Edinburgh, Scotland
EURONOISE 2009
October 26-28

The effect of ground on performance of Sonic Crystal Noise barriers

Anton Krynkin[1]
Olga Umnova [2]
School of Computing, Science & Engineering, University of Salford
Salford, Greater Manchester, M5 4WT, UK

ABSTRACT

Array of infinitely long cylindrical scatterers with main axis parallel to the reflecting plane (ground) is considered that results in treating problem as two-dimensional. This array, referred to as sonic crystal, is placed in acoustic medium with physical parameters of air. The acoustical properties of the ground are described by the Delany-Bazley empirical relationships. The interactions of waves scattered by each cylinder and reflected from the ground can be analysed by the previously developed numerical methods based on the Green theorem with the Green's function adjusted to the presence of the locally reacting ground. The results are compared with the multiple scattering in free acoustic space. It is shown that the insertion loss depends on the distance between the array and the ground as well as on the impedance of the ground.

[1] Email address. a.krynkin@salford.ac.uk
[2] Email address. o.umnova@salford.ac.uk

1. INTRODUCTION

The scattering by many obstacles placed in an unbounded acoustic medium has been studied for many years. There is a substantial amount of literature devoted to analytical and numerical techniques of solving the multiple scattering problems. The semi-analytical approach has been developed for the simple shapes such as circles in 2D and is based on the superposition of the solution form of a single scatterer[1,2]. For more complex geometry of obstacles the numerical approach such as Boundary Element Method (BEM) based on the method of the integral equations is more common[3].

These methods can be adapted to the bounded acoustic medium with the ground as a boundary. For the semi-analytical approach the method of images can be applied to construct the reflected field[4-6]. It is also necessary to modify the scattering field, i.e. waves scattered by the obstacles, to satisfy boundary conditions on the ground[5,6].

As for the integral equations method it is possible to modify the Green's function[7] so that the domain with impedance ground transforms into the unbounded acoustic medium. The result is that the boundary integral equations are only considered over the surface of the scatterers. With this approach the computational time can be relatively low compared to the full problem with the ground as an additional surface. The method has been widely used for prediction of performance of the noise barriers mounted on the ground[8,9].

In this paper we use the mentioned numerical method to analyse the performance of the array of rigid cylindrical scatterers suspended over the impedance ground. We also compare the performance of some sonic crystals in the bounded domain with the crystals placed into the unbounded acoustic environment. The main interest is to show how the presence of the impedance ground affects the so-called band-gaps where one can observe the maxima of positive insertion loss[10].

2. FORMULATION

We consider two-dimensional problem where finite array of N scatterers is placed into half-space D which is an acoustic medium bounded by the impedance ground. The acoustic properties are given by sound of speed c and density ρ.

The array is described by the finite periodic lattice where periodic cell defines the characteristic length of the array, referred to as periodicity L. For example, if the cell is square then the periodicity takes value of the square side, see Figure 1. Another characteristic length that influences the waves scattered by the obstacles is the distance h between the bottom row of scatterers and Ox axis which is the ground.

The solution of the problem is given by acoustic pressure $p(\mathbf{r_0},\mathbf{r})$ at the point of observation $\mathbf{r}$ and satisfies the Helmholtz equation

$$\Delta p(\mathbf{r}_0,\mathbf{r}) + k^2 p(\mathbf{r}_0,\mathbf{r}) = 0 \qquad (1)$$

subject to the Sommerfeld radiation condition

$$\frac{\partial p}{\partial r} - ikp = o\left(r^{-1/2}\right), \qquad (2)$$

where $\mathbf{r_0}$ is the position of the point source, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $k = \frac{\omega}{c}$ with angular frequency ω.

