MULTIPATH REDUCTION FOR SYNTHETIC APERTURE SONAR INTERFEROMETRY WITH 2XN ARRAY

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1 ABSTRACT

Synthetic aperture sonar interferometry technique that provides high resolution image of the seafloor can be challenging for shallow water environments due to multipath signal issues; (1) high correlation between the direct signal and the reflected signals both from the ocean surface and the seabed and (2) the limited scale of the vertical array restrict the performance of adaptive beamformer used for suppression of multipath effects. In this work, we present a space-alternating generalized expectation maximization algorithm for mitigating the multipath effects with two vertical hydrophone array. The space-alternating generalized expectation maximization (SAGE) algorithm is used to estimate a set of parameters of all the arrival signals such as time delay, Doppler shift, and azimuth and elevation angles. By applying this algorithm, the height of the seafloor can be provided through the estimated angle arrival of the direct signal from the seafloor. The SAGE algorithm shows better performance of the bathymetry estimation on the synthetic and measurement data of the SAS interferometry system. The adaptive beamformer is used for comparison.

2 INTRODUCTION

Multipath interference causes ghosts seen as a faded duplicate seafloor image when interferometry is used. These ghosts behave like the targets, and so the receiver has difficulty in isolating the correct targets. In addition, multipath causes limitation on SAS performance because of data driven micro-navigation techniques used to estimate the sonar trajectory[1].

Recent developments of the SAS interferometry in shallow water operation have heightened the need for high resolution image of the seafloor and target. For that purpose, an adaptive beamformer approach for multipath reduction using a small vertically-displaced array is proposed[2,3]. Ann E. A. Blomberg et al. presented a method for improved bathymetry estimation using the minimum variation (MV) adaptive beamformer which increase the signal-to-noise ratio by filtering the signals from unwanted directions. However, there are limitations of the performance in multipath reduction. First, near angle signals are difficult to separate because of the limited vertically displaced array. Second, the direct signal and multipath are strongly correlated, which implies a severe degradation of adaptive beamformer algorithm[4]. Third, the direct signal and multipath signals are overlapped, in which case it is difficult to distinguish the path of arrival signals.

To find one solution to reduce these limitations, we focus on a space-alternating generalized expectation maximization (SAGE) algorithm[5]. The SAGE algorithm is a high resolution method in mobile radio communication systems to estimate a set of parameters (amplitudes, time/delays, Doppler shifts, and incident azimuth and elevation angles) for direct plus multipath[6]. And this algorithm is a low complexity generalization of expectation-maximization (EM), which is an iterative method for finding the maximum likelihood estimation of parameters of interest[7].

The purpose of this paper is to present a SAGE algorithm for the high resolution target and seafloor image. This paper has been divided into four parts. The first part of this paper, in chapter 3, some definitions, notations, signal model, and problem formulation are established. Chapter 4 describes the method that is used to conduct the work. The results of the parameter estimate for bathymetry on the synthetic data of the SAS interferometry system are discussed in chapter 5. Finally, in chapter 6, an overall summary of the study as well as suggestions for further research are presented.

3 SIGNAL MODEL

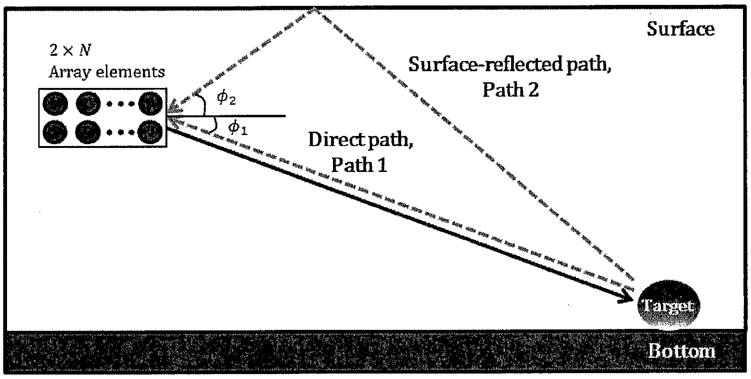

Figure 1. Sonar geometry in shallow water environment[3].

