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ON THE APPROXIMATION OF TOTAL ABSORPTION OF THE STREET OPEN CEILING AT LOW FREQUENCY

A. Pelat LAUM, CNRS, Université du Maine, avenue Olivier Messiaen, 72085 Le Mans, France.
B. Lihoreau LAUM, CNRS, Université du Maine, avenue Olivier Messiaen, 72085 Le Mans, France.

ABSTRACT

This work deals with numerical modeling of sound propagation in a U-shaped street canyon with flat building facades. The street is seen as an open waveguide and two 3D wave models are used : a parabolic equation model and a modal expansion model. The comparison between models shows a very good agreement. Then, we focus on the radiation condition at the opening of the street. In usual energetic approaches as ray tracing, the opening is assumed to be perfectly absorbing. Since this assumption is realistic in high frequency, the reflection phenomenon caused by the geometric discontinuity at the opening is still an open question at low frequency. This possible reflection is investigated through a parametric study of the acoustic longitudinal energy flux decay along the street.

1 INTRODUCTION

In modeling the sound propagation within an urban street canyon, one is particularly interested in considering the effects of the opening of the street on an infinite domain, the sky. Basically, the opening leads to radiation losses: the waves, partially confined within the street while they propagates are also radiated to the sky. As a consequence, the radiation losses contribute to the sound pressure level decay with the distance to the source, in addition to the atmospheric losses and other dissipation phenomenon (absorption at walls for example). This work aims at investigating the attenuation caused by radiation losses, independently from the other dissipation phenomenon. To adress the problem, the street is seen as an open waveguide. Indeed, the typical elongated shape promotes guided waves along the axis of the street. In contrast, the opening of this guide on an infinite domain - the sky - results in wave radiation.

Several formulations have been proposed to model the sound propagation within 3D street canyons. The most used, implemented in predictive software for acoustical engineering studies, are so called energetic approaches as ray tracing [1], image-sources [2], radiosity based method [3] or sound particules [4]. In these approaches, the opening of the street to the sky is modeled as a perfectly absorbing boundary. Hence, the geometry of the problem is equivalent to that of Fig. 1(a) where the height of the street above the ground is infinite. In the following, this geometrical configuration will be called "U-shaped" geometry. Since the assumption of total absorption at the street opening seems to be well adapted in the high frequency domain, one may question the validity of this assumption in the low frequency domain. Notably, we can refer to the behavior of a duct in wich the stationary waves are due to the almost total reflexion at the open extremity.

To investigate this question, we propose to compare the U-shaped geometry in Fig. 1(a) with the geometry of Fig. 1(b) where no assumption is made on the behavior of the wave at the street open ceiling ($z = d$). In this configuration, the street is seen as an uniform waveguide with a rectangular cross-section open on the infinite half space above, bounded by PML [5] to truncate the computational domain. To solve the problem in both geometry, we use two different wave approaches. On the one hand, the parabolic equation method wich has been succesfully used to study outdoor, long range, propagation problems. Recently, the PE method has been extended to model the propagation in a 3D open waveguide [6]. On the other hand, the modal expansion has been used in the 70's [7] to describe the propagation within a street seen as a guiding structure between two infinite parallel planes. Recently, the modal expansion has been extended to the case of 3D open waveguides by using leaky modes [8].

The paper is organized as follow. The section 2 presents the theoretical background of both two waves approaches. Then, the section 3 presents numerical results: the two methods are first compared and then, the assumption of total absorption is studied through a parametric study of the leakage of the street. Finally, the section 4 gives a discussion about the physical interpretation of the results, possible improvements to this work and concluding remarks.

