EARLY REFLECTOR GEOMETRY OPTIMIZATION METHODOLOGY

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1 INTRODUCTION

It is well known that adding early reflections in auditoriums raises the apparent loudness of the direct sound in a comfortable, natural way, much more agreeable than turning up the volume. The denser the early reflections, the better a person can hear. Reflectors in concert halls are vital to the quality of the sound that reaches the audience. Roof reflections can give a very unique and characteristic sound profile in a closed space .

In this paper a methodology to design the geometry of roof and/or side surfaces to evenly distribute the early reflections to the audience is presented. Through this methodology, an algorithm is developed to generate the acoustic shell by calculating the ideal curvature and the orientation of the surfaces for this purpose, while also retaining given geometric properties and constraints.

2 GEOMETRY OF REFLECTIONS

In closed spaces the perceived sound level can be substantially increased compared to free field conditions, since sound energy is redirected to the audience due to reflections. As the sound waves that travel into the air encounter higher impedance obstacles, like walls and ceilings, part of the sound energy bounces back to the room causing a lower amplitude sound wave to travel in the opposite direction. When these reflections reach the ear very shortly after the initial sound wave, they are not perceived as new sound, but as an enhancement of the direct sound.

The main part of the reflected energy, according to the law of reflections, will form a sound wave that will have the same angle of reflection as the angle of incidence, whereas the angles are measured relative to the perpendicular to the surface at the point where the wave strikes the surface. A simple model for simulating sound propagating from a speaker inside an auditorium would be to consider the source as a ray that travels in the room and reflects on its sound-reflective surfaces (surfaces that minimize the absorbed and diffused energy) to reach the audience and the audience as a vertically offset surface from the floor at the seating area.

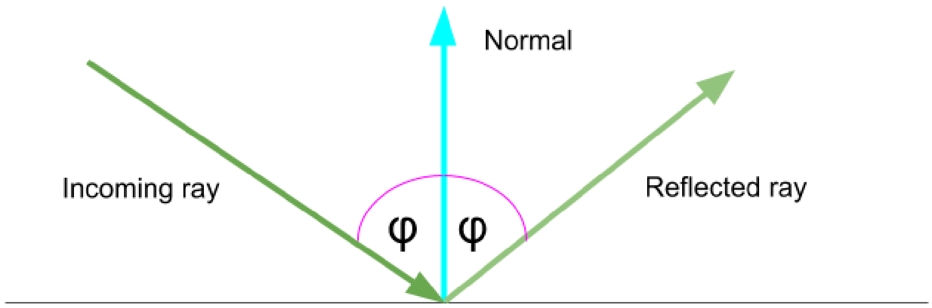

Figure 1, visualisation of law of reflection

In two dimensions, the problem of finding the curve that will uniformly distribute the first order reflections to the audience, is formed by the givens of a point that is the sound source (x_s,y_s) and of a curve that represents the audience - points ($x_r(t)$,$y_r(t)$) -. The task is to find the curve (ceiling for vertical 2D-sections, walls for horizontal 2D-sections) composed by points ($x_c(t)$,$y_c(t)$):