For operating SAS interferometry system in shallow water environment, the dominating path contributions are the direct path and first order surface-reflected path. Figure 1 shows the sonar geometry in shallow water environment. We consider only direct and first order surface-reflected path in this paper.

The sonar consists of two horizontal arrays vertically with N hydrophones. The received signal of complex amplitude γ_l, delay τ_l and incident azimuth ϕ_l can be written as

$$y(t,u) = \sum_{l=1}^{L} s_l(t,u) + n(t,u) \tag{1}$$

where

$$s_l(t,u) = a_l(\phi_l(t,u))p\left[t - \tau_l(t,u)\right] \tag{2}$$

and l is the multipath component.
Here, u is the spatial domain of the sonar position and $a_l(\phi_l(t,u))$ is the steering vector of an array consisting of $2 \times N$ hydrophones displaced by $\frac{2}{\lambda}$ in horizontal and 20λ in vertical. A pulsed chirp or linear frequency modulated (LFM) signal is defined as

$$p(t) = \gamma(t)exp(j\beta t + j\alpha t^2), \tag{3}$$

where $\gamma(t) = 1$ for $0 \le t \le T_p$ and is zero otherwise; T_p is the pulse duration. The instantaneous frequency is the derivative of its phase function with respect to time:

$$\omega_i = \frac{d}{dt}(\beta t + \alpha t^2) = \beta + 2\alpha t, \tag{4}$$

where α is the chirp rate and β is the modified chirp carrier frequency.
The received signal can be rewritten as follow:

$$y(t,u)=\sum_{l=1}^{L}a_l(\phi_l(t,u))\gamma_l(t,u)exp[j\beta(t-\tau_l)+j\alpha(t-\tau_l)^2]+n(t,u)\,. \tag{5}$$

The received signal $y(t)$ at the specific position can be expressed in vector notation as a superposition of the L multipath components corrupted by additive noise as

$$\begin{aligned}\mathbf{Y}(t) &\triangleq \left[\mathbf{Y}_1(t),\ldots,\mathbf{Y}_N(t),\mathbf{Y}_{N+1}(t),\ldots,\mathbf{Y}_{2N}(t)\right]^T \\ &= \sum_{l=1}^{L}\mathbf{s}_l(t;\theta_l)+\mathbf{N}(t)\end{aligned} \tag{6}$$

In (6), the vector $\mathbf{N}(t) \triangleq [\mathbf{N}_1(t),\ldots,\mathbf{N}_{2N}(t)]^T$ is the spatially and temporally uncorrelated noise and the $s_l(t;\theta_l)$ means the contribution of the lth multipath signal to the $2\times N$ received signals. The parameters of a single multipath signal with index l can be expressed by $\theta_l=[\phi_l,\gamma_l,\tau_l]$ which compose of the azimuth angle, amplitude, and time delay.

4 METHOD

4.1 Expectation-Maximization Algorithm

The EM algorithm is an iterative method for finding maximum likelihood estimation of parameters in statistical models. The EM algorithm consisting of an expectation step and a maximization step is ideally suited for problem to estimate the parameters of a probability distribution function[8]. It produces maximum likelihood estimation of parameters of observed signal in white Gaussian noise. The problem is to estimate the parameters $\theta_l=[\phi_l,\gamma_l,\tau_l]$ of lth multipath signal. As referred before, we consider only the direct and first order surface-reflected path: $l=1,\ 2$.

