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A wave-based analysis for acoustic transmission for fluid-filled elastic waveguides

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ABSTRACT

This paper describes an original numerical prediction technique developed for the analysis of coupled vibro-acoustic problems in fluid waveguides. Specifically it is is a wave-based method. Unlike the conventional element-based methods, this technique uses wave functions that satisfy the governing equations to describe the dynamic variables exactly. One advantage is that fine domain discretizations, used by element based methods near the fluid-structure interface typically, are no longer required. Hence the resulting model sizes are much smaller than element based methods yielding a more time-efficient prediction technique that may allow handling of mid-frequency applications. Another advantage is that dispersion relations between propagating wavenumbers and excitation wavenumbers are easily obtained and an example to show this is presented. Also a discussion on how the wave-based prediction technique can be used for coupled vibro-acoustic problems a cavity with a non-reflecting boundary and a complex silencer duct problem. Its beneficial characteristics compared to element-based methods are demonstrated through the validation study and transmission loss examples.

1. INTRODUCTION

A variational formulation for the two--dimensional Helmholtz equation in an acoustic waveguide is presented. The objective is to calculate sound transmission through local perturbations of a fluid or supporting structure located either within the fluid or at the fluid boundary. The case considered is that of wave propagation from an acoustic or structual source in a waveguide that is assumed to be straight and arbitrarily long with uniform rectangular cross-section. Parameters governing the fluid wavespeed are also allowed to vary piecewise along the longitudinal direction of the waveguide. The proposed approach can provide accurate solutions over domains of arbitrary length in a certain direction; namely, an infinite waveguide, a semi-infinite waveguide and a finite rectangular enclosure.

The problems studied here are specific to waveguides whose fluid domains only vary in width. For general domains there has been a significant effort in improving the numerical modelling capability for linear steady-state acoustic problems in two and three dimensions for interior or exterior acoustic domains. The most general local approaches are the finite element method (FEM) and the boundary element method (BEM). These discrete methods rely on spatial discretizations that are small compared to the wavelength of the problem. For finite and boundary element methods the underlying approximating functions are usually polynomials with compact support in the acoustic region. For high wavenumbers this choice leads to systems of equations with a large number of degrees of freedom. Whereas, for acoustic waveguide problems a judicious choice of basis functions, which have an underlying global nature, may lead to significantly less number of degrees of freedom. For the new spectral element method

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presented here the basis functions inherently possess some of the wave nature of the acoustic field.

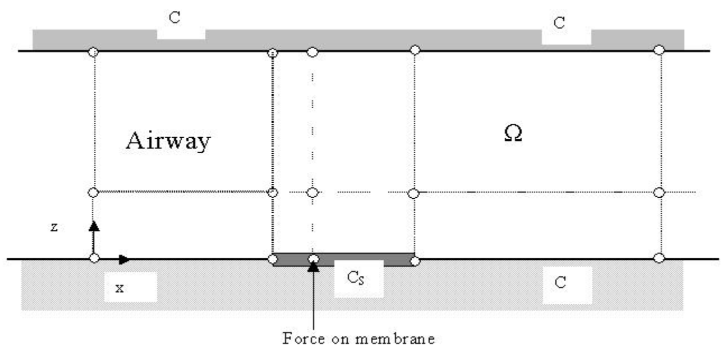

Figure 1: Duct geometry showing structure

The spectral finite element method (SFEM) applied to waveguide problems[1-3] is viewed as a merger of the dynamic stiffness method and the finite element method. The method is based on a variational formulation for non-conservative motion in the frequency domain.

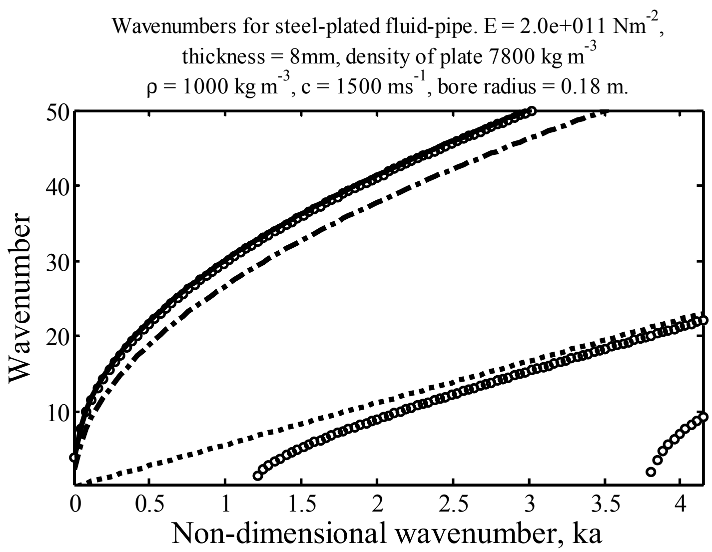

Figure 2: Disperson relations for beam in fluid-filled waveguide, depth R= 180 mm, Solid line (infinite beam coupled to half-space), dots (fluid wavenumber) dash-dot (structure wavenumber) and circles for SFEM fluid-filled pipe.

