OPTIMISING ACTIVE SONAR ARRAYS USING FINITE ELEMENT AND BOUNDARY ELEMENT METHODS

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1. INTRODUCTION

There is interest in developing high power, low frequency active sonar arrays for various applications. This can represent a considerable technical challenge as there may be many conflicting requirements e.g. optimal projector spacing for best acoustic performance and overall size and weight constraints to fit a platform. In such arrays, the projectors are generally close packed; so interaction between devices can be significant and must be accounted for when predicting array performance. This makes the design problem extremely difficult in general and achieving best performance within the imposed constraints can be very complicated. Therefore, an automated system for optimising performance is extremely desirable.

The coupled finite element/boundary element method provides accurate predictions of the radiated field [1] including the interaction effects. Here finite elements describe the structures and the boundary elements the acoustic fluid. Using the relative displacements of the transducers from an initial "guess" configuration, standard optimisation techniques can be applied in an attempt to achieve the required output field. By design the method can "encourage" power radiated in some directions and "discourage" power radiated in others. The method is demonstrated on two configurations of directional volumetric array.

2. THEORY

Suppose that there are n vibrating elastic structures in an infinite external fluid. If the structures are modelled by the finite element method and the acoustic medium by the boundary element method then the equations take the form,

$$\begin{bmatrix} [S]-\omega^2[M] & [T]^T \\ -\omega^2\rho[G][E]^T & [H] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix} \tag{1}$$

using the notation of Macey [2]. Both structural and acoustic parts of *(1)* can be naturally partitioned as nxn blocks. Let α_1, α_2, ..., α_m be the displacements of the structures from their initial configuration. Differentiation *(1)* with respect to α_i results in

$$\begin{bmatrix} [S]-\omega^2[M] & [T]^T \\ -\omega^2\rho[G][E]^T & [H] \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial\alpha_i}\{u\} \\ \frac{\partial}{\partial\alpha_i}\{p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \omega^2\rho\frac{\partial[G]}{\partial\alpha_i}[E]^T\{u\} - \frac{\partial[H]}{\partial\alpha_i}\{p\} \end{Bmatrix} \qquad (2)$$

Equation *(1)* is first solved to determine the structural displacements $\{u\}$ and surface pressures $\{p\}$, then using the same matrix factorisation, with a different right hand side *(2)* is solved to determine the sensitivities $\frac{\partial}{\partial\alpha_i}\{u\}$ and $\frac{\partial}{\partial\alpha_i}\{p\}$. The objective function is defined as

$$g(\alpha_1, \ldots, \alpha_m) = \sum_{j=1}^{k} w_j|p(x_j)|^2 \qquad (3)$$

where $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_k$ are positions in the fluid. If these points are taken in the far field then minimising g will tend to encourage acoustic energy to be radiated in directions with negative weighting factors and discourage radiation in directions with positive weighting factors. The pressure at the point $\underline{x}_j$ can be determined by a discretisation of the Helmholtz formula. Hence by differentiating with respect to the parameters α_i, the relative displacements of the structures, it is straightforward to determine the gradient of the objective function,

$$\underline{\nabla g} = \begin{bmatrix} \frac{\partial g}{\partial\alpha_1} \\ \ldots \\ \frac{\partial g}{\partial\alpha_m} \end{bmatrix} \qquad (4)$$

This can then be used in standard optimisation strategies (e.g. the method of steepest descent or the Fletcher Powell technique) to minimise g.

3. DIRECTIONAL VOLUMETRIC ARRAY EXAMPLES

Directional volumetric arrays are readily achievable with compact sources such as flextensional transducers [3] as they behave very much like ideal point sources. That is, they are very much smaller than a wavelength of the sound they transmit. In contrast, free flooding ring transducers are distributed sources. It is not possible to align two ring transducers $\lambda/4$ apart, as described by Helmer [3], as the diameter of a free flooding ring is typically of the order of $\lambda/2$ (where λ is a wavelength of sound in the water).