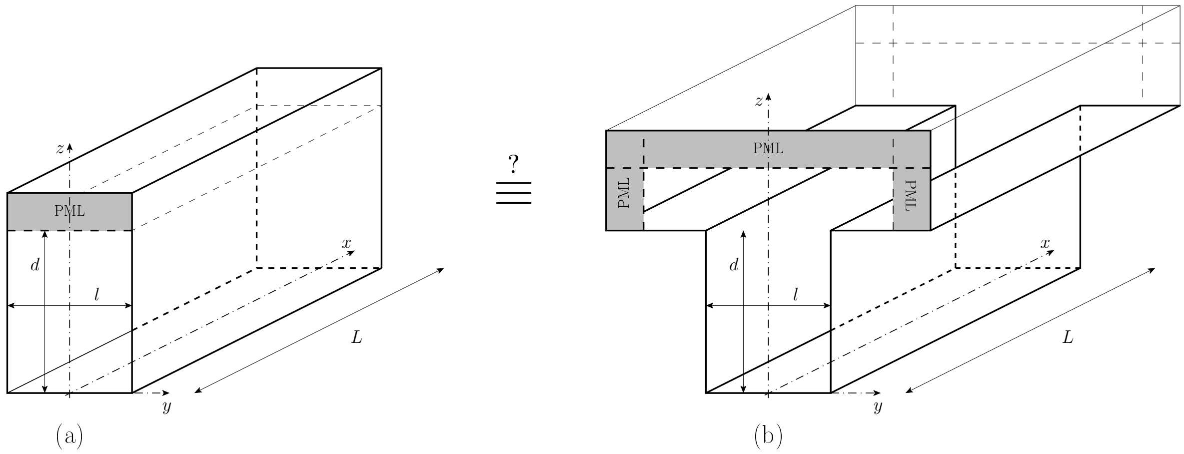

Figure 1: (a) The street is seen as a U-shaped uniform waveguide, of width l and height d, where a perfectly absorbing ceiling is considered ($z = d$). (b) The street is seen as an open uniform waveguide with a rectangular cross-section of same dimensions open on the infinite half space.

2 TWO WAVE APPROACHES FOR THE SOUND PROPAGATION MODELING IN A 3D UNIFORM OPEN CANYON

In both formulations, the equation to solve is the classic wave equation :

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] p'(x, y, z, t) = 0, \quad (1)$$

where p' is the acoustic pressure and c the wave celerity. In the frequency domain and taking into account the PML to bound the sky above the street, the wave equation to solve is

$$\left[\frac{\partial^2}{\partial x^2} + \frac{1}{\tau}\frac{\partial}{\partial y}\left(\frac{1}{\tau}\frac{\partial}{\partial y}\right) + \frac{1}{\tau}\frac{\partial}{\partial z}\left(\frac{1}{\tau}\frac{\partial}{\partial z}\right) + k^2\right] p(x, y, z) = 0, \quad (2)$$

where p is the complex acoustic pressure ($p' = pe^{-i\omega t}$), k is the wavenumber and τ is a complex scalar PML parameter fulfilling

$$\begin{cases} \tau = Ae^{j\beta} & \text{in the PML domain,} \\ \tau = 1 & \text{in the physical domain} \end{cases} \quad (3)$$

with $A > 0$ and $\beta \in]0, \pi/2[$. Neuman boundary conditions $\partial_n p = 0$, ∂_n the normal derivative are defined both for all the assumed rigid walls of the domain and for the outer bounders of the PML.

2.1 Parabolic equation

In the PE models [9], Eq. (2) is splitted into 2 equations for the outgoing wave (+) and the incoming wave (-). The equation for the outgoing wave is :

$$\left[\frac{\partial}{\partial x} - ikQ\right] p^+ = 0, \quad (4)$$

where Q is the operator defined by

$$Q^2 = 1 + \frac{1}{k^2}\left[\frac{1}{\tau}\frac{\partial}{\partial y}\left(\frac{1}{\tau}\frac{\partial}{\partial y}\right) + \frac{1}{\tau}\frac{\partial}{\partial z}\left(\frac{1}{\tau}\frac{\partial}{\partial z}\right)\right]. \quad (5)$$