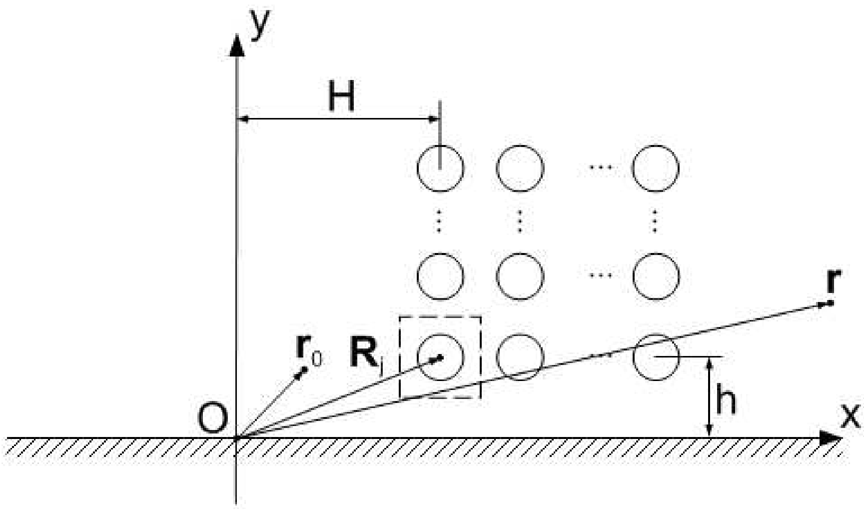


Figure 1: Example of the Sonic Crystal suspended over the ground

For the sake of simplicity all scatterers are assumed to be circles of the same radius. The position of each scatterer C_j is given by vector $\mathbf{R}_j$. On the surface ∂C_j of the j-th scatterer impedance boundary conditions are imposed as follows

$$\frac{\partial p}{\partial n} = ik\beta_s p, \tag{3}$$

where β_s is the normalized specific surface admittance which is independent of the acoustic pressure variations (i.e. medium is locally reacting). On the other hand β_s may depend on the frequency[7] and varies on the scatterer surface[9]. In case if $\beta_s = 0$ the surface of scatterer is regarded as rigid.

Applying relations (1)-(3) to the Green's theorem[7] the integral equation for p($\mathbf{r_0}$,$\mathbf{r}$) can be derived in the following form

$$\varepsilon(\mathbf{r})p(\mathbf{r}_0,\mathbf{r}) = G_\beta(\mathbf{r}_0,\mathbf{r}) + \sum_{j=1}^{N} \int_{\partial C_j} \left[\frac{\partial G_\beta(\mathbf{r}_s,\mathbf{r})}{\partial n(\mathbf{r}_s)} - ik\beta_s G_\beta(\mathbf{r}_s,\mathbf{r}) \right] p(\mathbf{r}_0,\mathbf{r}_s)ds, \tag{4}$$

where

$$\varepsilon(\mathbf{r}) = \begin{cases} 1, & r \notin \overline{C}_j, \\ 1/2 & r \in \partial C_j \end{cases} \tag{5}$$

and $G_\beta(\mathbf{r}_0,\mathbf{r})$ is the solution for the half-space in absence of the scatterers[7, eq. (2.1.2)] with the admittance β of the homogeneous impedance plane. Note, that in relation (5) the corner points of an obstacle are not defined due to the circular shape of the scatterers.

3. RESULTS

In this section all the results are computed in terms of the insertion loss that is defined as

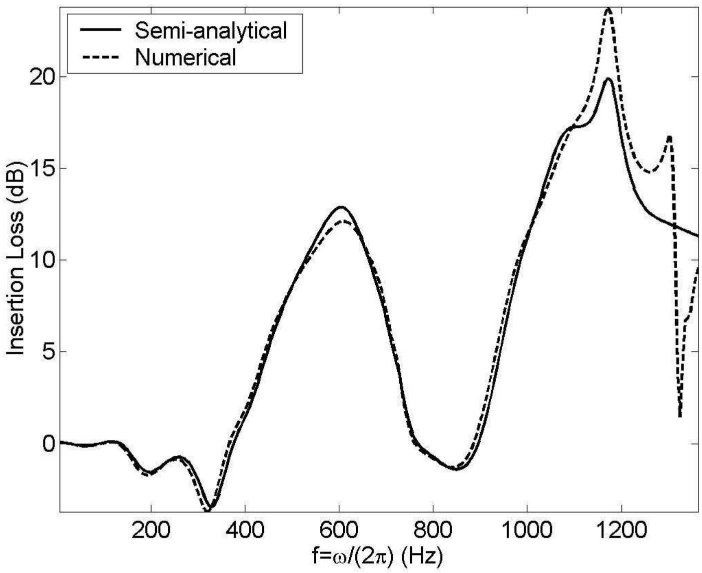
$$ILS = 20\log_{10}\left(\left|\frac{G_\beta}{p}\right|\right). \tag{6}$$

The relation (6) is supposed to be a function of angular frequency ω so that all other parameters such as position of the source and point of observation are fixed to the given values.

