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ON THE GENERALISED IMPEDANCE CONCEPT, ANALOGIES AND ACOUSTIC APPLICATIONS

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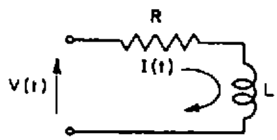
1. INTRODUCTION

The term impedance, universally identified by capital zetha (Z) was introduced in the electrical field in 1886 by the famous English Physicist Oliver Heaviside to describe the voltage to current ratio in a circuit comprising a resistor (R) and a inductor (L). (fig. 1),



$$Z = \frac{V}{I}$$

Fig. 1

It was a step forward to the Ohms law, soon extended to circuits including capacitors (C). The impedance concept has proved to be very useful to relate any voltage to current ratio. It has had a great development in electric network theory, as well as in other fields of physics including acoustics.

A particularity of acoustics is that the "systems" involved in most acoustic problems, do not have physically concentrated parameters as is the case of most electromechanical systems. In acoustics we have in general mechanical vibratory sources with concentrated parameters, immersed in continuous media with distributed parameters in which the vibratory energy is propagated by elastic waves that may reflect, diffracts, absorb or transmit energy in obstacles, or develop sound sensations in living species through the ear mechanism.

The electro-mechanical-acoustical analogies is a very useful tool to describe acoustical systems with concentrated parameters as can be the case of the Helmholtz resonators. The flow of acoustic waves through tubes and cavities are treated as mechanical systems with concentrated parameters provided that its physical dimensions be small as compared with the wave lengths.

This is not the case in many acoustical situations where in dealing with waves it is necessary to make appeal to the wave impedance concept and to phenomena better developed in the electromagnetic field (as is the case of the acoustic shadows by barriers, based on the edge diffraction theory of electromagnetic waves solved by Sommerfeld in 1896) and in particular in the transmission lines theory.

We will refer first to the concentrated parameters systems and then to electromagnetic waves and transmission lines, using the appropriate analogies for the applications to acoustic systems. In all cases we will consider linear and constant parameters, that besides of facilitating the equations formulation coincides with most of the common physical phenomena and processes.

2. GENERALISED IMPEDANCE

2.1.- Generalised impedance concept

Life and Nature in its most general meaning convey continuous process of propagation, transmission and transformation of energy. The transfer of energy implies sources and receivers and in most cases intermediate transmission systems.

Energy whatever its nature, can be defined and evaluated by the product of two factors, related respectively with its intensity and capacity characteristics.

So we have the following pair of factors for the different kinds of energy.

Energy	Electrical	Mechanical	Acoustical	Electromag.	Thermodin.
Factors	V,I	F.U	p.Q	ExH	T.S (entropy)
Power(watts)	Volt.amp.	N. m/s	$N/m^2.m^2.m/s$	Volt./m amp.m	kcal/s

As a generalisation of the impedance concept, the impedance of a system can be defined as an intrinsic "operator" that transforms the applied excitation function (ε) into a response function, (r) representing respectively the intensity and capacity factors of the energy involved, i.e.

$$Z = \varepsilon / r$$

The power Wa "absorbed" (no reflected) by the system, being the product of the excitation by the response, is governed by the impedance

$$\left.\begin{aligned} W_a &= \varepsilon . r \\ Z &= \frac{\varepsilon}{r} \end{aligned}\right\} W_a = \frac{\varepsilon^2}{Z} = Zr^2 \ (watts)$$

The impedance controls and "measure" the response of the system, and is called its input impedance.

2.2.- Impedance complex representation

The linearity of the systems allows the application of the additive principle to simplify all kind of excitations, and consequently of responses, to harmonic functions of time ψ_e and ψ_r of the form

$$\psi\ (\omega t) = \psi\ \cos\ \omega t = |\psi| \cos\ (\omega t + \varphi) = \mathbf{Real}\ |\psi|\ e^{j(\omega t + \varphi)}$$

and the impedance

$$Z(\omega) = \frac{\psi_e}{\psi_r} = \frac{|\psi_e|}{|\psi_r|} e^{j(\varphi_e - \varphi_r)} = |Z| e^{j\theta} = R + j\,X$$

a vector to be represented in the impedance complex plane (fig. 2a).