If there is no coupling between outgoing and incoming waves, Eq. (4) represents an exact equation of the evolution of the acoustic pressure. It is more convenient to work with a scaled variable $\phi = p^+ e^{-ikx}$ which leads to the one-way equation to solve :

$$\left(\frac{\partial}{\partial x} - ik(Q-1)\right)\phi = 0. \quad (6)$$

The solution of Eq. (6) is :

$$\phi(x + dx, z) = e^{ik(Q-1)dx}\phi(x, z). \quad (7)$$

The key point is to approxim the square root operator :

$$Q = \sqrt{1 + \mathcal{L}_y + \mathcal{L}_z}, \quad (8)$$

with $\mathcal{L}_y = \frac{1}{k^2\tau}\frac{\partial}{\partial y}\left(\frac{1}{\tau}\frac{\partial}{\partial y}\right)$ and $\mathcal{L}_z = \frac{1}{k^2\tau}\frac{\partial}{\partial z}\left(\frac{1}{\tau}\frac{\partial}{\partial z}\right)$. It can be approximated using a higher order Padé approximation:

$$\sqrt{1 + \mathcal{L}_y + \mathcal{L}_z} = 1 + \sum_{k=1}^{n_p} \frac{a_{k,n_p}\mathcal{L}_y}{1 + b_{k,n_p}\mathcal{L}_y} + \sum_{k=1}^{m_p} \frac{a_{k,m_p}\mathcal{L}_z}{1 + b_{k,m_p}\mathcal{L}_z}, \quad (9)$$

where n_p and m_p are the number of terms and a_{k,n_p}, b_{k,n_p} are real coefficients given in Ref. [10]. For 2D problems ($\mathcal{L}_y = 0$), the Padé series expansion allows a very wide angle propagation along z, the angular limitation depending on parameter n_p. In Eq. (9), we neglected terms in $\mathcal{O}(\mathcal{L}_y^{2n_p+1}, \mathcal{L}_z^{2m_p+1}, \mathcal{L}_y\mathcal{L}_z)$. Due to the term in $\mathcal{O}(\mathcal{L}_y\mathcal{L}_z)$, Eq. (7) does not, strictly speaking, have a wide-angle capability [11]. Thus, it seems coherent to make a Padé series expansion of order 1 with $a = 1/2$ and $b = 1/4$. In this case, Eq. (6) leads to:

$$\left(\frac{\partial}{\partial x} - ik\left(\frac{a\mathcal{L}_y}{1 + b\mathcal{L}_y} + \frac{a\mathcal{L}_z}{1 + b\mathcal{L}_z}\right)\right)\phi = 0. \quad (10)$$

The previous initial- and boundary- value problem is numerically solved using an alternating direction method [12], which requires numerical solutions for each of the following:

$$\frac{\partial \phi}{\partial x} = ik\frac{a\mathcal{L}_z}{1 + b\mathcal{L}_z}\phi, \quad (11)$$

$$\frac{\partial \phi}{\partial x} = ik \frac{a\mathcal{L}_y}{1 + b\mathcal{L}_y} \phi. \tag{12}$$

The alternating direction method allows us to compute only 2D problems (xz- planes for Eq. (11) and xy- planes for Eq. (12)). These equations are solved using a Crank-Nicholson integration in range and a finite difference method in depth and width.

2.2 Coupled modal-finite element method

In the coupled modal-finite element method [13, 14], the transverse domain (y, z) of the canyon is discretized by using a standard FEM. A N-nodes triangular mesh is generated on the cross-section and a first order interpolating polynomial $\psi_n(y, z)$ is defined at each node n of coordinates (y_n, z_n) so that the pressure field is developed on the basis of polynomials $\psi_n(y, z)$:

$$p(x, y, z) = \sum_{n \geq 1} P_n(x) \psi_n(y, z) = {}^{\mathrm{t}}\vec{\psi}\vec{P}, \tag{13}$$

where the components of $\vec{P}$ are the values of the pressure at nodes (y_n, z_n): $P_n(x) = p(x, y_n, z_n)$.