We assume the background noise N is the $2\times N$ dimensional vector of white Gaussian noise. The covariance and the variance is considered to be known. Using the conditional probability density function, we can obtain the likelihood function for our signal model via[9]

$$p(\mathbf{Y};\theta)=\frac{1}{(\pi\sigma_n^2)^{2\cdot N}}\exp\left(-\frac{\left\|\mathbf{Y}-\sum_{l=1}^{n}\mathbf{s}_l(t;\theta_l)\right\|_F^2}{\sigma_n^2}\right), \tag{7}$$

whereis the Frobenius norm of a matrix. The maximum likelihood estimation of $\theta,=[\theta_1\ \ldots\ \theta_L]$ is given by $\hat{\theta}=\arg\max_{\theta}\{p(Y;\theta)\}$.

Estimation of this problem has limitations since the objective function has no analytical solution for the global maximum, $p(Y;\theta)$ generally is not a concave function of θ and the complexity of the EM algorithm for $\theta_l=[\phi_l,\gamma_l,\tau_l]$ is $2\times L$ dimensional nonlinear optimization procedure.

4.2 Parameter Estimation using SAGE Algorithm

To circumvent above tradeoff between convergence rate and complexity, we use the SAGE algorithm which estimates the parameters of each wave θ_l sequentially instead of estimating the parameters of all wave θ in parallel in one iteration EM algorithm step. This SAGE algorithm is based on the unobservable data $X(t)$ and the observable data $Y(t)$ which is a function of the complete data[10]. (See Figure 2)

The basic concept of the SAGE algorithm is the hidden data space[5,9]. The parameters of θ_l is split into several smaller subsets θ_s and the maximum likelihood estimation for parameters is calculated in each subset while keeping the parameters in other subsets fixed. We choose hidden data space as one noisy wave $X_l = S_l + N_l$, where N_l is white Gaussian noise with variance $\beta_l \sigma_n^2$.

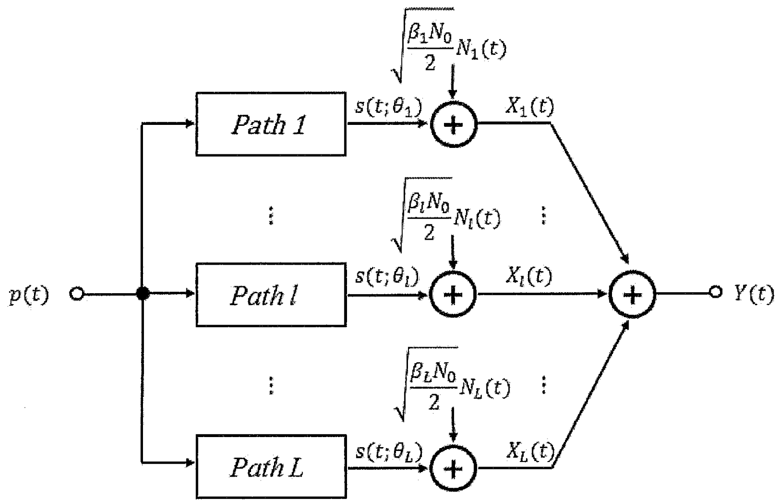

Figure 2. Relation between the complete and incomplete data[10].

This choice of the hidden data space leads to a fast convergence rate and low complexity due to sequential updating and one-dimensional optimization procedures. This stochastic mapping of the hidden data space to the observable signal is $Y = X_l + \sum_{l'=1, l' \neq l}^{L} S_{l'} + N_{l'}$. We choose the sequence of parameter vector $\theta_S(\mu)$ (μ is the parameter update index) as $\theta_S(1) = [\tau_1]$, $\theta_S(2) = [\phi_1]$, $\theta_S(3) = [\gamma_1]$, . $\theta_S(4) = [\tau_2]$, $\theta_S(5) = [\phi_2]$, $\theta_S(6) = [\gamma_2], \cdots$ .For fast convergence we set $\beta_l = 1$ [10].
Since complete data cannot be measurable, it has to be estimated by observable data. Thus, the expectation step can be written as

$$\begin{aligned} \hat{X}_l(t;\hat{\theta}_{l'}) &\triangleq E_{\hat{\theta}_{l'}} \{X_l(t)|Y(t)\} \\ &= Y - \sum_{l'=1, l' \neq l}^{L} S_{l'}\left(\hat{\theta}_{l'}\right) \end{aligned} \tag{8}$$