The SFEM has been used to study vibration in beam frame—works[1], beam--stiffened railway cars[2] and for fluid-filled pipes[3]. Finnveden used the spectral pipe elements for various cases including assessment of approximate theory[4]. Use of a variational formulation for the spectral method provides a natural basis for approximations and a simple tool for combination with standard finite elements. In the following spectral finite elements are combined to solve a waveguide fluid-structure problem.

2. VARIATIONAL FORM

The geometry of the general problem is illustrated in Figure 1. A prismatic duct of uniform cross section lies along the x axis of a Cartesian coordinate system. It comprises an airway with sound speed c_0 and density ρ_0 carrying an acoustic fluid and surrounded by a locally reacting liner. The wall of the duct is formed either by a rigid boundary or by a number of membrane plates. One such plate is depicted in Figure 1, but the analysis holds for any number of membranes, and applies, for example, to rectangular ducts whose walls are entirely formed in this way. Solutions are sought for the acoustical pressure (potential) in the duct, p(.), and the normal displacement of the membrane, W(.) where ω is the frequency of the disturbance and $k=\omega/ c_0$ is the acoustic wavenumber. The applied rigidity of the plate is given by D_S and m_S is its mass density per unit area and $k_S^4 = m_S\, \omega^2/ D_S$ is the beam wavenumber. The pressure and displacement are coupled through a kinematic constraint on the fluid-structure interface, C_S. It is relatively simple to establish the functional for the structural-fluid problem:

$$L(W,\varphi)=\rho\int_{\Omega}\left(-k^2\varphi^a\varphi+\nabla\varphi^a\cdot\nabla\varphi\right)dxdy$$
$$-\rho\omega\int_{C_s}\left(\varphi^a W+W^a\varphi\right)dx-\rho\int_{C}\left(\frac{\partial\varphi}{\partial n}\varphi^a+\frac{\partial\varphi^a}{\partial n}\varphi\right)dx \quad (1)$$
$$+\int_{C_s}\left(D_s\frac{d^2W}{dx^2}\frac{d^2W^a}{dx^2}-m_s\omega^2 W W^a\right)dx$$

The underlying differential equations which emerge from the functional above are the Helmholtz equation in the airway and the second order membrane equation. “Natural” boundary conditions arise provided and the potential is free to vary over C and C_s and provided also that the class of functions from which it is drawn is continuous at the interface. The “natural” boundary conditions associated with the variation of W(.) correspond to zero bending and force condition at the longitudinal ends. To model clamped conditions in the finite element formulation, fro example, the class of element shape functions require extension to include any “essential” constraints.

In acoustic waveguides with constant cross-section the solutions of the equations of motion, i.e. Helmholtz equation, are exponential terms describing wave propagation in an axial direction with corresponding cross--sectional modes. Polynomials are derived using a procedure developed by Finnveden[3] in to describe fluid motion in the cross-section of pipe structures. Inserting the approximating functions into the variational formulation (1) , see[5] for details for the pure acoustic case, evaluating certain derivatives and integrals in the z-direction, expressions for the dynamic stiffness matrices are obtained. a little matrix algebra a $(N+1)x(N+1)$ system of second order differential equations for the unknown functions, $S=(W,p)$, follows

$$[K_4]\frac{d^4S}{dx^4}+[K_2]\frac{d^2S}{dx^2}+k^2[K_0]S=0 \quad (2)$$

Since the system of equations (2) has constant matrix coefficients the solution may be written in the form:

$$S(x) = X\, e^{\lambda x}$$

where X is a vector representing wave amplitudes and λ wavenumbers determined by a suitable matrix eigenvalue problem. The form of the PEP (Polynomial Eigenvlaue Problem) is not regular and needs some regularization. The details are omitted here. Using the definition for propagating waves and polynomials in the z-direction it is possible to form appropriate trial or wave influence functions over a finite arbitrary length fluid-structure element. Dynamic stiffness matrices may be combined for fluid-structure elements and fluid only elements to model problems with various geometries and material parameters.