The Admittance $Y = \frac{1}{Z}$, as the geometrical inversion of a vector, corresponds to the vector 1/Z in the figure 2b

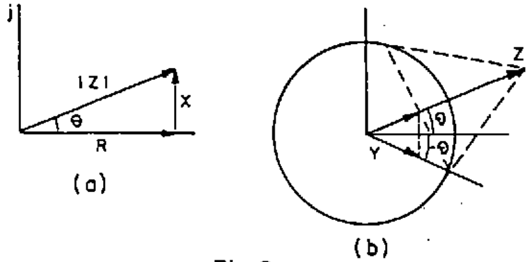


Fig. 2

3..IMPEDANCE ANALOGIES

3.1.- Lumped parameters systems: Electrical, Mechanical and Acoustical impedances

The energetic processes involve phenomena of dissipative, inertial and potential nature . The power transmission through physical components is regulated by well known physical laws that relate its particular reactions in said processes, Coulomb, Hook, Newton, Ampere, Faraday and Maxwell being the main names involved.

The dissipative phenomena being directly related to the response, and the kinetic and potential one to its derivative and integral respectively, the following general equations can be established

$$\text{Dissipative laws,} \quad \psi_e = \mathbf{A}\psi_r$$

$$\text{Dynamic laws,} \quad \psi_e = \mathbf{B}\frac{d\psi_r}{dt}$$

$$\text{Potential laws,} \quad \psi_e = \frac{1}{\mathbf{C}}\int\frac{d\psi_r}{dt}$$

The constants ***A B*** and 1/***C*** corresponds with physical system components deduced directly by application of these laws to different fields.

The next table shows the respective analogous components in the electrical mechanical and acoustic fields.

Component / Energy	Dissipative **A**	Kinetic **B**	Potential **1/C**	Excit Ψ_e	Resp. Ψ_i
Electrical	Resistance, R	Inductance, L	Capacitance, C	V	I
Mechanical	Mech. Friction, R_m	Inertance, m	Mec.Compliance, C_m	F	U
Acoustical	Flow Resist., R_A	Ac. Inertance, M_A	Ac. Compliance, C_A	p	Q

When dealing with oscillatory functions and in particular with harmonics functions, the dynamic and potential laws, as containing time derivatives and integral of the functions, have imaginary components, $d\Psi/dt = j\,\omega\psi$ and $\int\Psi dt = \Psi/j\omega = -j\,\Psi/\omega$.

Heaviside, applying the Cauchy operational calculus introduced an elegant procedure to solve integro-differential equations mainly in the electric field by means of the operator $p = d/dt$ and $1/p = \int dt$.

For harmonic functions,

$$p = j\omega \quad \text{and} \quad 1/p = 1/j\omega = -j/\omega$$

If we go back to the original series electric circuit, with lumped R L and C components (Fig. 3), excited by an harmonic voltage $V = |V|\,e^{j\omega t}$

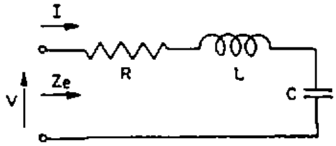


Fig.3

Fig.3

The Kirckoff's voltage law, $V = V_R + V_L + V_C$, gives $V = RI + L\,dI/dt + 1/C \int I dt$ and applying the Heaviside operator p

$$V = RI + pLI + I/pC = I\,(R + pL + 1/pC)$$

and the input impedance results

$$Z_i = V/I = R + pL + 1/pC = R + j\,(\omega L - 1/\omega C)$$

This is the well known electrical series impedance $Z = R + jX = |Z|\,e^{j\theta}$ where

$$|Z| = \sqrt{R^2 + X^2} \quad \text{and} \quad \text{arctg}\,\theta = \frac{X}{R}$$