And, the maximization step is obtained via

$$\begin{aligned} \hat{\tau}_l &= \arg\max_{\tau_l} \left\{ \frac{\left| \mathbf{a}\left(\hat{\phi}_l\right)^{\mathrm{H}} \hat{\mathbf{X}}_l\left(\mathbf{p}(\tau_l)\right)^* \right|^2}{2N\beta_l\sigma_n^2} \right\}, \\ \hat{\phi}_l &= \arg\max_{\phi_l} \left\{ \frac{\left| \mathbf{a}\left(\phi_l\right)^{\mathrm{H}} \hat{\mathbf{X}}_l\left(\mathbf{p}(\hat{\tau}_l)\right)^* \right|^2}{2N\beta_l\sigma_n^2} \right\}, \\ \hat{\gamma}_l &= \frac{\mathbf{a}\left(\hat{\phi}_l\right)^{\mathrm{H}} \hat{\mathbf{X}}_l\left(\mathbf{p}(\hat{\tau}_l)\right)^*}{2N}. \end{aligned} \tag{9}$$

Here, $\mathrm{p}(\tau_l)$ is a transmitted signal delayed by τ_l. The expectation step and the maximization step are performed iteratively until the algorithm converges. For a region which is close enough to a local maximum, the SAGE algorithm has to be initialized.

Note that the SAGE algorithm is designed only to estimate the wave parameters, but not the number of multipath. As referred before, the number of waves L is predetermined to 2.

5 RESULTS ON SYNTHETIC DATA

To validate the proposed algorithm, some results on synthetic data are shown. Under the circumstance in Figure 1, a SAS interferometry system with $1.9\,cm \times 6.1\,cm$ spatial resolution is simulated with a $2 \times N$ hydrophones displaced by $\frac{2}{\lambda}$ in horizontal and 20λ in vertical. The synthetic data is shown for the parameters quoted in Table 1.

Table 1. The specifications for the SAS interferometry simulation system

parameter	value	unit	Definition
f_c	80	kHz	Center frequency
f_0	20	kHz	Baseband frequency
T_p	2	ms	Pulse duration
c	1500	m/s	Sound speed
du	0.03	m	Spacing between pings
Nu	766		Number of pings
N	10		Number of horizontal hydrophones
L	10	m	Synthetic aperture
Δ_x	$2/\lambda_{max}$	m	Spacing between horizontal hydrophones
Δ_y	$20\lambda_{max}$	m	Spacing between vertical hydrophones
H	10	m	Shallow water depth
h	6	m	SAS height
R	50	m	Cross range

Table 2. The specifications for the SAS interferometry simulation system

A stationary point target is located on the seafloor. We assume that the amplitude of direct path and first order surface reflected path signals for point target are equal. Then, we estimate only time delay τ_l and incident azimuth ϕ_l in this simulation. The signal to noise ratio which denotes the direct signal to background noise ratio is $5\mathrm{dB}$.

In this simulation, Overlapping multipath components are considered. The overlapping means that two signals cannot be distinguished in time domain. The estimation results may be observed in Figure 3 & Figure 4, where $T_p = 2\ ms$.

