $f_{\text{ctrl}}$ grating lobes may still occur but they don’t interfere with the main lobe and don’t affect the direct sound coverage of the audience. Grating lobes can be attenuated by using directional loudspeakers, but this will limit the steering capabilities of array.

Over the past decades, several beamforming technologies have emerged, each making different design and performance trade-offs. The two most established types will be discussed in more detail now.

### 2.2.1 Mechanical beamforming

The first type is mechanical beamforming, used in Line Array technology. In its original form\(^2\)\(^6\), a line array comprises a number of relatively large loudspeaker elements mounted in a curved line (typically, J-shaped), all driven in phase. In order to reduce spatial aliasing, directional waveguides are employed to minimize driver interference at high frequencies. The vertical dispersion is governed by the curvature of the array. The horizontal coverage angle is fixed and defined by the radiation pattern of the individual elements. In order to change the vertical dispersion, the splay angles between the cabinets need to be adjusted.

More recently, the spectral and spatial performance of a line array can be optimized by processing the input signals to the loudspeakers. However, after it is deployed, the direction and width of the beam cannot be altered much due to the narrow vertical dispersion of the HF waveguides and the curvature of the array.

### 2.2.2 Electronic beamforming and steering

The second technology discussed here is electronic beamforming, mostly used in beam-steered columns (either analog or digital). Typically, the array consists of many, densely spaced, wide dispersion drivers. In order to obtain a constant beamwidth over a wide frequency range, a ‘constant-$\lambda$’ method\(^6\) is used, i.e., reducing the effective length of the array with decreasing wavelength using frequency shading filters. This approach can be extended to two dimensions, enabling horizontal as well as vertical control of the opening angle. The downside to frequency shading is the reduction in SPL due to the low array efficiency at high frequencies.

The principle of beam steering is explained geometrically in Figure 1, showing a linearly spaced array of N drivers.

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By applying increasingly longer delays to the signal of drivers 0 to N-1, the wavefront is tilted. Note that in the steering direction, the driver contributions are in phase, resulting in constructive interference. The required delay for driver $n$ is given by the distance $d$ between the drivers and the steering angle $\theta$

$$\tau_n = n \tau = n \frac{d}{c} \sin(\theta) \quad \text{with } n = [0, 1, ..., N - 1]$$  \hspace{2cm} (5)

An interesting feature of DSP-controlled arrays is the possibility of emitting multiple beams, steered in different directions using several parallel processing chains, which isn’t possible with mechanically steered arrays.

### 2.3 Wave Field Synthesis

Wave Field Synthesis (WFS) is a spatial audio rendering technique that places virtual sound sources in real space. Spherical wave fields are synthesized by many separately driven loudspeakers (secondary sources) distributed along a horizontal line or across a (vertical) plane. In theory, WFS is capable of delivering a physically correct reproduction of the auditory scene. Unlike conventional techniques (like stereo or surround), WFS does not rely on stereophonic principles (i.e., phantom sources) creating an acoustic ‘illusion’ over a very small area in the center of the loudspeaker setup, known as the ‘sweet spot’. WFS allows listeners to perceive the correct direction of the virtual sound source, regardless their position in the reproduced sound field.

WFS is based on the Huygens’ principle (published in the year 1690) which states that the propagation of a wave from a source through a medium can be derived by combining the contributions (wavelets) of secondary sources, positioned along the wavefront. Later, the Huygens principle was quantified by the Kirchhoff-Helmholtz and Rayleigh representation theorems, which describe the reconstruction of a primary source field by a distribution of secondary sources (loudspeakers) on an arbitrarily shaped closed surface or infinitely large plane. For practical reasons, the general 3D WFS formulation can be transformed into a so-called 2½D solution\textsuperscript{3,4} using horizontal line arrays instead of planar arrays. In this paper the full 3D solution is considered using planar arrays.
Essentially, the driving signals of the secondary sources are delayed and attenuated versions of a pre-filtered virtual source signal. Note that each loudspeaker is driven with the full range source signal, i.e., no frequency shading is applied across the array.

Usually, the virtual sources are placed behind the WFS array. The acoustic ‘field of view’ for a given virtual source is limited by the size of the array, and can easily be found by drawing the ‘lines of sight’ between the source point and the edges of the WFS array, as shown in Figure 2. In this example, each virtual source inside the indicated source area can be localized by every listener in the listener area. Just like with real point sources, the SPL of virtual point sources drops with 6dB per distance doubling.

A unique feature of WFS is the possibility to place virtual sound sources in front of the array, so-called focused sources. The rendering area of a focused source can be found by drawing lines from the edges of the array through the focus point. Obviously, the sound won’t be localized correctly between the array and the focus point, but beyond the focus point the localization is valid throughout the listener area.