The real and imaginary parts of the vector impedance, are related with the components of the system, what makes possible to evaluate the response of the circuit; $I = V/Z$ and the power delivered to the circuit $W = V^2/Z = ZI^2$

By substitution of the corresponding components in each field, the following input impedance are obtained,

Electrical series circuit: $Z_e = V/I = R + j(\omega L - 1/C\omega)$

Mechanical series circuit: $Z_m = F/U = R_m + j\,(\omega m - 1/C_m\omega)$

Acoustical series circuit: $Z_A = p/Q = (F/S)/SU = Z_m/S^2 = R_m/S^2 + j(\omega\, m/S^2 + 1/C_m S^2)$

$= R_A + j\,(\omega\, M_A - 1/\omega\, C_A)$

The acoustic components are deduced from the mechanical ones by dividing the Resistive and Inertial components by the square of the surface (S^2) to which the acoustic pressure is applied, and multiplying by S^2 the potential element or mechanical compliance.

The units are:

$$Z_e = \frac{V}{I} = \frac{Volts}{amp.} = \Omega_e \text{ , electrics ohms}$$

$$Z_m = \frac{F}{U} = \frac{Newton.s}{m} = \Omega_m \text{ , mechanics ohms}$$

$$Z_A = \frac{p}{Q} = \frac{Newton/m^2}{m^3/s} = \frac{N.s}{m^3.m^2} = \Omega_A \text{ , Acoustics ohms or Rayl s/m}^2$$

The Acoustic inertance, $M_A = \frac{m}{S^2} = \frac{N.s^2}{m^5}$

The Acoustic compliance $C_A = C_m.S^2 = \frac{m}{N}.m^4 = \frac{m^5}{N} = \frac{V}{B}$

(B = Bulk Modulus of elasticity, N/m^2)

(V, m^3)

3.2.- Distributed parameters systems: Acoustic waves and impedances

Condensed matter in any of its states and aggregation grades, contains the three basic components **A, B, C,** involved in the transmission of energy, the difference being that these components are spatially distributed, instead of concentrated.

Any physical perturbation in the continuum medium is propagated by waves. This is the case of acoustics waves, that propagate mechanical energy transmitted to or through the medium by matter vibration.

The acoustic hypothesis simplifies the medium to the isotropic non dissipative, linear and homogeneous case, which facilitates the formulation of the physical laws of mass and momentum conservation, that together with the Bulk elasticity modulus $B = -\frac{\partial P}{\partial V / V}$ results in the well known wave equation

$$\nabla^2 \psi = 1/c^2 \ddot{\psi}$$

where ψ is any variable defining the physical perturbation (displacement, velocity acceleration or pressure time variations) and c is the speed of wave propagation $c = \sqrt{B/\rho}$, ρ is the density of the medium.

The solution of this equations is a wave function of the general form

$$\psi\ (r.t) = \psi\, e^{-\Gamma r + pt}$$

with complex constants:

ψ (amplitude) = $|\psi| e^{\gamma\varphi}$

Γ(propagation constant) = $\alpha + j\beta$

p (oscillation constant) = $\xi + \omega t$

Γ and p, for the Acoustic hypothesis and harmonic excitation, reduces to $\Gamma = j\beta$, $p = \omega t$ and the solution is written

$$\psi(r\ t) = \psi\, e^{j(\omega t - \beta r + \varphi)},$$

which defines for any spatial position r_i an harmonic oscillation with time, and at any instant t_i, a spatial sinusoidal state of oscillation.

The variable ψ use to be the acoustic pressure p because of the facility to measure it through microphones or hydrophones in fluids. In solids use is made of vibrometers to measure displacement, velocity or acceleration.

The wave fronts defined by the surfaces of constant phase $\phi = \omega t - \beta r + \varphi$ = Const. depends on the geometry of the excitation, resulting in plane, spherical or cylindrical wave.