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The authors have studied a number of different configurations of ring transducer array. In this case a small scale conceptual study was conducted using DERA test rings made of solid PZT4 piezoelectric ceramic. The dimensions of the ceramic rings are as follows:-

Outer Radius = 57.15mm, Inner Radius = 50.8mm, Axial Height = 28.0mm

These rings are silvered on the inner and outer surfaces and poled in the radial direction. They are waterproofed using a 5mm layer of polybutadiene and resonate at 7.5kHz in water.

4. DIAMOND ARRAY EXAMPLE

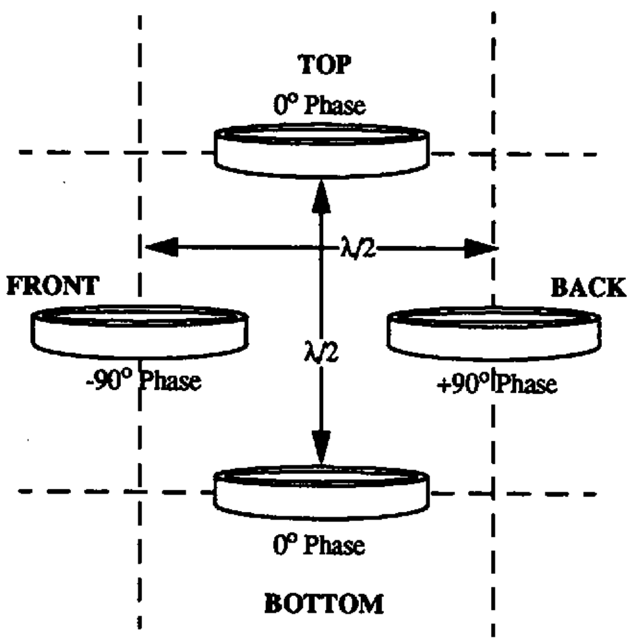

Figure 1 : Schematic Diamond Array Configuration

The example presented here, to demonstrate the technique, is a Diamond Array of 4 free flooding ring projectors shown in Fig. 1. This type of array has previously been studied extensively by the authors [4][5] as, being close packed and having strong acoustic interactions, it provides a technically challenging array configuration on which to illustrate advanced numerical modelling methods.

Ideally the array horizontal and vertical separation should be $\lambda/2$ at the centre frequency of operation (as shown schematically in Fig. 1) to achieve the best directionality across the operating band of the array. However, this is not achievable using the DERA test rings as the outer diameter (including the Polybutadiene coating) significantly exceeds $\lambda/2$ at the resonance of the rings.

5. THE MODEL OF THE ARRAY

Exploiting the axisymmetry inherent in a single ring is impossible in this case and a full 3D approach is necessary. A three dimensional model of a 180° section of a small scale free flooding ring transducer was constructed from twenty noded piezoelectric brick elements [6]. As a one-to-one coupling is required between the acoustic boundary element patches and the wet surfaces of the structural elements, the density of the structural mesh is determined by the requirements of the acoustic mesh. The mesh should have as few degrees of freedom as possible to keep the run time to a minimum, while being sufficiently fine for an accurate representation of the surface acoustic pressure field up to the highest frequency of interest [7].

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The largest dimension is in the circumferential direction which requires to be divided into 18 elements. As only frequencies up to 15kHz are to be considered the wall thickness and axial length of the ring are small relative to a wavelength, so one element is sufficient in each case. The mesh for this array is shown in Fig. 2.The rings were all driven by an alternating voltage applied across the electrodes with the phases shown.