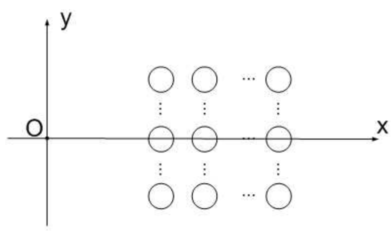
We first validate the solution of equation (4) with the well-known semi-analytical results developed in the previous works[1,2]. In doing so the Green's function in (4) has to be simplified to the case of unbounded acoustic medium that is

$$G_\beta(\mathbf{r}_0,\mathbf{r}) = -\frac{i}{4}H_0^{(1)}\left(k\left|\mathbf{r}-\mathbf{r}_0\right|\right) \tag{7}$$

In Figure 2 the results computed in terms of the insertion loss show the interactions of the scattered waves so that one can clearly see two positive peaks related to first and second band-gaps. The accuracy of the numerical method is within 5% of the semi-analytical method below $f = 1000\ Hz$ and it is deteriorating around the second peak (i.e. $f \approx 1150 Hz$). The increasing difference in solutions can be reduced by taking more elements on the surface of the scatterers that increases computational time. We can also observe the so-called spurious resonance in the vicinity of $f \approx 1300 Hz$ indicating non-uniqueness of the solution of equation (4). The latter can be avoided by modifying the Green's function (7)[3,12].



(a)

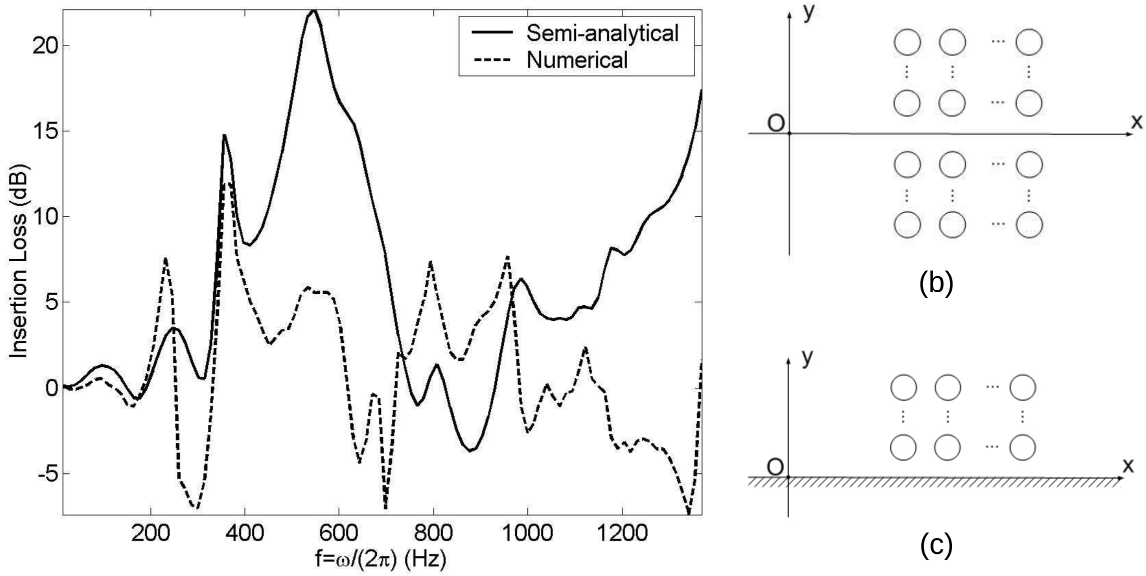


(b)

Figure 2: Square array of N=15 rigid scatterers of radius 0.1 m placed into the unbounded acoustic medium symmetrically with respect to Ox axis. The periodicity of the array is L=0.3 m, the source is at the origin $\mathbf{r_0}$=(0,0) m, point of observation is at $\mathbf{r_0}$=(0,10) m and horizontal distance to the array H=1.5 m. Solid line is the semi-analytical solution; dashed line is the numerical solution.

The multiple scattering in the unbounded acoustic medium (see Figure 3 (b) for the array geometry) can also be compared with the results for the half-space, geometry is shown in

Figure 3(c). In Figure 3(a), the results for free-space depicted as solid line clearly illustrate the presence of the band-gap in the vicinity of $f \approx 573\ Hz$. By introducing boundaries into the acoustic medium this effect may deteriorate. In particular, dashed line in Figure 3(a) illustrates performance of the sonic crystal in the presence of the ground. These results are computed for the impedance ground modelled by the Delany-Bazley empirical relationships[7, eq. (1.2.11)] where flow resistance σ=25e+4 Pa s/m^2 corresponds to the grass covered soil. It is seen that interaction of the scattered waves with the ground reduces positive insertion loss in frequency interval related to the first band-gap (i.e. in the vicinity of $f \approx 573\ Hz$).