For $\frac{d\phi}{dt} = 0$ and $c = \frac{dr}{dt}$ results $c = \frac{\omega}{\beta}$

The specific acoustic impedance is defined by the ratio of the acoustic pressure (excitation) to the associate velocity of oscillation (response)

$$Z = \frac{p}{u}$$

For the basic plane wave geometry, $p(xt) = |P|\, e^{j(\omega t - \beta x + \varphi)}$ and $u(xt) = |U|\, e^{j(\omega t - \beta x + \varphi)}$

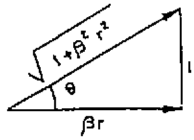
The linearized Euler equation, $\frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t}$

results in $Z = p/u = \rho c$, a real quantity

For other geometries the ratio p/u depends of r and β and is in general complex. In the case of spherical waves, the absolute magnitude of Z results

$$|Z| = \frac{|p|}{|u|} = \rho c \frac{\beta r}{\sqrt{1+\beta^2 r^2}} \quad \text{with } \operatorname{tg}\theta = \frac{1}{\beta r}$$

which corresponds to the vector diagram



Therefore $\cos\theta = \frac{\beta r}{\sqrt{1 + p^2 r^2}}$ and $\frac{|p|}{|u|} = \rho c \; cos\theta$

For large values of βr (high frequencies or long distances) $\cos\theta \rightarrow 1$ and the ratio $\frac{|p|}{|u|} = \rho c$ coincides with the ratio $Z = p/u$ in plane waves.

The constant ρc, a significant property of the medium is called the characteristic impedance Z_o of the medium. As

$$c = \sqrt{\frac{B}{\rho}} \qquad Z_o = \rho c = \sqrt{\rho B}$$

ρ, B are two distributed parameters of the medium related with the inertial and potential energy.

4. WAVES AND TRANSMISSION LINES ANALOGIES

4.1.- Electromagnetic field impedance

Once more the development in other field as in electromagnetism, can be useful in acoustics.

The electrical transmission lines is a particular case of electromagnetic waves "guided" through wires, at low frequencies.

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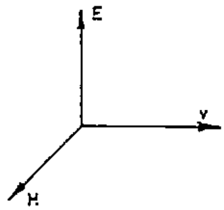
The electromagnetic waves vectors are the solutions of the Maxwell equations relating the spatial variation of the Electric intensity field vector E (Volt/m) and the magnetic, H(amp.m).

$$\left.\begin{aligned}\nabla^2 E - \Gamma^2 &= 0\\ \nabla^2 H - \Gamma^2 &= 0\end{aligned}\right\}$$

With the propagation constant $\Gamma = \sqrt{j\omega\,\mu(g + j\omega\,\varepsilon)} = \alpha + j\,\beta$ function of the field distributed parameters g ε μ. The vectors E and H oscillate harmonically with time as corresponds to harmonic excitation of the field.

The solution for E and H are propagating waves of the form, $E(xyz) = E\,e^{\cdot\Gamma r}$, $H(xyz) = He^{\cdot\Gamma r}$ the instantaneous values of E and H being $Ee^{j\omega t}$ and $H.e^{j\omega t}$.

The vectors E and H have different values and directions according to the type of wave associated to the excitation. In the wave front, the vectors E and H, and the speed of propagation v are orthogonal.



In general the vectors EHv are not trirectangular and 6 wave impedances are defined in three directions, $Z_{XY} = \frac{E_X}{H_Y}$, $Z_{XZ} = \frac{E_X}{H_Z}$, $Z_{YZ} = \frac{E_Y}{H_Z}$ and the opposites Z_{YX}, Z_{ZX} and Z_{ZY}.

For perfect dielectric media or in vacuum, g = 0, and the ratio $\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$, defines the intrinsic <u>characteristic impedance of the medium</u>, and $\Gamma = j\omega\sqrt{\mu\varepsilon}$, the propagation constant.

In general, $\eta = \sqrt{\frac{ju\mu}{g + ju\varepsilon}}$ and $\Gamma = \sqrt{j\omega\mu + (g + j\omega\,\varepsilon)}$.

