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THE APPLICATION OF HIGHER-ORDER ADAPTIVE FINITE ELEMENTS IN VIBRO-ACOUSTIC ANALYSES

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1 INTRODUCTION

Vibro-acoustic analysis has become a mainstream engineering tool in the quest for quieter cars, railway vehicles, and aircrafts. Moreover, for acoustics applications, the engineers are often interested in modelling the sound field over the entire audible frequency range (20 to 20,000 Hz), which means solving the Helmholtz equation over a large frequency range. Following the nature of the solution, the requirements in terms of mesh resolution vary drastically over this frequency range. In conventional finite element (FE) formulations, smaller wavelengths translate into smaller element sizes, therefore, leading to extremely large system of matrices and lengthy computations. Furthermore, in conventional FE formulations—with 1st- or 2nd-degree polynomial shape functions—typically 6 to 8 elements per wavelength are required for the accurate representation of the sound wave. Therefore, ideally, a separate mesh should be generated for each frequency of interest: i.e. an optimum mesh (h refinement in hp-FEM) for each frequency step. On the contrary, it is common practice to use a single FE mesh, with element density suited to represent the sound waves at the highest frequency of interest. Such practical considerations are compounded by more fundamental limitations of the conventional finite element methods at mid to high frequencies. In particular, the pollution effect (i.e. the cumulative build-up of dispersion error over the computational domain) leads to a rapid increase in numerical error at high frequencies [1]. The dispersion error is the difference between the theoretical wavenumber k and the wavenumber $\breve{k}$ actually observed in the numerical model. The presence of this pollution error enhances even further the differences in mesh resolutions required for low and high frequencies.

This paper presents an efficient implementation of the higher-order finite elements, which keeps finite element mesh density same throughout the frequency range of interest, and yet maintains the accuracy of the solution, by increasing the polynomial order (p refinement in hp-FEM) of the finite elements. A key feature of the proposed method is the ability to select automatically the order of interpolation in each element so as to obtain a target accuracy for each individual frequency, while minimizing the cost. This is achieved using a simple local a *priori* error indicator. For simulations involving several frequencies, the use of hierarchical shape functions leads to an efficient strategy to accelerate the assembly of the finite element model.

This p-adaptive higher-order finite element method has been implemented in the FEM AO (Adaptive Order) solver of the LMS Virtual.Lab™. The performance of the FEM AO, compared against conventional FEM, is illustrated using three different examples: a. prediction of acoustic transfer function for pass-by-noise simulation; b. prediction of fan noise for an aero-engine; c. prediction of transmission loss of an industrial muffler. The examples show that FEM AO delivers equally accurate results 2 to 20 times faster compared to a conventional FEM and with a more efficient use of in-core memory.

2 THE PRINCIPLE OF HIGHER-ORDER ADAPTIVE ELEMENTS

2.1 Higher-order polynomial functions

An alternative to reduce the pollution error is to increase the order of interpolation of the polynomial basis functions. Higher-order approximations lead to significantly reduced resolution requirements, which in turn result in smaller numbers of degree of freedom (DoF) for a specific Helmholtz