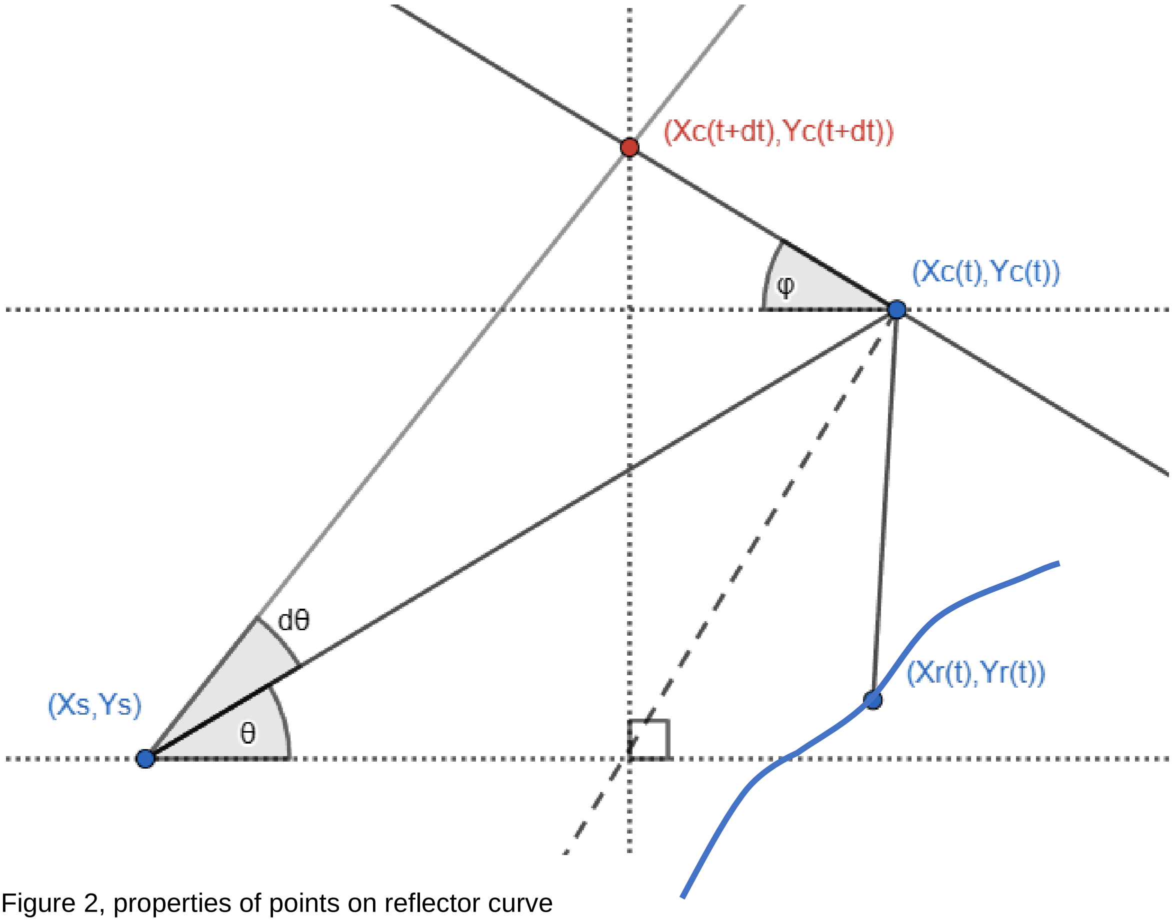

Figure 2, properties of points on reflector curve

3 ANALYTIC SOLUTION

Setting an infinitesimal change in angle θ, the next point at the reflector curve is consider still lying in the perpendicular of the bisector of the incident and the reflected ray.

$\varphi = \theta - \frac{\lambda}{2}$, where:

$$\tan\lambda = \frac{H}{L-d},$$

$$d = \sqrt{(x_c - x_s)^2 + (y_c - y_s)^2},$$

$$L = (x_r - x_s)\cos\theta + (y_r - y_s)\sin\theta$$

$$H = -(x_r - x_s)\sin\theta + (y_r - y_s)\cos\theta$$

Hence, setting $a(t) = \tan\varphi = \tan\left(\theta(t) - \frac{\operatorname{atan}\frac{H}{L-d}}{2}\right)$:

$$\frac{\Delta Y_c}{\Delta X_c} = a(t)$$

$$x_c(t+dt)\tan\theta(t+dt) - x_c(t)\,\tan\theta(t) = x_c(t+dt)\tan a(t) - x_c(t)\,a(t)$$

$$x_c(t+dt) = x_c(t)\frac{\tan\theta(t) - a(t)}{\tan\theta(t+dt) - a(t)}$$

$$x_c(t+dt) - x_c(t) = x_c(t)\frac{\tan\theta(t) - a(t) - \tan\theta(t+dt) + \alpha(t)}{\tan\theta(t+dt) - a(t)}$$