For free space, $\mu_0 = 4\pi x\ 10^{-7}$ Henry/m, $\varepsilon_0 = \frac{1}{36\pi}\ x\ 10^{-9}$ Farad/m

$$\eta_0 = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi\ \Omega,\quad v_0 = \frac{1}{\sqrt{\mu.\varepsilon.}} \approx 3\ x\ 10^{8}\ \text{m/s}$$

4.2.- Analogies between electromagnetic and transmission line waves propagation

When writing the Maxwell equations for uniform plane waves or TME (Transverse electromagnetic) and choosing the equiphase plane yz, the equations have the form

$$\frac{dEy}{dx} = -\,j\omega\mu H_z = Z\,H_z$$

$$\frac{UH_z}{dx} = -\,(g + j\omega\varepsilon)\,E_y = YE_y$$

These equations are analogous to the equations for homogeneous electric lines

$$\frac{dV}{dx} = -(R + j\,\omega\,L)\,I = Z\,I$$

$$\frac{dI}{dx} = -(G + j\omega\,C)\,V = Y\,V$$

where Z and Y are the distributed series impedance and shunt admittance for unit length (fig. 8). The pairs E_y H_Z and E_Z H_y (and the acoustics p u) are equivalent to the pair V I in electric lines with distributed parameter for unit of length RLG C, the parameters equivalencies being

$$\begin{bmatrix} \mu & \equiv \rho \equiv & L \\ g & \equiv - \equiv & G \\ \varepsilon & \equiv B \equiv & C \end{bmatrix}$$

These equivalencies between plane waves and electric lines together with the previous lumped electroacoustic analogies, allows with some restrictions on wave length *vs* geometry, the transmission of acoustic waves in open or limited spaces systems to be treated as equivalent transmission lines in lumped parameters circuits.

5. ENERGY TRANSMISSION

5.1.- Equivalent source circuit

The transfer of energy implies a source (active system) and a receiver (passive circuit, or load) and in many cases a transmission system in between.

The receiver has been characterised, for any kind of energy, by a Dipole with its input impedance. $Z_i = \frac{\text{excitation}}{\text{response}} = |Z|\, e^{j\theta}$

To define the source, we go back to the electrical network theory and apply the Thevenin theorem, (1883) that states that any active linear network is equivalent to a Dipole consisting of a generator with the open circuit voltage V_s in series with its internal impedance Z_s as measured with all its sources in short circuit. (fig. 4a)

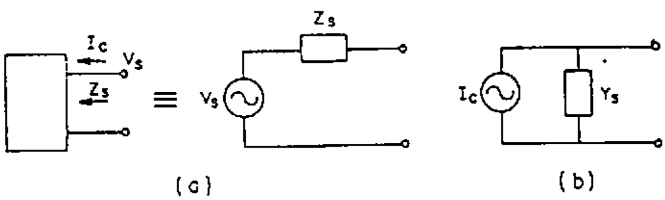


Fig. 4

Sometimes it is convenient to use the equivalent dual current source $I_c = \frac{V_s}{Z_s}$ in parallel with the admittance $Y_S = \frac{1}{Z_S}$ (fig. 4 b)

5.2.- <u>Energy transmission by direct coupling</u>

For direct coupling between source and load the equivalent electrical circuit of fig. 5 holds, where the voltage V_s drives a current I through the whole circuit.

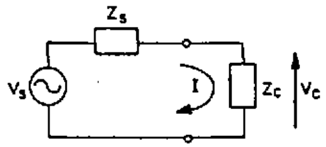


Fig. 5

The power delivered to the load $W_c = V_c.I$, can be solved in terms of both V_s (the excitation intensity factor) and the impedances Z_s and Z_c. It is straightforward from the circuit that

$$\left.\begin{array}{l} W_c = V_c.I \\ Z_c = \dfrac{V_c}{I} \end{array}\right\} \quad W_c = Z_c I^2 = V_s^2 \frac{Z_c}{(Z_s + Z_c)^2} = V_s^2 \frac{1}{(1+\tau)^2} \quad watts \qquad \tau = Z_2/Z_1$$

$$\left(I = \frac{V_s}{Z_s + Z_c} \right)$$

i.e. the power "received" is a function of the ratio Z_2/Z_1.