Following, Eq. (2) is reformulated in its discrete form as

$$\vec{P}'' + \left(M^{-1}K + k^2\right)\vec{P} = 0, \tag{14}$$

where $''$ denotes the second derivative with respect to x, and mass and stiffness matrices M and K are given by, respectively,

$$M_{mn} = \int_S \tau \psi_m \psi_n \, \mathrm{d}y\mathrm{d}z, \tag{15a}$$

$$K_{mn} = -\int_S \frac{1}{\tau} \left(\frac{\partial \psi_m}{\partial y}\frac{\partial \psi_n}{\partial y} + \frac{\partial \psi_m}{\partial z}\frac{\partial \psi_n}{\partial z} \right) \mathrm{d}y\mathrm{d}z, \tag{15b}$$

A general solution of Eq. (14) can be written as function of the eigenvalues α_i^2, $i \geq 0$, and eigenfunctions ν_i of the matrix $M^{-1}K$:

$$\vec{P} = \Upsilon\left(D(x)\vec{C}_1 + D(L - x)\vec{C}_2\right), \tag{16}$$

where $\Upsilon = [\nu_0, \nu_1, \nu_2, \ldots]$ and $D(x)$ is a diagonal matrix given by

$$D_i(x) = e^{\mathrm{j}k_i x}, \tag{17}$$

with $k_i = \sqrt{k^2 - \alpha_i^2}$ ($\Re\{k_i\} \geq 0$, $\Im\{k_i\} \geq 0$). $\vec{C}_1$ and $\vec{C}_2$ are constant vectors determined by the end conditions at $x = 0$ and $x = L$. Let assume that the condition at $x = 0$ is defined as a given pressure field $p(x = 0, y, z) = p_0 = {}^t\Phi\vec{P}_0$ and the condition at $x = L$ as an admittance matrix Y_L fulfilling $\vec{Q}(L) = Y_L\vec{P}(L)$, with $\vec{Q}$ the vector of the components of $\partial_x p$ in the basis $\{\psi_n\}$. Then, the constants $\vec{C}_1$ and $\vec{C}_2$ are

$$\vec{C}_1 = (I - \delta)^{-1}\Upsilon^{-1}\vec{P}_0, \tag{18a}$$

$$\vec{C}_2 = -D^{-1}(L)\delta\vec{C}_1, \tag{18b}$$

where

$$\delta = D(L)\left(Y_L\Upsilon + \Upsilon\Gamma\right)^{-1}\left(Y_L\Upsilon - \Upsilon\Gamma\right)D(L), \tag{19}$$

with $\Gamma_{ij} = \mathrm{j}k_i\delta_{ij}$. Note that the admittance matrix in the input plane $x = 0$ can be written as function of the output admittance matrix Y_L:

$$Y_0 = \Upsilon\Gamma(I + \delta)(I - \delta)^{-1}\Upsilon^{-1}. \tag{20}$$

3 NUMERICAL RESULTS

3.1 Parameters of the study

In both methods, the acoustic field is computed downstream from a starting field $p(x=0,y,z)=p_0$, function of the transverse coordinates (y,z), defined in the street input plane ($x=0$). In the following results, the starter is chosen as an incident gaussian beam given by:

$$p_0 = e^{-\dfrac{(y-y_s)^2+(z-z_s)^2}{2\sigma^2}} \tag{21}$$

where y_s and z_s are the position of the center of the gaussian and σ is a parameter controlling the width of the gaussian. This kind of starter can be seen as an approximation of the far field pattern generated by a point source. Nevertheless, the radiation pattern of a gaussian beam is not omnidirectionnal, the directivity being driven by the divergence of the beam given by $\Theta \simeq 2c/\pi f\sigma$ rad.