$$\lim_{\mathrm{dt}\to 0}\left(\frac{x_c(t+dt) - x_c(t)}{dt}\right) = -\lim_{\mathrm{dt}\to 0}\left(\frac{x_c(t)}{\tan\theta(t+dt) - a(t)}\frac{\tan\theta(t+dt) - \tan\theta(t)}{dt}\right)$$

$$\frac{\partial x_c(t)}{\partial t} = \frac{x_c(t)}{\tan\theta(t) - a(t)}(\tan\theta(t))'$$

$$\frac{\partial x_c(t)}{\partial t} = \frac{x_c(t)}{\tan\theta(t) - a(t)}(1 + \tan^2\theta(t))\frac{\partial\theta}{\partial t}$$

When there is a known point on the reflector curve, the above differential equation can be solved numerically for x_c. The other coordinate, y_c, is then calculated by:

$$y_c(t) = (x_c(t) - x_s)\tan\theta(t) + y_s$$

4 METHODOLOGY AND ALGORITHM

The differential equation derived in the previous chapter can easily be numerically solved by implementing an algorithm, especially when the angle $\theta(t)$ step is linear, therefore the term $\partial\theta/\partial t$ is constant. A linear step corresponds to evenly distributed reflected areas to the target region. Solutions for vertical mid-axis cross-sections of a typical small auditorium and of a larger concert hall composed of two seating areas, are shown in figures below.

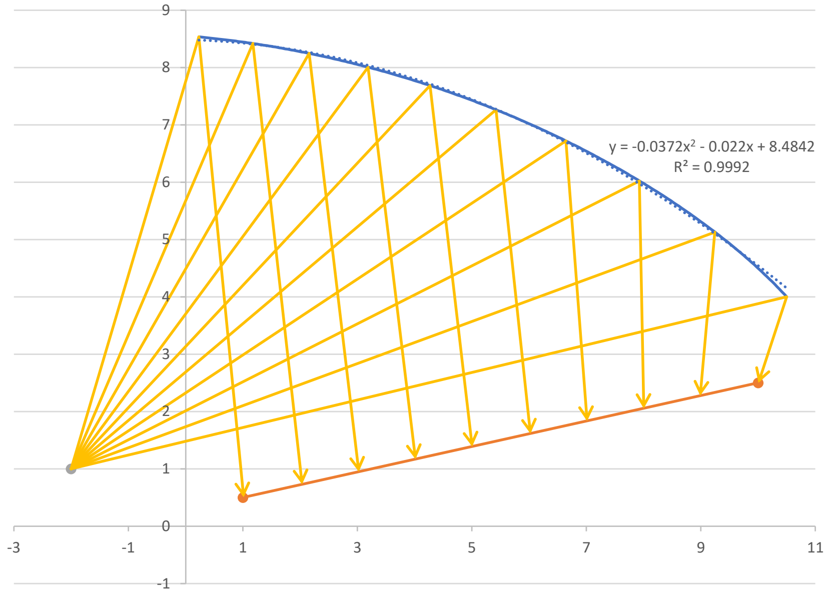

Figure 3, resulting reflector curve for vertical mid-axis cross-section of a typical small auditorium