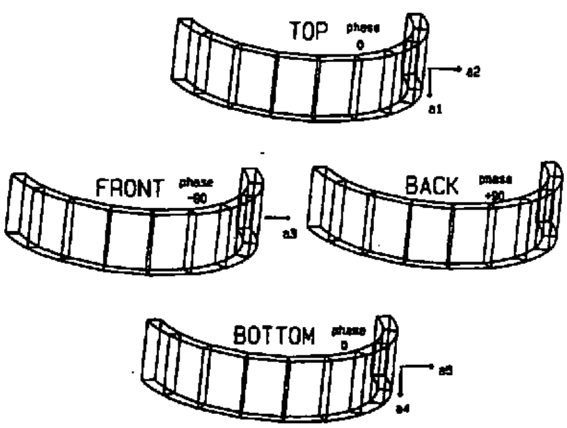

Figure 2 : Mesh of a Small Scale Four Ring Diamond Array Driven with 90° Phase Difference.

The initial starting configuration had the acoustic centres of opposite rings 141.42mm apart. This is λ/2 spacing at 5.3kHz, well below the required operating band of the array. At the optimal frequency of 5.3kHz, this produced a classic cardioid beam pattern in the horizontal plane and a useful vertical beam pattern with one major forward beam and two small rear facing lobes at about 40° to the horizontal [4]. However, at 7.5kHz, the resonance of the rings, and at 10kHz, the centre frequency of the operating band, the beam patterns are not so useful and the front-to-back ratio is much less, due to significant rear facing lobes.

6. OPTIMISATION

The optimisation parameters are shown in Fig. 2, listed as α_1 to α_5. The back ring was given no freedom to move. The top and bottom rings were given two degrees of freedom to move in the plane of symmetry. The front ring was only permitted to move towards and away from the back ring, in the plane of symmetry. Constraints were applied, to prevent unphysical configurations such as rings overlapping. The constraints used were as follows:-

α_1 a maximum of 50mm (to prevent the top ring contacting the back and front rings);

α_3 a maximum of 20mm (to prevent the front ring contacting the back ring);

α_4 a minimum of -50mm (to prevent the bottom ring contacting the back and front rings).

The objective function to be minimised was $-|p_1|^2 + |p_2|^2$ where p_1 is the pressure at 100m from the array in the front direction and p_2 is the pressure at 100m in the back direction. The desired effect is to increase the output in the front direction while decreasing the output in the back direction. The method of steepest decent was used to optimise the array for two separate cases 7.5kHz and 10kHz.

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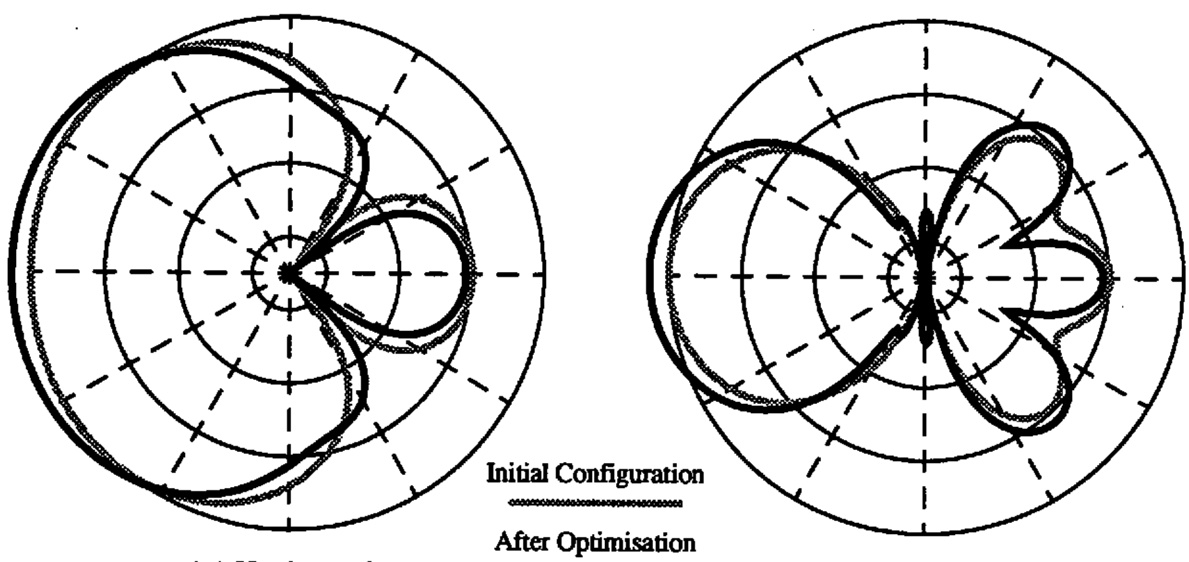
For the 7.5kHz case the parameter values were