In the numerical results presented below (section 3.3), the fixed parameters are the center position of the gaussian beam chosen as $(y_s,z_s)=(0,0.1)$ m to approach typical source positions as vehicles and the heigth of the street $d=10$ m. The varying parameters are then the width of the street $l=5,10,20$ m and the frequency of the gaussian beam $f=62.5,125,250$ Hz. To get comparative results, we adjust σ so that Θ remains constant ($\sigma=1.44$ for $f=125\ Hz$).

The main purpose of this work is to sudy the energy losses through the opening of the street to the sky. This radiation losses can be evaluated by measuring the decay of longitudinal energy flux along the street. Hence, we define the integrated indicator $W(x)$, homogeneous to a power, as:

$$W(x) = \frac{1}{S}\int_0^d \int_{-l/2}^{l/2} \frac{1}{2}\mathbb{R}\{pv^*\}\mathrm{d}y\mathrm{d}z \tag{22}$$

where v is the longitudinal velocity and $S=ld$ is the rectangular cross section.

3.2 Comparison between the two waves approaches

In this section, the results between both methods are compared in the case of a street canyon (Fig. 1(b)) of fixed dimensions $l=d=10$ m, for the three chosen frequencies $f=62.5,125,250$ Hz.

First, the comparison concerns the pressure field. Fig. 2 shows horizontal maps of the pressure field (modulus) at height $z=z_s$ from the ground. For each chosen frequency of the incident gaussian beam, the sound fields computed from the two methods display very similar interference pattern of the field. For example, on the line defined by $(y,z)=(y_s,z_s)$, the solutions are in good agreement. Nevertheless, one can see slight discrepancies due to the differences between the two discretization schemes of the problem. On the one hand, for computation cost reasons, the thickness of the mesh in the modal-FE method is much more lower than in the parabolic equation method. This relative sub-discretization of the transverse domain tends to stiff the problem. As a consequence, while the overall pattern of the fields between the two methods are very close, locally, the locations of the interferences may be slightly different. On the other hand, at low frequency ($f=62.5$ Hz), even using a very thin spatial step ($\lambda/20$) in the finite difference scheme (parabolic equation), the number of discretization points is not as large as needed to ensure the stability of the method and typical high frequency Gibbs oscillations appear.