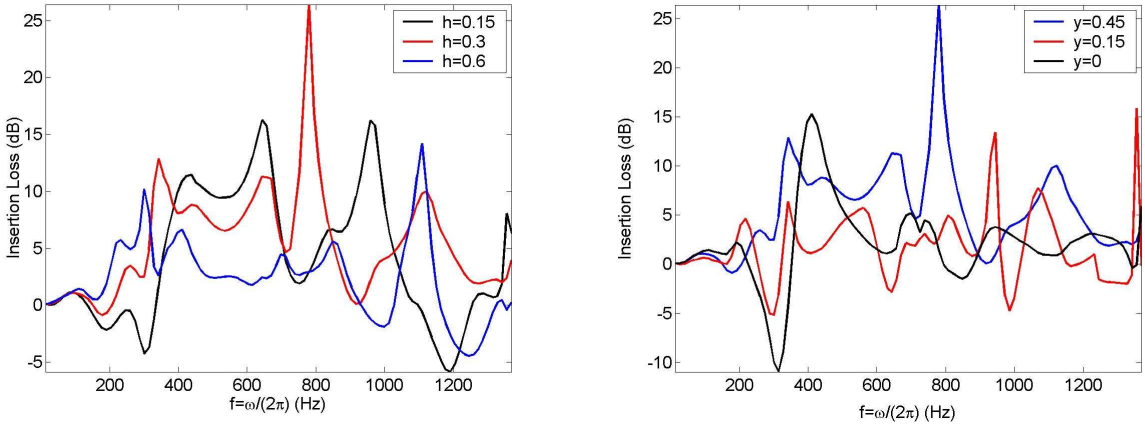
(a)

Figure 3: Comparison of the results corresponding to the free-space (solid line) and the half-space (dashed line). In both cases the array is square with periodicity L=0.3 m and radius of rigid scatterers 0.1 m, the source is at the origin $\mathbf{r_0}$=(0,0) m, point of observation is at $\mathbf{r_0}$=(10,0) m and horizontal distance to the array H=1.5 m. Solid line represents insertion loss for structure (b) which is the array of N=30 scatterers; Dashed line represents insertion loss for structure (c) which is the array of N=15 scatterers suspended over the impedance ground (i.e. σ=25e+4 Pa s/m^2) at a distance h=0.3 m.

It is now interesting to see how the observed influence of the ground differs with variation of the involved geometrical parameters. In Figure 4, distance to the ground h and coordinates of the point of observation **r** are varied. The results presented in Figure 4(a) illustrate the deterioration of first band-gap effect when h is increasing and **r** is fixed. As is expected the best performance of the sonic crystal in terms of the maximum insertion loss is observed for the minimum distance h=0.15 m (i.e. black line). On the contrary, the results in Figure 4(b) obtained for the fixed h and variable **r** show that the band-gap effect may be clearly present when h is less than distance of the point of observation to the ground.

Finally, we analyse the influence of ground acoustic properties on the observed insertion loss. Coloured lines in Figure 5 represent computed insertion loss for the different type of the impedance ground defined by the values of σ which correspond here to grass (i.e. σ=250000 Pa s/m^2), hay (i.e. σ=168000 Pa s/m^2) and mineral wool (i.e. σ=20000 Pa s/m^2).

It can be seen that lower values of the flow resistance may completely eliminate the band-gap effect whereas the ground with acoustic properties close to that of rigid surface maintains better performance of the sonic crystals in the frequency intervals related to the band-gaps.



(a) (b)

Figure 4: Insertion loss against frequency computed for the square array of N=15 rigid scatterers of radius 0.1 m with periodicity L=0.3 m. The source is on the ground at $\mathbf{r_0}$=(0,0) m and horizontal distance to the array H=1.5 m. (a) Variation of the distance from the lower row of the scatterers to the ground (i.e. Ox axis) with **r**=(10,0.45) m; (b) Variation of coordinates of the point of observation **r**=(10, y) m.