In Figure 3 the time delay estimate of direct and surface reflected signals is compared with the true parameters as sonar moves along its track. (a) shows the time delay estimate of direct signal for scenario with $\tau_1 \approx 0.0667$ and (b) is the time delay estimate of first order surface reflected signal for scenario with $\tau_2 \approx 0.0681$. In the bottom of Figure 3, the relative time delay between direct and surface reflected signals are estimated for scenario with about 0.0014 which is smaller than time duration of transmitted signal. From the data in Figure 3(a) & 3(b), no significant error of estimates in time delay was found. However the observed difference of time delay between paths is too small

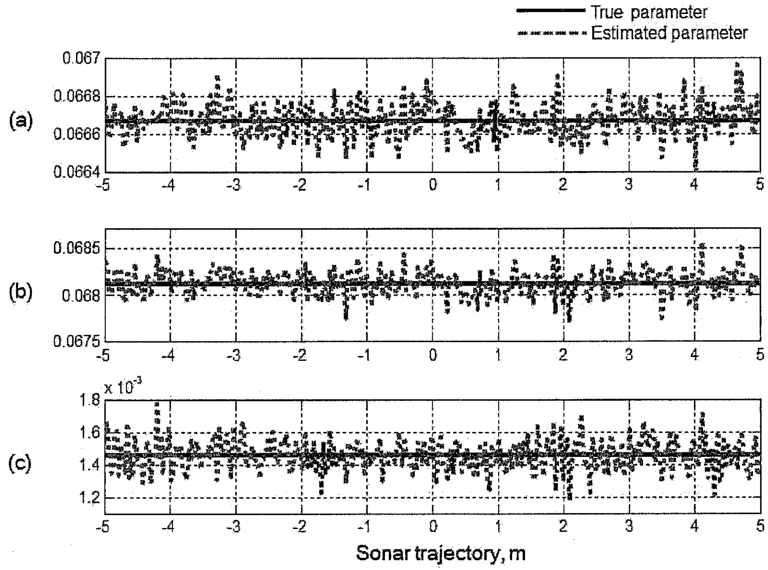

Figure 3. Time delay of the direct and surface reflected signals versus sonar trajectory: (a) Time delay of the direct signal; (b) Time delay of the surface reflected signal; (c) relative time delay. The unit is sec.

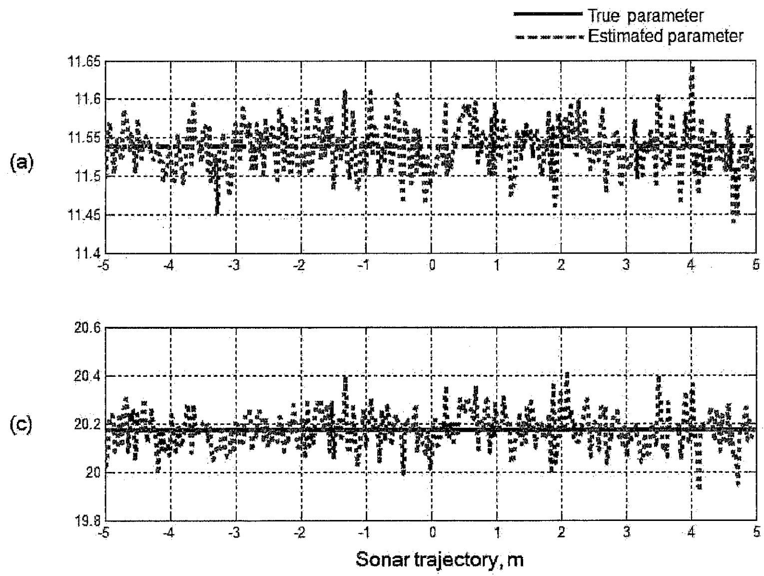

Figure 4. Azimuth of the direct and surface reflected signals versus sonar trajectory: (a) Azimuth of the direct signal; (b) Azimuth of the surface reflected signal. The unit is degree.

to decompose each path.

To find the arrival direction of desired signal for reconstructing image, we have estimated the incident azimuth angle of direct and surface reflected signals. The Figure 4 demonstrates the performance of estimate using SAGE algorithm for the incident azimuth angle for a scenario with $\phi_1 = 11.53°$ and $\phi_2 = 20.17°$. It can be observed that the estimate error is smaller than $1°$ which means two signals can be separate correctly to steer the beam in each of these directions. And SAGE algorithm is robust to find the most likely angle of desire signal when seafloor is not flat.