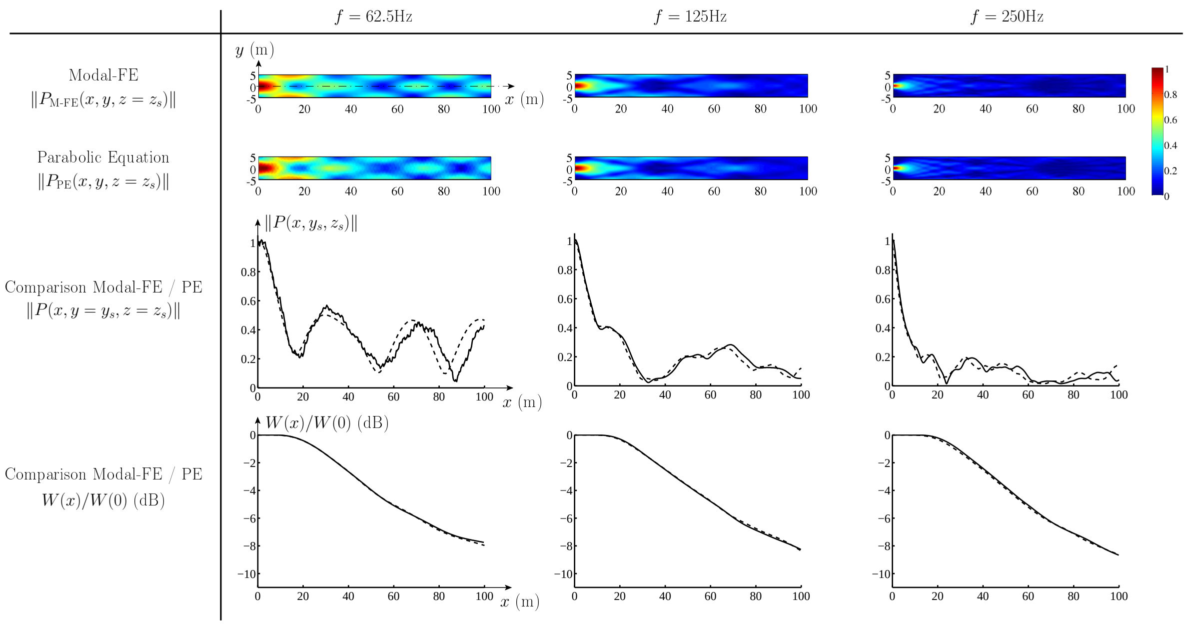

Figure 2: Comparison of the results between the two wave approaches in the case of a street canyon (Fig. 1(b)) of fixed width $l = 10$ m and height $d = 10$ m at three given frequencies $f = 62.5, 125, 250$ Hz. For each frequency, the results are presented as, first, a horizontal map of the pressure field (modulus) at a height $z = z_s$ from the ground, second, the pressure field (modulus) along a line defined by $(y, z) = (y_s, z_s)$ (— : PE / - - - : modal-FE), third, the decay of the integrated indicator $w(x)$ function of x (— : PE / - - - : modal-FE).

Second, the comparison concerns the results obtained for the integrated indicator $W(x)$ defined in Eq. (22). Fig. 2 shows the decrease of the dimensionless value of $W(x)/W(x = 0)$, for each studied case. Results obtained with both methods are nearly similar.

This comparison shows that even if the two methods provides very similar results, the modal-FE method is preferable for computations at low frequency while the parabolic equation method is preferable at higher frequencies. In the following study of the leakage of the street, we will use the modal-FE method at $f = 62.5$ Hz and the parabolic equation method at $f = 125$ Hz and $f = 250$ Hz.

3.3 Study of the leakage

The goal of this section is to compare the acoustic energy flux along the street computed for both geometries of Fig. 1. Results are summarized in Fig. 3 for several frequencies and street widths. All curves exhibit a similar behavior: a stationary phase followed by a monotone decrease. The stationary phase can be explained by the gaussian starter directivity and would not exist with a ideal omnidirectional starter. The acoustic energy flux decrease can by explained by the acoustic leakage across the opening of the street. In all cases, the acoustic energy flux decrease is slower for the geometry of Fig. 1(b) which shows that some acoustic energy is reflected at the opening of the street.

To simplify the lecture of the results, the difference ΔW between the acoustic flux in both geometries are given at 100 m of the source. One can see that, for a given street width, the slower is the frequency, the bigger is ΔW. In another hand, for a given frequency, the weaker is the street width, the bigger is ΔW. Interesting cases are those for whom the ratio $\frac{l}{\lambda}$ is constant (cases a.3-b.2-c.1 or a.2-b.1 or b.3-c.2) and where ΔW is nearly the same. This observation tends to show that the dimensionless ratio $\eta = \frac{l}{\lambda}$ is a significant parameter to quantify the contribution of the acoustic reflection at the opening. In conclusion, if η is too small, the approximation of total absorption at the