Differentiating W_c with respect to τ and equating to zero, the maximum value of W_c is for $\tau = 1$

i.e. for $Z_c = Z_s$ (impedance matching) and $(W_c)_{max} = \dfrac{V_s^2}{4Z_c}$ watts

The maximum coupling or impedance matching is of primary importance in the transfer of energy. In acoustics is used in the positive or negative sense according if it is desired to transmit the maximum energy, as is the case of the ear mechanism, or to reflect or absorb the maximum as in isolation systems. At the end, the energy not reflected has to be transmitted, transformed or dissipated, the sink elements been the resistances.

5.3.- Transmission of energy through an intermedium system: four terminals or tetrapoles.

In most cases between the source and the load it is intercalated a transmission system that can be generalised to be a tetrapole (fig. 6a).

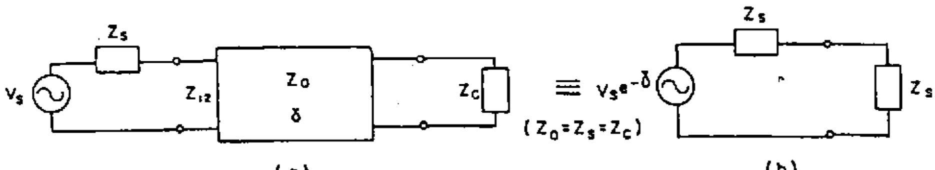


Fig. 6

In the tetrapole are defined different impedances. The more importants from the point of view of energy transmission are its characteristic impedance Z_0 and the input impedance Z_{12} as a function of the load Z_c.

Z_0 is an intrinsic characteristic, defined as the impedance seen in the primary poles when the secondary is loaded with this same impedance Z_0. Consequently Z_0 coincides with the impedance of an "infinitive" chain of equal tetrapoles as represented in the figure 7.

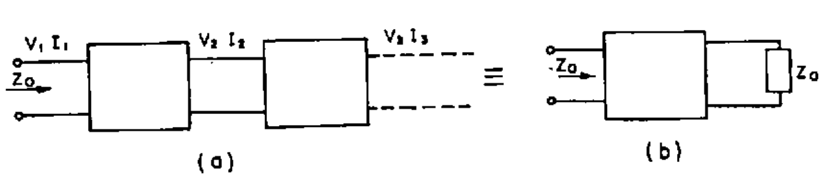


Fig. 7

It follows directly from the chain that

$$Z_0 = \frac{V_1}{I_1} = \frac{V_2}{I_2} = \ldots \quad and \quad \frac{V_1}{V_2} = \frac{V_2}{V_3} = \ldots \frac{V_n}{V_{n+1}} = e^{\gamma}$$

where γ is the transmission constant. in general complex, $\gamma = \alpha + j\beta$, with α the attenuation constant and β the phase constant. For Harmonic excitation $\beta = k = \frac{\omega}{c}$

The insertion of a tetrapole in the case of matched impedances, $Z_0 = Z_s = Z_c$, is then equivalent to a direct coupling with the voltage source attenuated to $V_s e^{-\gamma}$ (fig. 6b). The transmited energy results

$$W_c = \frac{V_s^2}{4Z_c} e^{-2\gamma}$$

In the general case of the tetrapole loaded with $Z_c \neq Z_0$ the input impedance can be evaluated by the well known equation in network theory

$$Z_{12} = Z_0 \frac{Z_c \operatorname{ch}\gamma + Z_0 \operatorname{sh}\gamma}{Z_c \operatorname{sh}\gamma + Z_0 \operatorname{ch}\gamma}$$

for $Z_c = 0$ $Z_{12} = Z_{1c}$ (short circuit input impedance) $= Z_0 \operatorname{th}\gamma$

for $Z_c = \infty$ $Z_{12} = Z_{10}$ (open circuit input impedance) $= Z_0 \operatorname{cth}\gamma$

and the following useful relations are deduced

$$Z_0 = \sqrt{Z_{1c} Z_{10}} \quad ; \quad e^{-\gamma} = \sqrt{\frac{Z_{10} - Z_0}{Z_{10} + Z_0}} = \sqrt{\frac{Z_0 - Z_{1c}}{Z_0 + Z_{1c}}}$$

i.e. Z_0 and γ can be obtained in terms of the short and open circuit impedances. Z_{1c} and Z_{10}

