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# ACOUSTIC PERFORMANCES CALCULATION OF DISSIPATIVE MUFFLER WITH NON-ISOTHERMAL SHEAR FLOW USING THE WAVE ENVELOPE METHOD

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## 1. INTRODUCTION

The sound propagation and attenuation, in a finite duct presenting an axial variation of wall impedance and in the presence of real flows i.e. viscous (shear flows) and non-isothermal fluids, are complex conditions often encountered in the aerodynamic and aeronautic industries. Munjal [1] and Cummings [2] presented comprehensive reviews of works involved in prediction methods for the performance of flow duct silencers. Recently, some complicated muffler geometries (Peat [3] and Seybert [4]) and complex material model for dissipative muffler (Allard [5], Glav [6] and Hansen *et al.* [7]) were investigated. Here, the wave-envelope technique developed by Baumeister [8] to alleviate the numerical calculation and the model of reflection coefficient and transmission loss calculation developed by Craggs [9] to evaluate the performance of mufflers are improved and adapted to the case of uniform cross-section dissipative muffler with mean shear flow and temperature gradients [10].

## 2. GENERAL EQUATIONS

Assume an uniform cross section dissipative muffler with two-dimensional rectangular coordinates in which an acoustic wave field is propagating in conditions of mean shear flow and temperature gradient. The propagation equations are given by the following system [11]

$$\begin{cases} M_0U_x(\partial p/\partial x) + (\partial u/\partial x) + (\partial v/\partial y) + j(kL)p = 0 \\ T(\partial p/\partial x) + M_0U_x(\partial u/\partial x) + M_0(\partial U_x/\partial y)\, v + j(kL)u = 0 \\ T(L/D)^2(\partial p/\partial y) + M_0U_x(\partial v/\partial x) + j(kL)\, v = 0 \end{cases} \qquad (1)$$

where $kL = 2\pi L/\lambda$ is the dimensionless frequency ($kD = (kL)D/L$), $\lambda$, $D$ and $L$ are respectively the wavelength and the transverse and longitudinal dimensions of the duct. $p$, $u$ and $v$ designate respectively adimensional acoustic pressure, longitudinal velocity and transverse velocity. The flow has only a single component $U_x$ according to the longitudinal direction $x$ and is a function of the transverse coordinate $y$ of the duct only, the temperature field $T$ is linear and depend on two variables, $x$ and $y$. $M_0$ is the Mach number and $j$ is given by $j^2 = -1$.

The continuity of acoustic displacement has been adopted here to represent wall conditions. So, we have :

$$(D/L)(kL)v = \pm\left\{A(kL)p - jM_0AU_x(\partial p/\partial x) - jM_0U_x(\partial A/\partial x)p\right\} \quad (2)$$

where $A = 1/Z$ is the acoustic wall admittance. The expression of wall impedance in the case of a thin material with localized reaction is given by $Z = \left(1/\sqrt{T}\right)\left[R + j\cot\left\{(kL)\left(e/\sqrt{T}\,L\right)\right\}\right]$, where $R$ is adimensional wall resistance and $e$ is the thickness of the material.

At the duct inlet, the boundary condition is expressed in the following form, on the assumption that there is a source placed in $x = 0$

$$u(0,y,t) = f(y)\exp(j2\pi t) \quad (3)$$

where $f(y)$ represents the transverse variation of the axial component of acoustic velocity. At the duct end, the boundary condition can be represented by an impedance implying an anechoic condition [11]

$$\zeta_e = p(1,y)/u(1,y) \quad (4)$$

## 3. FINITE DIFFERENCES WAVE ENVELOPE METHOD

Because of the waves propagate along axial direction, each acoustic perturbation $f$ $(p,u,v)$ is written as $f(x,y) = \bar{f}(x,y)\,e^{-j(kL)\beta x}$ and then each partial derivative with respect to $x$ is written as $\partial f/\partial x = \left(\partial\bar{f}/\partial x - j(kL)\beta\bar{f}\right)e^{-j(kL)\beta x}$. $\beta$ is the propagation constant calculated numerically in the case of an infinite hard-walled duct containing an inhomogeneous medium [12]. The new variables $\bar{f}$ are the envelopes of the wave with slower variation according to the $x$, and their calculation is performed numerically by the finite-difference method and expressed in matrix system form with complex values, as follows:

$$[A]\{\delta\} = \{B\} \quad (5)$$

where $[A]$ is the matrix having a dimension $3\times N_1\times N_2$ ($N_1$ and $N_2$ are the grid points number in the $x$ and $y$ direction respectively), for which the coefficients are known; $\{\delta\} = \left\{\bar{p},\bar{u},\bar{v}\right\}$ is the unknown vector and $\{B\}$ represents the column vector containing the inlet condition or the source term. The system is solved by the $LU$ factorization technique.