![Figure 2: Source and listener areas for different virtual source positions, behind as well as in front of the WFS array.](image)

### 3 A WAVE PERSPECTIVE ON BEAMFORMING

#### 3.1 Coverage and aperture

In section 2.3 it was shown that the acoustic field of view is limited by the aperture of the array, i.e., the solid angle occupied by the array as seen from the virtual source. By placing the virtual source closer or further away from the array, the coverage angle in both directions can be increased or decreased. In addition, by shifting the virtual source horizontally or vertically, the wave can be steered sideways, or up and down, respectively. It should be realized that this simple geometric interpretation is only valid in the far field of the array.

Note that by changing the distance between the virtual point source and the array, the horizontal and vertical opening angle cannot be set independently. The ratio of the opening angles in both directions is governed by the width and height of the array. This limitation could be overcome by
applying a suitable 2D amplitude tapering function, but that would reduce the effective array size, which is undesirable. By using a proprietary delay method, the curvature of the wavefront can be curved independently in the horizontal and vertical direction, resulting in an ‘ellipsoidal’ shape as opposed to spherical.

### 3.2 Inverse Wavefront Shaping

This ‘wave perspective’ on beamforming, introduced in the previous section, can be taken further by synthesizing complex-shaped wavefronts. This novel and unique technique can be best described as ‘Inverse Wavefront Shaping’. As an example, consider the geometry in Figure 3, consisting of a rectangular audience plane (green) covered by a planar loudspeaker array (black). By calculating the delay and gain for each driver in the matrix array according to an undisclosed, patent-pending procedure, the curvature and the corresponding divergence of the wavefronts (blue meshes) are spatially optimized to the size and shape of the coverage area. The intensity at each point on the wavefront (indicated by the density of the blue meshes) is matched to the corresponding distance to the audience, ensuring an optimum driver utilization and a uniform coverage.

![Figure 3: Expansion of a spatially optimized, complex-curved wavefront (blue meshes) matching the size and shape of the audience area (green), illustrating the concept of ‘Inverse Wavefront Shaping’.

The synthesized wavefronts can be interpreted as being emitted by a spatially extended, curved virtual source behind the array, which is different from conventional WFS where virtual point sources are used. An interesting observation is that the curvature of the wavefronts increases towards the bottom, allowing for an increasingly wide dispersion towards the front of the audience.

### 3.3 3D Beam Optimization

Numerical beam optimization has been a proven concept for more than 20 years. Working backwards from the desired levels at the coverage and avoidance areas, the optimum driving filter for each output channel can be calculated. To achieve this, a suitable spatial phase function has to be chosen. Naively, a simple radial phase function, with its origin at the acoustic center of the array, would be an obvious candidate. However, this leads to a non-optimum array efficiency at high frequencies, as only a small ‘stationary phase’ zone’ around the acoustic center contributes to the total output. In order to solve this problem, a more educated, spatially optimized phase function is composed using the ‘Inverse Wavefront Shaping’ method, described in section 3.2.

The combination of Inverse Wavefront Shaping and numerical optimization is the foundation of HOLOPLOT 3D Audio-Beamforming. This technology allows for precise shaping and steering of...
beams in two dimensions, providing high array efficiency, optimum coverage and spectral consistency across the audience. The sheer amount of output channels in a matrix array, varying from a few tens to many thousands, sparked the development of an efficient algorithm, capable of solving huge optimization problems.

To show the versatility of the 3D Audio-Beamforming technology, a few application examples will be given, deploying the HOLOPLOT X1 Matrix array\(^\text{\textregistered}\). The X1 series features two Audio Modules, Modul 96 (MD96) and Modul 80-S (MD80-S). The MD96 is a 2-way design, integrating 96 drivers in a two-layered matrix design. The MD80-S is a 3-way design with 80 drivers in its first two matrix layers and a subwoofer driver in its third layer. Each Audio Module integrates a powerful DSP, providing individual signal processing (FIR+IIR) to every single loudspeaker driver. A front view of both module types is shown in Figure 4.

Figure 4: Front view of the HOLOPLOT X1-MD96 (left) and MD80-S (right) Audio Modules (grille removed).

First, a so-called Coverage Beam is optimized for a fan-shaped audience area, as shown in Figure 5. For demonstration purposes the ‘tip’ of the fan is pointing away from the array, which poses a more difficult scenario for the optimization. The audience is surrounded by an avoidance area with outer dimensions of 60x60 m. The array is a 5x4 (rows x columns) Matrix, populated with a mix of 20 X1-MD96 and X1-MD80-S modules, with in total 1,860 output channels.

Figure 5: SPL distribution of an audience area with an atypical shape using a 5x4 HOLOPLOT X1 Matrix array.
A single Matrix array can generate multiple sound fields simultaneously, each with its own content, equalization, level, shape, and target area. In order to demonstrate this feature, a beam is projected to one zone while the neighboring zone is avoided. The dimensions of each zone are 20x40 m. In this example an acoustic separation of 20 dB can be realized between the two zones in the speech frequency range, as shown in Figure 6. By projecting a second beam (not shown here), with reversed coverage and avoidance zones, different language audience areas can be created for example.