Figure 3 & 4 are quite revealing in several ways. Unlike the adaptive beamformer (MVDR, MUSIC) which provide high resolution under high SNR and uncorrelated sources, and degrades performance under snapshot-starved data[11,12,13], SAGE algorithm resolves the direct surface reflected signals with high resolution in detection of coherent arrivals.

6 CONCLUSION

This paper has dealt with the multipath problem for SAS interferometry in shallow water environments. In order to achieve reliable SAS interferometry performance, we have estimated the parameters of direct and multipath signals using SAGE algorithm.

Based upon the synthetic data, the SAGE algorithm has showed better performance than adaptive beamformer approach for reducing multipath signals using $2 \times N$ array elements. Further analysis will show the reconstructed bathymetry image for comparison of the performance with adaptive beamformer and the computation load for comparison of the number of iteration cycles using EM algorithm.

Further research is recommended for high dimensional L, the number of multipath components. In addition to expanding the SAGE algorithm, the measurement data of SAS interferometry will be used to verify this algorithm.

7 REFERENCES

1. A.Bellettini and M.A.Pinto., 'Theoretical Accuracy of Synthetic Aperture Sonar Micronavigation Using a Displaced Phase-Center Antenna', IEEE J. Ocean Eng. 27(4) 780-789. (2002).
2. Ann.E.A.Blomberg and M.Hayes., 'Multipath reduction for bathymetry using adaptive beamforming', Oceans 2010 IEEE. 1-6. (2010).
3. Q.Chen, W.Xu, X,Pan, J. Li, 'Wideband multipath rejection for shallow water synthetic aperture sonar imaging', IET Radar Sonar Navig. 3(6) 620-629. (2008).
4. S.Rougerie, G.Carrie, L.Ries, F. Vincent, R. Pascaud, E. Corbel and M. Monnerat , 'Multipath mitigation methods based on antenna array', Proc. ITM'11. 596-605. California (2011).
5. J.A.Fessler, A.O.Hero, 'Space-Alternating Generalized Expectation-Maximization Algorithm', IEEE Transection on Signal Processing. 42(10) 2664-2667. (1994).
6. B.H.Fleury, M.Tschudin, R.Heddergott and D.Dahlhaus, 'Channel Parameter Estimation in Mobile Radio Environments Using the SAGE Algortihm', IEEE J.Selected Areas in Communications. 17(3) 434-450. (1999).
7. M.Feder and E.Weinstein, 'Parameter Estimation of Superimposed Signals Using the EM Algorithm', IEEE Transactions on Acoustics Speech Signal Processing. 36(4) 477-489. (1988).
8. T.K.Moon, 'The Expectation-Maximization Algorithm', IEEE Signal Processing Magazine. 13(6) 47-60. (1996).
9. F.Antreich, J.A.Nossek and W.Utschick, 'Maximum likelihood delay estimation in a navigation receiver for aeronautical applications', Aerospace Science and Technology. 256-267. (2008).
10. B.H.Fleury, M.Tschudin, R.Heddergott, D.Dahlhaus, and K.I.Pedersen, 'Channel Parameter Estimation in Mobile Radio Environments Using the SAGE Algorithm', IEEE J. Selected Area in Commuications, 17(3) 434-450. (1999).
11. H. Van Trees, Optimum Array Processing (Detection, Estimation, and Modulation Theory, Part IV) (Wiley-Interscience, New York, 2002)
12. J. Capon, "High-resolution frequency-wavenumber spectrum analysis," Proc. IEEE 57, 1408–1418 (1969).

13. R. Schmidt, "Multiple emitter location and signal parameter estimation," IEEE Trans. Antennas Propag. 34, 276–280 (1986).