$$\alpha_1 = -6.436\text{mm},\ \alpha_2 = -3.155\text{mm},\ \alpha_3 = 20\text{mm},\ \alpha_4 = 6.533\text{mm},\ \alpha_5 = 3.1\text{mm}$$

after 14 iterations. The comparison of vertical directivities before and after optimisation is shown in Fig. 3.

The dynamic range of the directivity polar plot is 35dB and each of the concentric rings represents 10dB. It can be clearly seen that the optimisation has had the desired effect and improved the front-to-back ratio by around 4dB by increasing the output on axis in the front direction while also reducing output in the back direction. This has been at the expense of slightly increasing the rear facing lobes off axis.



(a) Horizontal *(b) Vertical*

Figure 3 : Beampatterns of a 4 Ring Diamond Array at 7.5kHz.

The 10.0kHz case is the more challenging problem as the wavelength is much shorter which would imply that much closer packing of the projectors is required than is physically possible. Here the parameter values were

$$\alpha_1 = -17.78\text{mm},\ \alpha_2 = -10.22\text{mm},\ \alpha_3 = 20\text{mm},\ \alpha_4 = -17.79\text{mm},\ \alpha_5 = 10.24\text{mm}.$$

after 4 iterations. The comparison of vertical directivities before and after optimisation is shown in Fig. 4.

An even more significant improvement in the front-to-back ratio, of the order of 8dB, was achieved in this case, again by increasing the output on axis in the front direction while reducing output in the back direction. As with the 7.5kHz case, this was at the expense of slightly increasing the rear facing lobes off axis, although, coincidentally, the endfire beams were advantageously reduced. However, more significant may be the fact that the vertical beam

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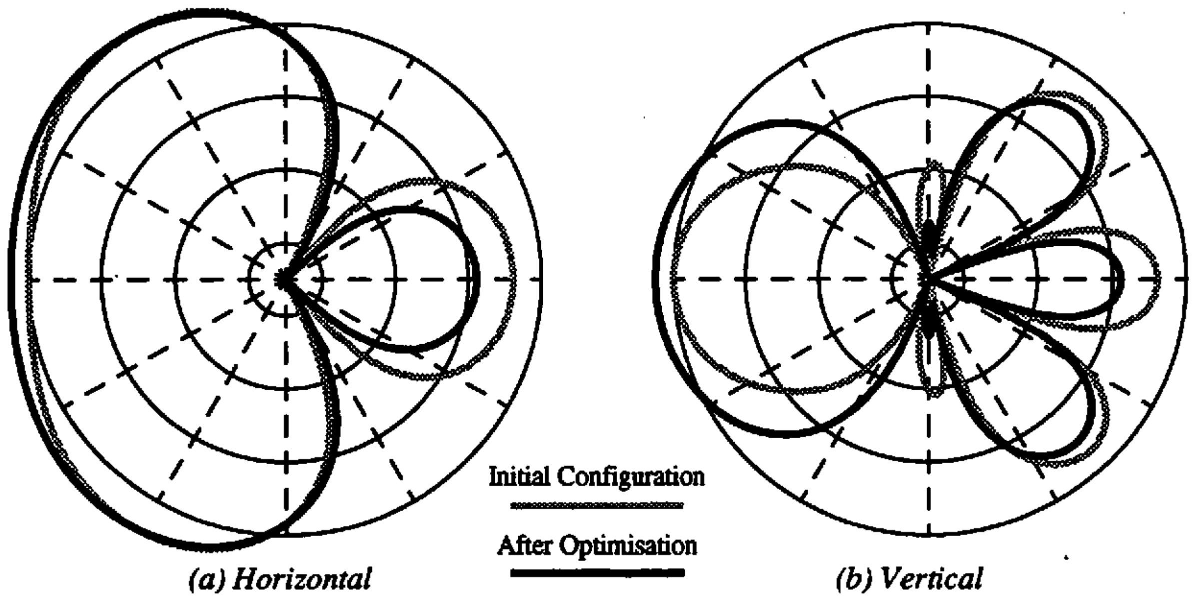