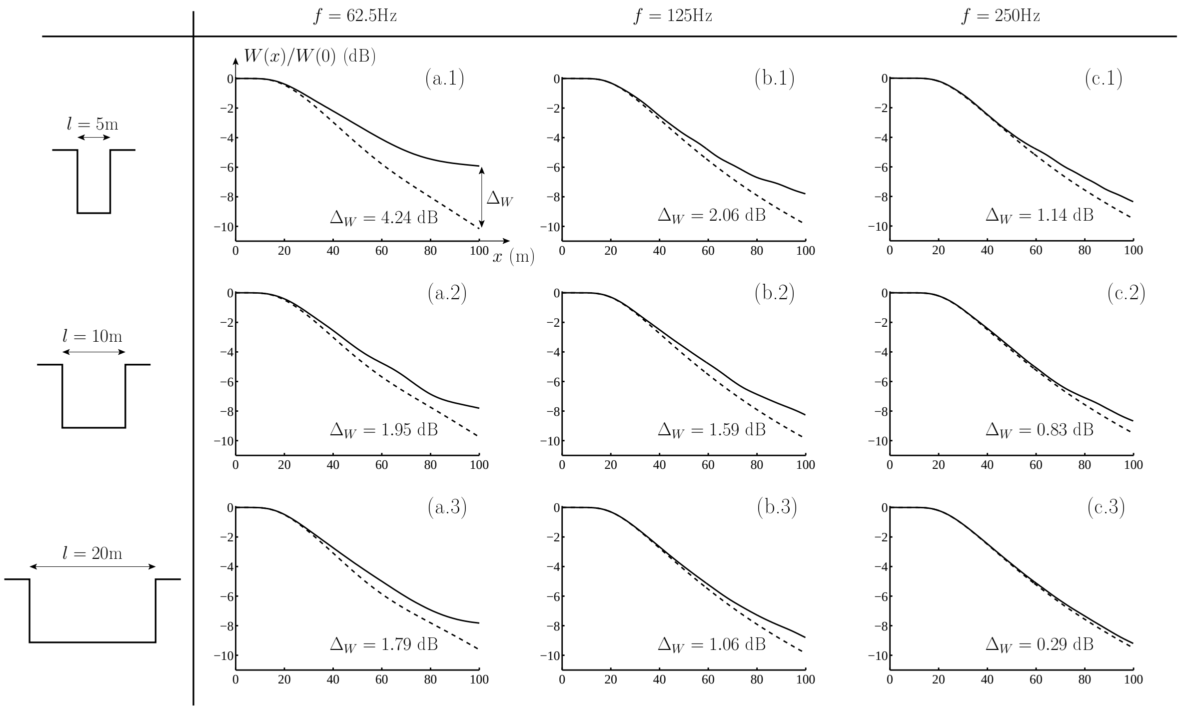

Figure 3: Acoustic energy flux decrease along the street. Configurations of Figs. 1.a (- - -) and 1.b (—) are compared for 3 frequencies $f = 62.5, 125, 250\ Hz$ and 3 street widths $l = 5, 10, 20\ m$.

opening is no longer verified: there exists a wave guiding effect between the ground and the ceiling, in addition to the one between both façades. This result is not surprising if one consider the case $\eta \to 0$ corresponding to an open pipe situation where the reflection is total.

4 DISCUSSION

In this paper, we have proposed two wave methods to study the acoustic propagation along a street. The comparison of these methods showed a very good agreement. Next we pointed out that assumption of total absorption at the opening is not valid in all situations. Indeed, the dimensionless ratio seems $\eta = \frac{l}{\lambda}$ to drive the acoustic reflection at the opening. For example, one can arbitrarily determine the following criterion: the total absorption hypothesis is no longer valid if it leads to a leakage difference ΔW higher than 1 dB at 100 m from the source. Thanks to results of Fig. 3, it means that the hypothesis is not valid below 125 Hz for a 20 m width street or below 250 Hz for a 10 m width street.

Many improvements can be provided to this work. First of all, the study of acoustic leakage through the opening is led by the acoustic energy flux calculus. This physical quantity may not fit with perceptive sensations. We use this quantity because it frees the calculus from interferences (see Fig. 2) which make uneasy the dicrease estimation. Another indicator could be the acoustic level at ears height obtained after adding acoustic intensity for several frequencies in an octave band. Anyway and whaterer the criterion used, it will be appreciable to work with more realistic street geometries including building facades irregularities and ground impedance. At least, it would be interesting to study the impact of the micrometeorological effects as refraction or convection on the reflection at the opening.

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