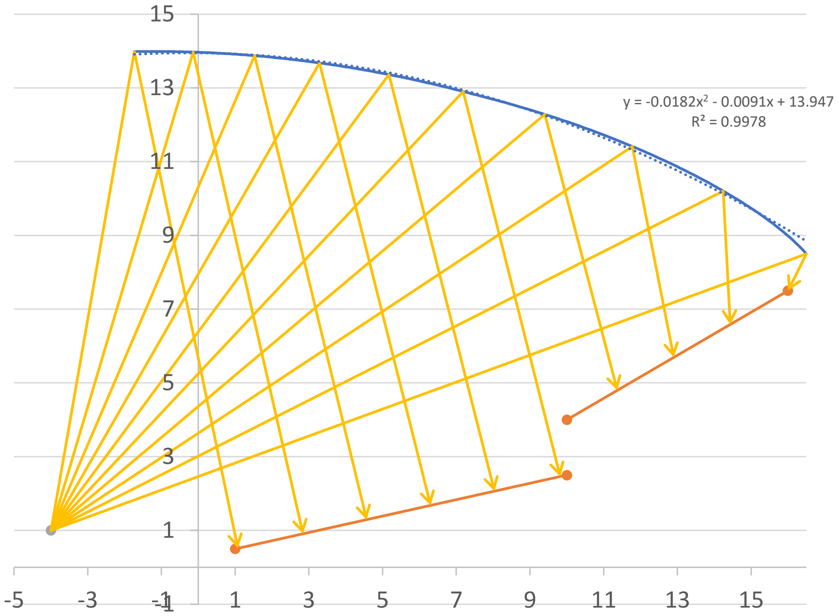

Figure 4, resulting reflector curve for vertical mid-axis cross-section of a concert hall composed of two seating areas

The figures displayed in the previous page show rays casted at evenly distributed angles which are reflected to equidistance intervals at the seating areas, hence proving the accuracy of the algorithm. Two important conclusions can easily be derived from the results: a) the resulting height for a single continuous curve to act as a reflector for all of the audience is very high and not feasible for construction, especially for small to medium auditoriums, and b) each **curve fits extremely well to the shape of a parabola**, which seems sensible considering the reflecting property of the focus of parabolas when casting rays parallel to their axis (although the focus point of the curve-fitting parabola in this case has nothing to do with the source point).

To undertake the height issue, non-continuous surface solutions can be sought, i.e. a break can be issued every time the reflector height exceeds a certain height. In order not to lose purposeful surface for reflections, the endpoints of sequential reflector surfaces should lie on a plane passing through the source point. The implementation of the above to the algorithm is shown in the figures below:

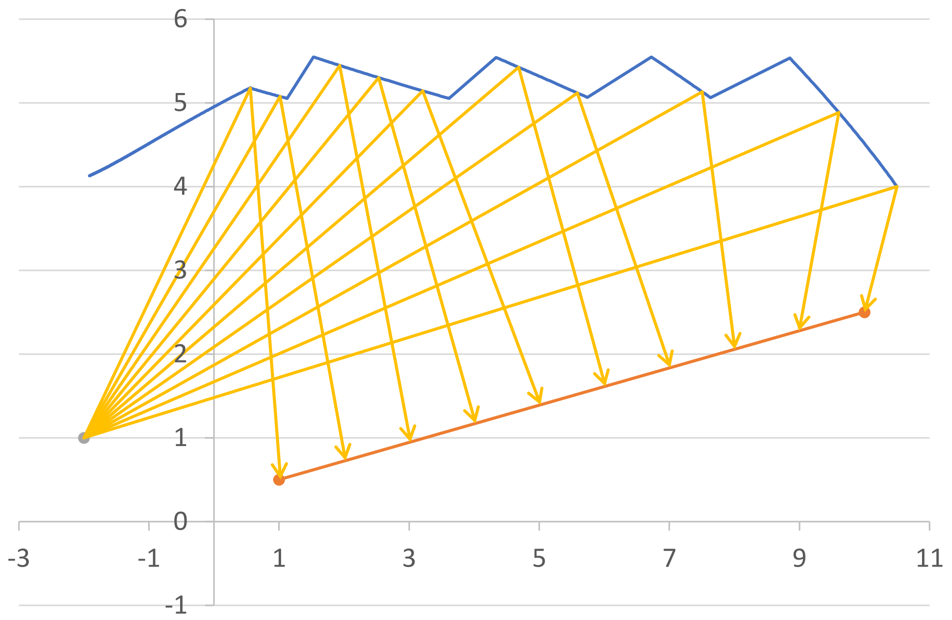

Figure 5, non-continuous surface solution for Figure 3 case

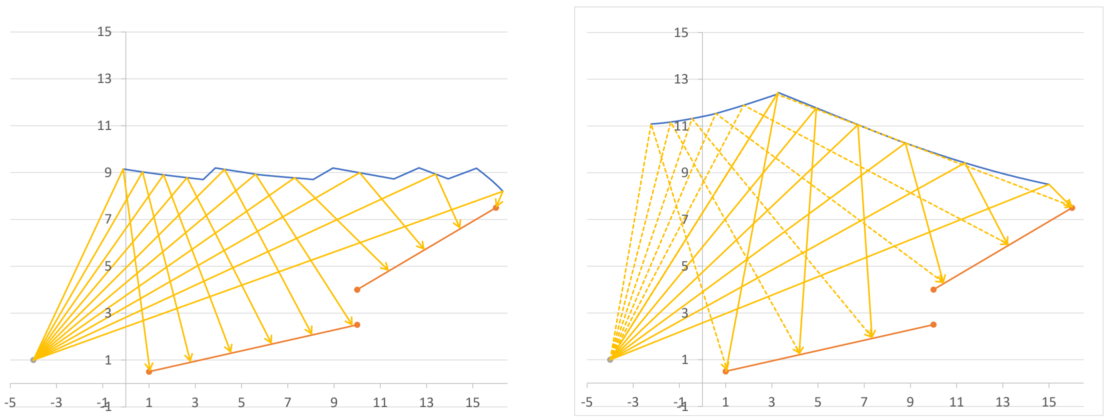

Figure 6, non-continuous surface solutions for Figure 4 case

The same technique can be used for the side walls:

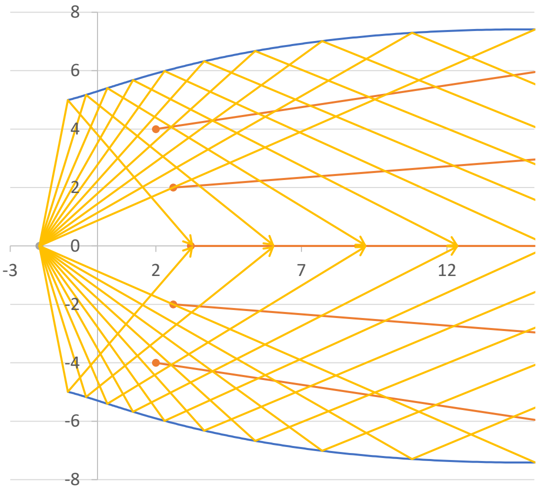

Figure 7, resulting reflector curve and rays for side walls in concert hall

The concept can be applied to three dimensions, where polar vertical cross-sections around the source point are solved and the results are combined to form a surface mesh for the ceiling reflector:

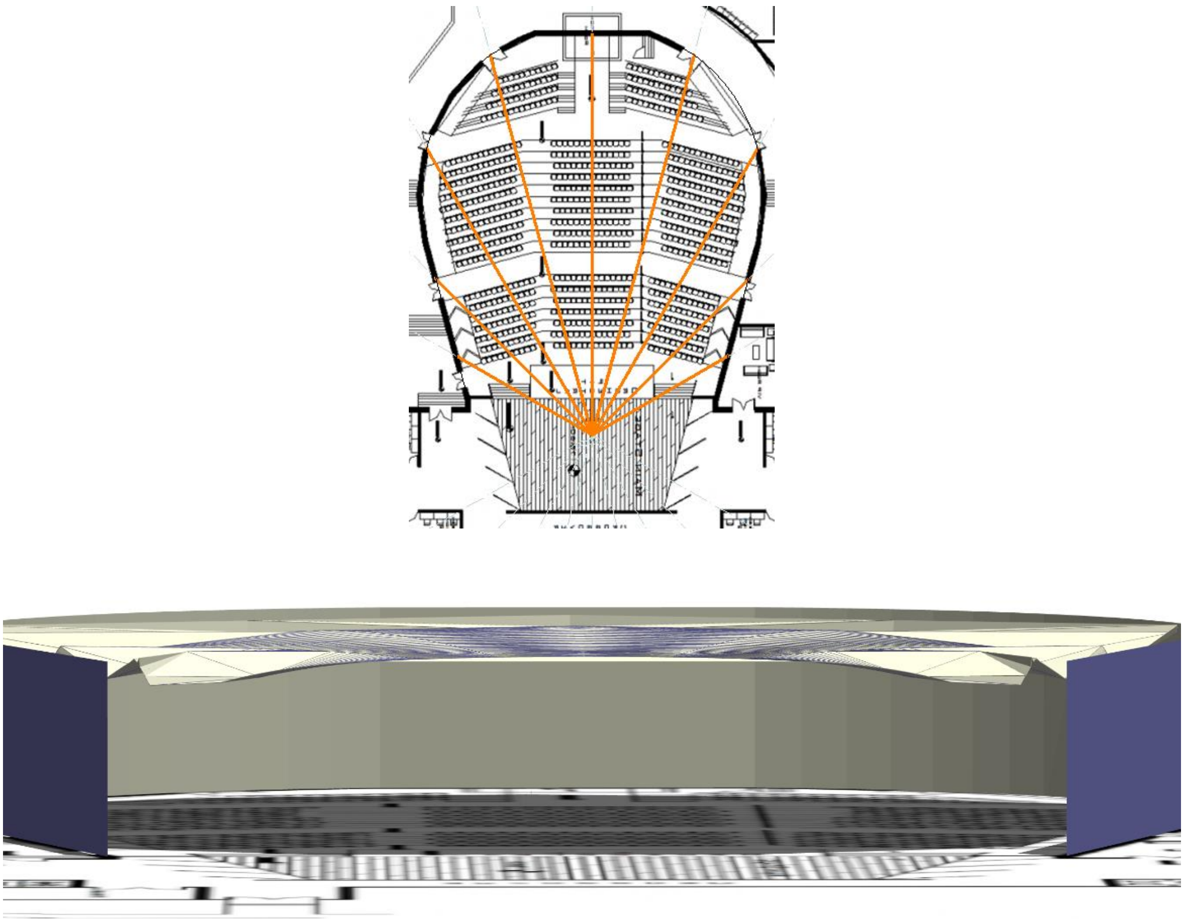

Figure 8, resulting surface mesh by combining algorithm’s results from multiple vertical sections

5 DISCUSSION

As shown, the proposed algorithm delivers vital tools for taking meaningfully into account the acoustics during early stages of auditorium design, by providing methods to generate or check the geometry regarding its ability to generate the beneficiary early reflections. The derived differential equation exposes that the results will be more accurate for standing sound sources, such as speeches during lectures and congresses or music from orchestras, than for moving sound sources, such as actors in a theatrical play.

It was also revealed that, when the design is based on long continuous sections of reflectors, parabolic shape surfaces provide more uniform distribution of reflected rays towards the audience than planar surfaces. Early outcomes from sensitivity analysis (comparisons of results in Figure 9 and Figure 10) show that reflections towards back seats are less sensitive to divergence of the sound source from the designed location and indicate that longer uniform sections of the reflectors provide more uniform distribution even when the noise source is moved closer or further away from the audience in comparison with the designed spot. In addition, when having sources closer to the audience from the designed spot, then the reflections are amplified at the back seats. Hence this aids to achieve higher signal to noise ratio, a valuable benefit for these areas in medium and large halls, where direct sound is notably attenuated.