Figure 4 : Beampatterns of a 4 Ring Diamond Array at 10.0kHz.

width of the main beam has been significantly increased. For some applications this may be deemed undesirable.

For simplicity, the objective function used here only considered the front and back directions on axis. It would be possible to incorporate other directions in the objective function or even base the objective function on a ratio rather than a difference of pressures.

7. A FURTHER EXAMPLE

The Diamond Array example, discussed above, did not permit much scope for the optimisation code to demonstrate a dramatic change in the array. A further example was constructed in an attempt to show a more significant improvement in performance. A simple two ring vertical line array, with $\lambda/2$ spacing at 7.5kHz produces a beam pattern which is omnidirectional in the horizontal plane. Starting with this configuration and applying a 90^o phase shift between the transducers, the optimiser was again set to minimise the objective function $-|p_1|^2 + |p_2|^2$, to achieve the best front to back ratio.

The optimisation procedure then modified the array as shown in Figure 5. Again constraints were applied, to prevent unphysical configurations such as rings overlapping. Here the parameter values were

$$\alpha_1 = -46.5\text{mm},\ \alpha_2 = -41.7\text{mm},$$

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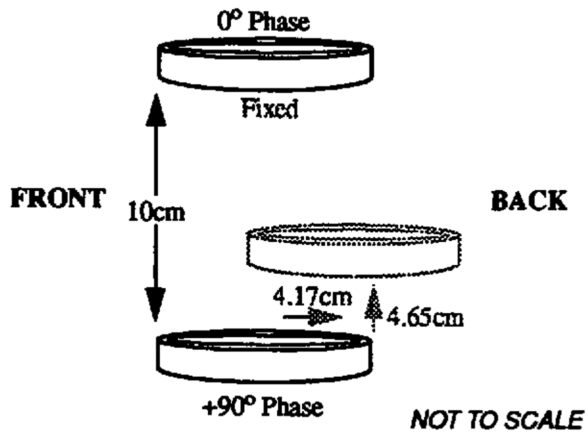
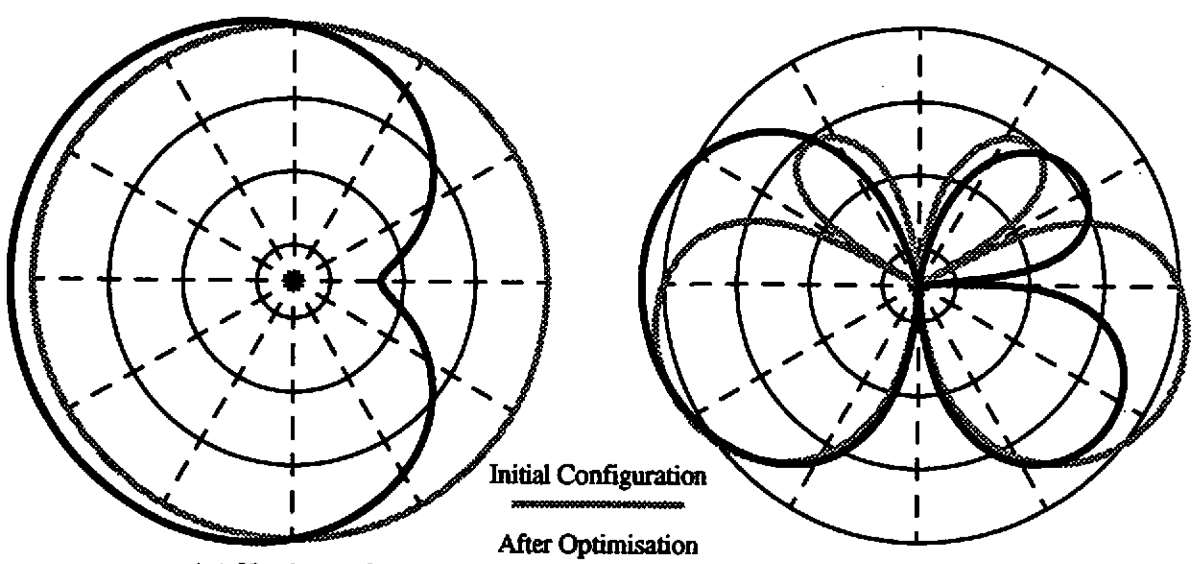