![Figure 6: Acoustic separation between a coverage and a neighboring avoidance area using a 5x4 HOLOPLOT X1 Matrix array.](image)

The same Matrix array is also capable of tightly focusing audio content onto small groups of people, either by placing focused virtual sources in the audience, or by creating narrow beams, each targeting a small area. This feature is demonstrated in Figure 7 using three focused beams projected onto a large audience area of 40x40m.

![Figure 7: Projection of three focused beams onto a large audience area using a 5x4 HOLOPLOT X1 Matrix array.](image)

### 3.4 Wave Field Optimization

From the literature it’s known that the application of WFS in indoor spaces can have a negative effect on the spatial rendering quality⁴. Depending on the acoustics, the interaction of the synthesized sound field with the room can be quite strong due to the omnidirectional characteristics.
of the virtual point sources. Also, the rapid level drop with distance of the direct sound is not always desirable in large spaces.

Wave Field Optimization can strongly reduce these effects. The process is very similar to beam optimization; the user specifies which areas should be covered or avoided. In addition, the desired virtual source position must be defined as an extra optimization parameter. It will be shown that in this way, an accurate source localization can be combined with a uniform coverage and a strong rejection towards the boundaries of the space.

As an example, consider a 4x12 HOLOPLOT Matrix array, populated with 48 X1-MD96 modules. Figure 7 shows the optimized wave field of a virtual source (green dot) for a single frequency (1 kHz) as well as the corresponding SPL distribution. For the optimization, the horizontal audience plane is set as the target area, with a uniform pressure amplitude, while the ceiling is defined as an avoidance area, i.e., with zero amplitude. For visualization purposes, a vertical, sectional plane is added, which is not used in the optimization. It's important to realize that the spherical wave expansion from the virtual source is only enforced at the audience. In the vertical plane no conditions apply to the spatial phase response.

From the plots a few interesting observations can be made. At the audience plane, a circular wave pattern can be recognized, originating from the virtual source position. The coverage is very uniform, only tapering off slightly towards the edges. In the vertical plane the wavefronts have a more straight profile with a large amplitude. These strong, directional waves are forming a narrow beam in the vertical plane, staying away from the ceiling, which is clearly visible in the SPL plots. In fact, the generated wave field can be interpreted as a cylindrical wave, emitted by a virtual line source, indicated with a short, green line segment through the virtual source point.

Based on the normal vectors of the wavefronts, it's evident that the sound will be correctly localized horizontally. In the vertical plane the perceived direction will slightly shift upwards with increasing receiver distance. Note that a similar vertical image shift is found with J-shaped line arrays.

**Figure 7:** Optimized wave field from an virtual source using a 4x12 HOLOPLOT X1 Matrix array. In the upper three plots different views of the synthesized wave field are shown. The lower three plots show the SPL distribution.

### 3.5 Discussion

It has been shown that, from a wave perspective, 3D Audio Beamforming and Wave Field Optimization are closely intertwined. Principles and features which were considered to be unique for one technology have been incorporated into the other, and vice versa. Their use cases remain different though, which has clear implications on the design of the sound system. In beamforming the main goal is to achieve a uniform coverage. The size of the array
should be mainly selected based on the required SPL and the desired directivity control at low frequencies (Eq.1).
In Wave Field Optimization the primary goal is to provide accurate and precise source localization everywhere in the audience. For this use case the size of the array is mainly defined by the required ‘field of view’, as seen from the virtual sources (section 2.3).

4 SUMMARY

In this paper a new way of creating and controlling sound for both functional and creative applications has been introduced. Building on the principles of Beamforming and Wave Field Synthesis, 3D sound fields reproduced by dense matrix arrays, can be controlled precisely in every direction.

The unique combination of ‘Inverse Wavefront Shaping’ and numerical array optimization is the foundation of HOLOPLOT 3D Audio-Beamforming. This technology allows for precise shaping and steering of beams in two dimensions, providing optimum coverage and spectral consistency across audience areas of any shape and size. Unwanted reflections from hard surfaces, or sound spill into neighboring areas can be strongly reduced by including avoidance areas.

HOLOPLOT 3D Wave Field Synthesis is capable of delivering a physically correct and realistic reproduction of the auditory scene. It enables accurate sound localization at every listening position.

The performance of a WFS system can be even further improved by Wave Field Optimization. By shaping the synthesized wave field, an accurate sound localization can be combined with a uniform coverage and minimum interaction with the room, which is not possible with conventional WFS.

A single HOLOPLOT Matrix array can generate multiple sound fields simultaneously, each with its own content, equalization, level, shape, and target area. This opens up new degrees of freedom in system design and content creation for Public Address as well as Spatial Audio.

5 REFERENCES

8. www.holoplot.com