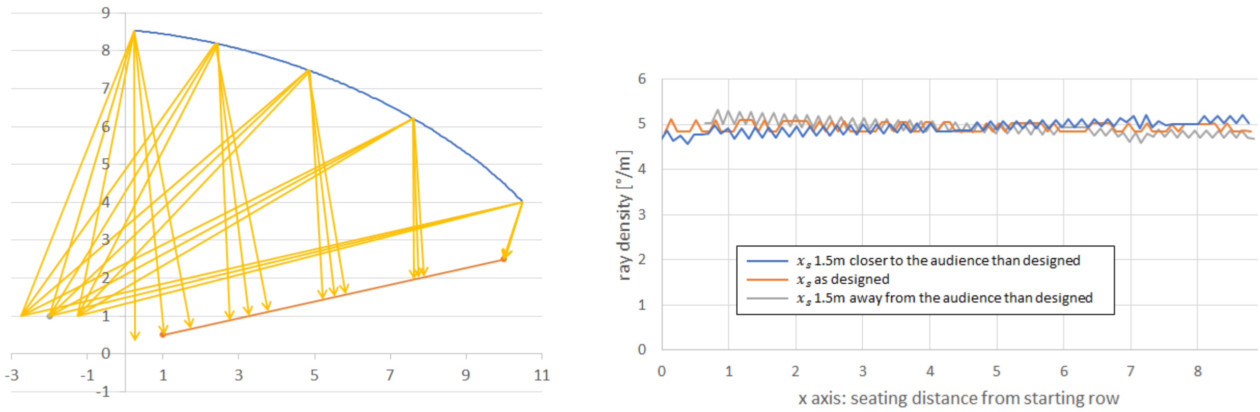

Figure 9, reflection ray density along audience (x=0 for front seats, x = 8.9 for rear seats) vs various locations of sound source on stage, case of uniform reflecting surface

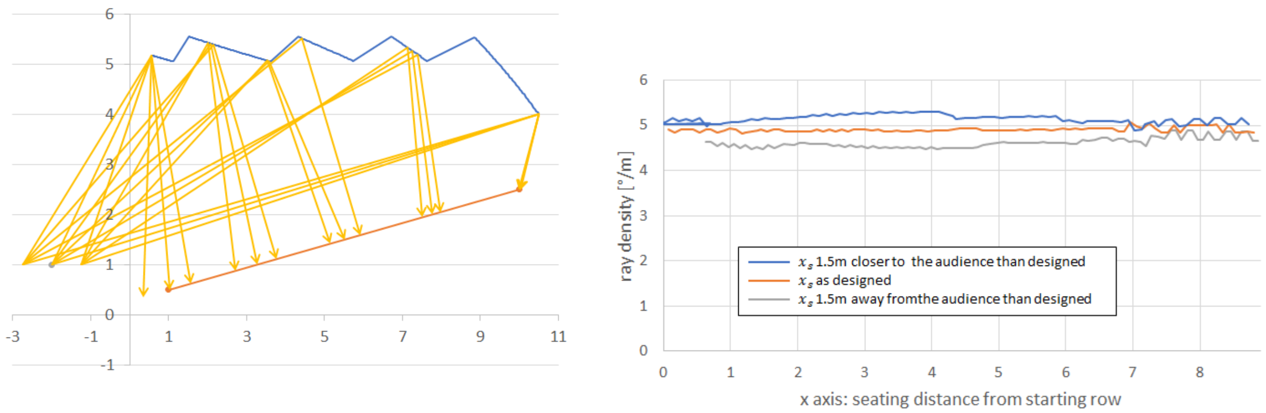

Figure 10, reflection ray density along audience (x=0 for front seats, x = 8.9 for rear seats) vs various locations of sound source on stage, case of non-continuous reflecting surfaces

The methodology can also be modified so as to favourite reflections to rear seats rather than uniform distribution, in order to incorporate the sound attenuation with distance, using a non-linear change in angle θ.

It is upon on-going study to further develop the algorithm to optimise the geometry not only for early reflections, but also for other critical acoustic properties such as early decay time, acoustic clarity, SNR and reverberation time.

Another important stage of the development for the algorithm is how to optimize for multiple point sources or area sources.

6 REFERENCES

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