Figure 5 : Result of Optimisation of a Two Ring Array for Best Front-to-Back Ratio at 7.5kHz.

The resulting horizontal and vertical beampatterns are shown in Figures 6 (a) and (b). The optimisation successfully modified the array from one which was omnidirectional in the horizontal plane to one which produces a classic cardioid beam pattern in the horizontal plane. Examining the horizontal and vertical beampatterns it can be seen that a useful front to back ratio is achieved (in excess of 25dB) at the resonance of the rings.



(a) Horizontal *(b) Vertical*

Figure 6 : Beampatterns of a Two Ring Array at 7.5kHz.

8. CONCLUSIONS

The results presented here indicate that the optimisation code is a useful tool for designing sonar arrays with particular directional requirements. Although only a very simple scheme was used here, it would be possible to develop more sophisticated objective functions to control side lobes and beam widths. Weighting functions can be applied to the different parameters to enhance behaviour preferentially in certain directions. As well as optimising for spatial distribution of transducers, the code could be modified to optimise phase differences or amplitudes of the drive voltages to enhance array performance further.

9. REFERENCES

[1] GALLAHER, A. B., BONIN, Y. B., FAVRE, M. "Comparison of British, French and Canadian Predictive Methods for a Ring Transducer" - Proc. UDT'95, July 1995.

[2] MACEY, P. C. "Finite Element/Boundary Element Modelling Techniques Applied to Ring Transducers with Viscoeleastic Coating " - Proc. 3rd UK Transducer Materials and Transducers Workshop 1995, Ferroelectrics Vol. 187, No 1-4 (1996).

[3] HELMER, E. "Directional Sound Sources for Active Towed Array Sonars" - Sonar Transducers '95 Conference Proc. I.o.A. Vol. 17 Part 3, 1995.

[4] GALLAHER, A. B. "Performance Prediction of a Volumetric Array of Free Flooded Ring Projectors" - Numerical/Analytical Methods for Fluid-Structure Interaction Problems, Proc. IoA, Vol. 18, Pt. 10, December 1996.

[5] MACEY, P. C. and GALLAHER, "Sonar Array Optimisation using Finite and Boundary Elements - Proc.UDT'98, July 1998.

[6] PAFEC "PAFEC-FE Level 8 Data Preparation Manual" - PAFEC Ltd., 1994.

[7] HARDIE, D. J. W. and GALLAHER, A. B. "Review of Numerical Methods for Predicting Sonar Array Performance" - IEE Proc. Radar, Sonar and Navigation, Vol. 143, No. 3 (Special issue on Recent Advances in Sonar), June 1996.