5.4.- Transmission line *vs.* tetrapole

The above relations defining Z_0 and γ for symmetric tetrapoles, can be applied to transmission lines with distributed series impedances $Z = R + j\omega L$ and shunt admittance $Y = G + j\omega C$ per unit length of line, with $Z_0 = \sqrt{Z/Y}$ and transmission constant, $l\sqrt{ZY} = \gamma l$ for a line of length l, (fig.8)

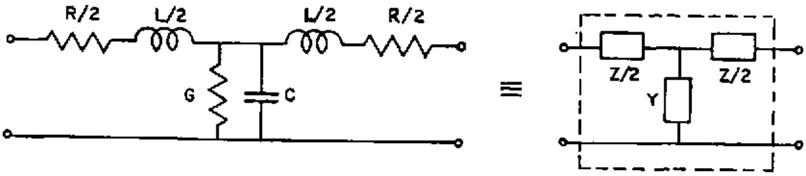


Fig.8

Therefore the transmition lines can be used as tetrapoles to define transmission systems with distributed parameters and contribute to the impedance coupling between source and load by varying the length of the line.

The input impedance of the coupling line will be that of a tetrapole with Z_0 and γl,

$$Z_{12} = Z_0 \frac{Z_c \; ch\gamma l + Z_0 \; sh\gamma l}{Z_c \; sh\gamma l + Z_0 \; ch\gamma l}$$

This equation leads to the followings with only th or cth

$$Z_{12} = Z_0 \frac{Z_c + Z_0 \, th\gamma l}{Z_c \, th\gamma l + Z_0} \quad and, Z_{12} = Z_0 \frac{Z_c \, cth\gamma l + Z_0}{Z_c + Z_0 \, cth\gamma l}$$

Because of the particular shapes of both functions th and cth. (fig. 9 a, b) it can be easily visualized how Z_{12} varies with the lenght of the line and with the load Z_c.

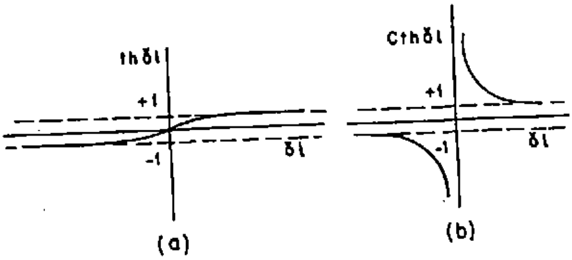


Fig.9

$l >> 1$, $\text{th}\,\gamma l \rightarrow 1$, $Z_{12} \rightarrow Z_0$

$Z_c = 0$ $Z_{12} = Z_0\,\text{th}\gamma l$

$\gamma l >> 1$, $\text{cth}\,\gamma l \rightarrow 1$, $Z_{12} \rightarrow Z_0$

$Z_c = \infty$, $Z_{12} = Z_0\,\text{cth}\,\gamma l$

6. ACOUSTIC APPLICATIONS OF IMPEDANCE ANALOGIES

6.1.- Impedances coupling

In acoustics a common case is to have a layer of absorbing material on a rigid surface, (fig. 10 a).

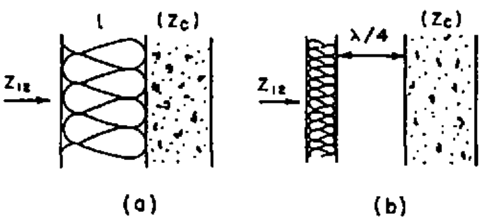


Fig. 10

In this case $Z_c \cong \infty$ and $Z_{12} = Z_0\,\text{cth}\ \gamma l$. In order to have $Z_{12} \cong Z_0$, l has to be large.