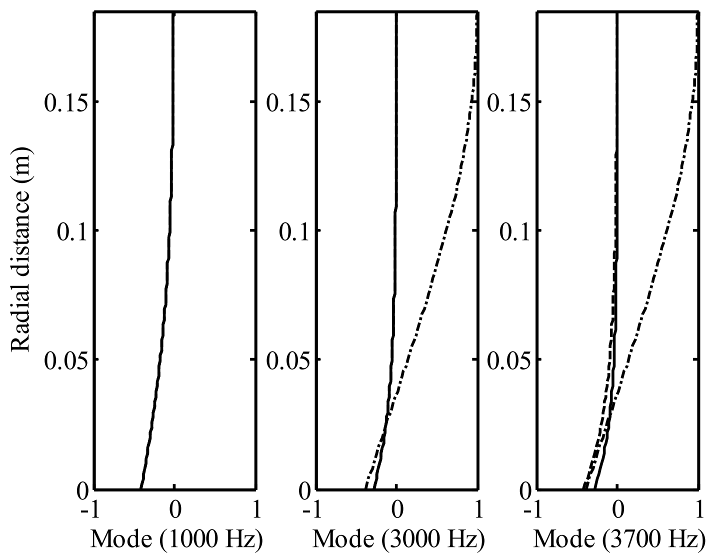

Figure 3: Cross-sectional modes corresponding to wavenumbers in Figure 2. Total height of duct is 185 mm ρ=1000 kgm^{-2} per unit length, c=1500 ms^{-1}. The plate is assumed to have a rigidity $D_S = 2x10^{11}\ Nm^{-1}$ and mass density per unit area m_S = 7850 kgm^{-2}

3. RESULTS

The results presented here are for a uniform fluid-structure duct. The dimensions of the duct are, for input/output duct H=31 mm, total depth of muffler R=185mm, width W=400mm enclosing a fluid ρ=1000 kgm^{-2} per unit length, and wave speed for the fluid c=1500 ms^{-1}. The beam is assumed to have a rigidity $D_S = 2x10^{11}\ Nm^{-1}$ and mass density per unit area m_S = 7850 kgm^{-2}. Figure 4 shows transmission coefficient results for the fluid-filled flexible pipe included in a rigid tube as inlet and outlet. Using the Spectral Finite Element Model (SFEM) results are shown for the fluid-filled pipe against an "analytical solution" (dashed line) for acoustic problem. The transmission coefficient represents the ratio of energy transmitted to the input energy.

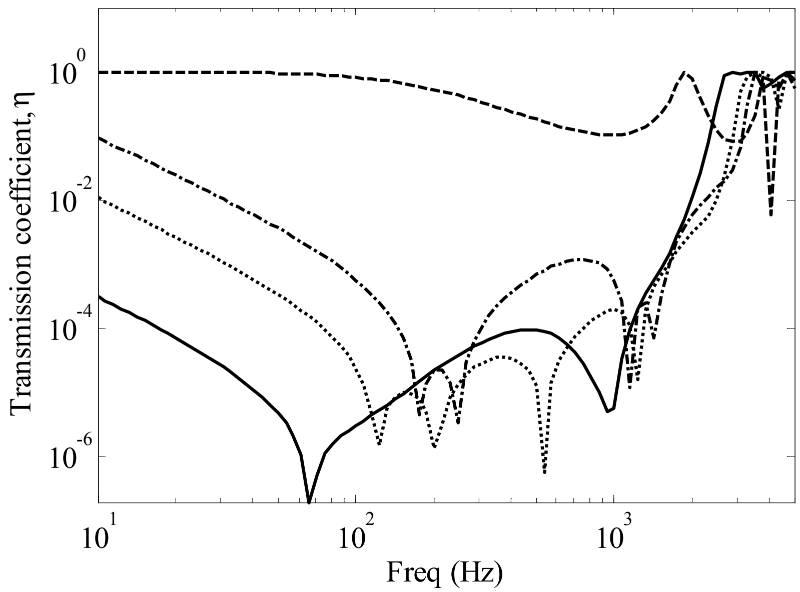

Figure 4: Transmission coefficient for fluid filled pipe of height H=31 mm with flexible muffler of width W=0.4, and total height R=18.5 mm. Thickness of beams are from below h = 5mm (solid), 9mm (dotted), and 13mm (dash-dotted).

4. CONCLUSIONS

In conclusion, the variational approach provides a useful method of computing dispersion relations and forced response solutions in lined, flexible ducts of uniform cross section. The spectral element approach has been proven to be a natural tool for predicting structural acoustic responses in waveguide or duct structures.

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