## 4. REFLECTION COEFFICIENT AND TRANSMISSION LOSS

In the case of an homogeneous medium, Craggs [9] developed a model for calculating transmission loss. He assumed that the incident wave is progressive and arrives from the infinite, and that the reflected wave propagates in the opposite direction toward the infinite (Fig. 1). Thus, he expressed pressure, in its intake condition, as the sum of a known incident pressure (plane wave) and an unknown reflected wave.

$$f=\left[f^{+}(y)\,e^{-j(kL)\beta^{+}x}+f^{-}(y)\,e^{+j(kL)\beta^{-}x}\right]e^{j2\pi t} \qquad (6)$$

In our case, for which the medium is inhomogeneous with mean shear flow, the situation is more difficult since the incident and reflected waves are no longer plane waves. Thus, the modeling of waves arriving from the infinite or going toward the infinite is not possible analytically. The magnitude $f^{+}$ and the downstream propagation constant $\beta^{+}$ are determined numerically in the direction of the flow, and the upstream propagation constant $\beta^{-}$ in the opposite direction [12].

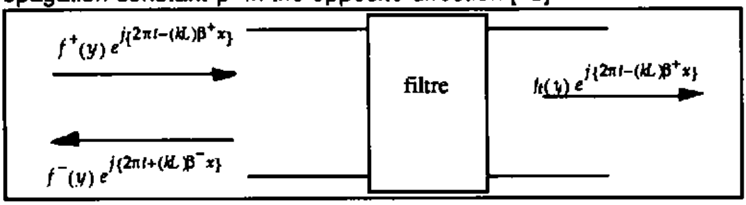
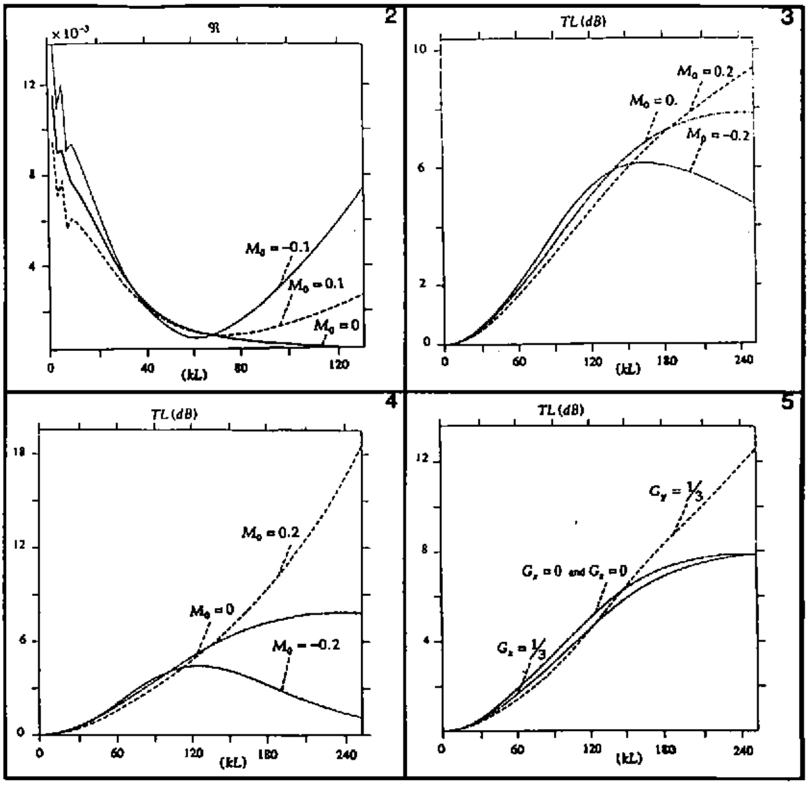



Figure 1. Model of reflection coefficient and transmission loss calculation

The reflection coefficient $\Re$ at the muffler entry always remains low, even with an inhomogeneous medium, as can be noted in Figure 2. This is in perfect agreement with predictions since dissipative mufflers cause little acoustic wave reflection. However, transmission loss $TL$ undergoes considerable modification. The absorption of acoustic energy provides for maximum attenuation at the resonance frequency of the elementary resonators which constitute the treated section. In the presence of turbulent and laminar flows (Fig. 3 and 4), this attenuation peak is displaced toward high frequencies for positive Mach numbers and toward low frequencies for negative Mach numbers. This is in agreement with empirical results in the literature (Munjal [1], chap. 6). These phenomena are more marked for laminar flow, which is normal given the thickness of the boundary layer. Attenuation increases with a transverse temperature gradient $G_y$ ($T_0 = 300K$ and $T_1 = 400K$), whereas absorption has a tendency to decrease in the presence of an axial gradient $G_x$ ($T_1 = 400K$ on the left, $T_0 = 300K$ on the right) (Fig.5).

## CONCLUSION

This work is an attempt to improve the wave-envelope technique and the model of the reflection coefficient and transmission loss calculation, because all parameters determined analytically in the literature and non adapted to inhomogeneous medium are calculated numerically here. The results above show that the transverse gradients of mean flow and temperature refract acoustic waves toward the walls (liner) and are responsible for the increase of acoustic energy attenuation.




Figures 2, 3, 4 and 5 numerical results ($L = 24D$)

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