To reduce the thickness of the layer and to have good coupling with the low impedance of the air use is made of an intermedium ear space equivalent to an open line ($Z_c \cong \infty$), with $l = \lambda/4$ (fig. 10 b) what makes $Z_{12} = 0$:

$$\gamma_o l = j\beta l = j\frac{\omega}{c}\ \frac{\lambda}{4} = j\frac{\pi}{2} \quad \text{and} \quad Z_{12} = Z_2 \,\text{cth}\ j\frac{\pi}{2} = Z_2 \frac{1}{j\tan g\frac{\pi}{2}} = 0$$

by adjusting $l \cong \lambda/4$, $Z_{12} \cong Z_0$, and the incident acosutic wave will practically have not any reflection. The system have a maximum selective absorption, for frequencies multiples of $\lambda/4$.

6.2.- Acoustic wave transmission between three media

To resume the analogical impedance applications in acoustics, the general case of transmission of sound energy between three media, will be considered. (fig 11)

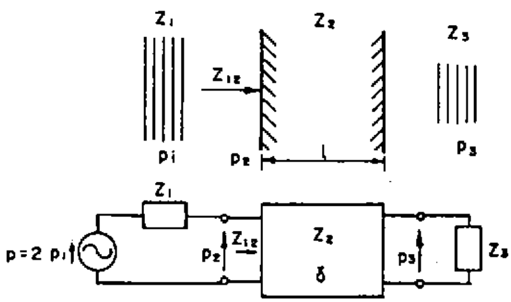


Fig.11

The equivalent circuit consists in a pressure source connected to a tetrapole and a passive dipole, defined by their impedances Z_1, Z_2, Z_3, the open circuit source pressure $p_0 = 2pi$ and the transmission constant γ of the intermedium system.

The first step is evaluate Z_{12} to deduce p_2 and then p_3. For Z_{12} applies the tetrapole formula in function of Z_2, Z_3 and $\gamma\, l$

$$Z_{12} = Z_2 \frac{Z_3\, ch\gamma l + Z_2\, sh\gamma l}{Z_3\, sh\gamma l + Z_2\, ch\gamma l}$$

In the pressure source circuit, the acoustic pressure p_2 over Z_{12} is deduced from the volumevelocity $Q = Su$.

$$\left.\begin{aligned} p_2 &= Z_{12}\, Q \\ Q &= \frac{2\, p_i}{Z_1 + Z_2} \end{aligned}\right\} \quad p_2 = p_i \frac{2Z_{12}}{Z_1 + Z_2}$$

The transmission system attenuates p_2 by $e^{-\gamma}$, therefore

$$p_3 = p_2\, e^{-\gamma} = p_i \frac{2Z_{12}}{Z_1 + Z_2} e^{-\gamma}$$

and the acoustic power delivered to the load Z_3,

$$W_3 = \frac{p_3^2}{Z_3} = p_i^2 \frac{4\, Z_{12}^2}{Z_3(Z_1 + Z_2)^2} e^{-2\gamma}$$

The transmission efficiency η,

$$\eta = \frac{W_3}{W_1} = \frac{W_3}{(2p_i)^2 / Z_1} = \frac{4\, Z_1\, Z_{12}^2}{Z_3\, (Z_1 + Z_2)^2} e^{-2\gamma}$$

If the source and load media are the same, $Z_1 = Z_3$ and η simplifies to

$$\eta = \left(\frac{2\, Z_{12}}{Z_1 + Z_2}\right)^2 e^{-2\gamma}$$

In the case of $Z_1 = Z_2 = Z_3 = Z_{12}$, and $\gamma = 0, \rightarrow \eta = 1$, i.e. 100% efficiency, as expected for a transmission without discontinuities and therefore without reflections.

It can be concluded that the generalisation of the impedance concept and the use of analog circuits helps in visualising and solving transmission energy problems between systems involving wave propagation, with particular application in acoustics.

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