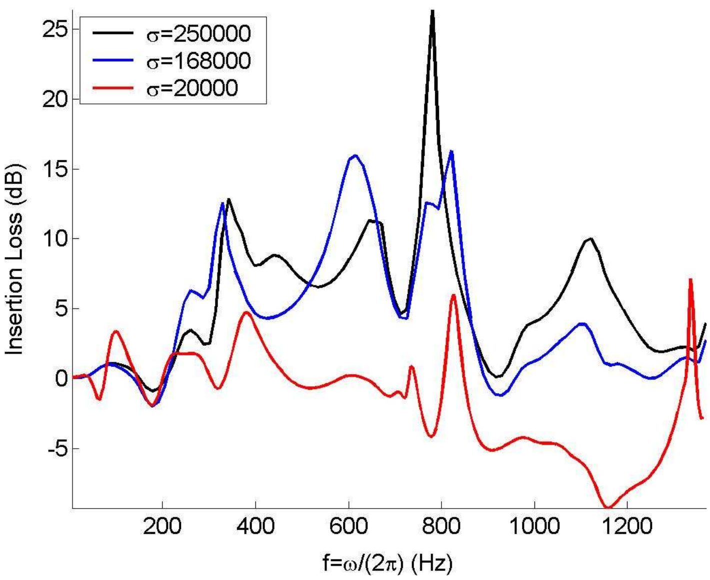


Figure 5: Insertion loss against frequency computed for the square array of N=15 rigid scatterers of radius 0.1 m with periodicity L=0.3 m. The source is on the ground at $\mathbf{r_0}$=(0,0) m, the point of observation is at **r**=(0.45,10) m, distance to the ground h=0.3 m and horizontal distance to the array H=1.5 m. The coloured lines correspond to the different values of flow resistance.

4. CONCLUSIONS

In this paper the method of integral equations is applied to two dimensional scattering problem where waves are scattered by an array of circular scatterers and reflected from the impedance ground. It is observed that the effect of band-gaps, associated with the eigenvalue problem of infinite doubly periodic array of scatterers, is strongly affected by the presence of the ground. In fact, the existence of this effect depends on the geometrical parameters of the problem (i.e. distance from array to the ground, coordinates of the source and point of observation) as well as on the properties of the impedance ground such as flow resistance.

From the results obtained in this paper we observe better performance of sonic crystals when flow resistance is high that makes ground almost rigid. The improvements can also be achieved if sonic crystal is closer to the ground than the point of observation.

The future work requires the analytical analysis of the proposed models that would allow us to find more effective configurations of the arrays. Supporting all typical behaviour of the periodic structures in the free-space, these configurations must maintain better performance in terms of the insertion loss over the chosen frequency interval.

ACKNOLEDGMENTS

Authors are grateful to the UK Engineering and Physical Sciences Research Council for supporting their work (grant EP/E063136/1).

REFERENCES

1. F. Zaviska, The deflection of electro magnetic waves on parallel, infinite long orbital cylinder, Annalen der Physik, **40**, pp. 1023-1056, (1913).

2. C.M. Linton and D.V. Evans, The interaction of waves with arrays of vertical circular cylinders, *Journal of Fluid Mechanics*, **215**, pp. 549-569, (1990).

3. P.A. Martin, Multiple Scattering: interaction of time-harmonic Waves with N Obstacles, Cambridge University Press, 2006, pp. 152-281.

4. P. Boulanger, K. Attenborough, Q. Qin and C.M. Linton, Reflection of sound from random distributions of semi-cylinders on a hard plane: modes and data, *Journal of Physics D: Applied Physics*, **38**, pp. 3480-3490, (2005).

5. K.M. Li, W.K. Lui and G.H. Frommer, The diffraction of sound by an impedance sphere in the vicinity of a ground surface, *The Journal of the Acoustical Society of America*, **115**, pp. 42-56, (2004).

6. R. Borghi, F. Gori. M. Santarsiero, F. Frezza and G. Schettini, Plane-wave scattering by a set of perfectly conducting circular cylinders in the presence of a plane surface, *Journal of the Optical Society of America A*, **13**, pp. 2441-2452, (1996).

7. S.N. Chandler-Wilde, Ground Effects in environmental sound propagation, PhD thesis, v.1, 1988.

8. D.H. Crombie and D.C. Hothersall, The performance of multiple noise barriers, *Journal of Sound and Vibration*, **176**, pp. 459-473, (1994).

9. D.H. Crombie, D.C. Hothersall and S.N. Chandler-Wilde, Multiple-Edge Noise Barriers, *Applied Acoustics*, **44**, pp. 353-367, (1995).

10. J.V. Sanchez-Perez, D. Caballero, R. Martinez-Sala, C. Rubio, J. Sanchez-Dehesa, F. Meseguer, J. Llinares and F. Galvez, Sound attenuation by a two –dimensional array of rigid cylinders, *Physical Review Letters*, **80**, pp. 5325-5328, (1998).

11. M.E. Delany and E.N. Bazley, Acoustical properties of fibrous absorbent materials, *Applied Acoustics*, **3**, pp. 105-116, (1970).

12. F. Ursell, On the exterior problems of acoustics, *Proceedings of the Cambridge Philosophical Society*, **74**, pp. 117-125, (1973).