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Acoustic propagation in variable sound speed profiles

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ABSTRACT

An important topic in the area of airborne sound propagation is the prediction of sound propagation above an impedance ground with an atmospheric profile whose sound speed varies with height. Even if this problem is simple in concept, it leads to complications for general velocity profiles. This work illustrates the existence of a large class of realistic atmospheric profiles for which analytical solutions exist to be used as benchmark solutions for numerical methods. Spectral finite element results are discussed for sound propagation in a half-space situated above a ground surface impedance.

1. INTRODUCTION

Analytical benchmark solutions are required for situations that go beyond the squared refraction index linear model in order to test numerical methods that are required for even more complicated situations. Asymptotic limits, like a large distance between transmitter and receiver, are of particular interest since numerical methods may then be inefficient.

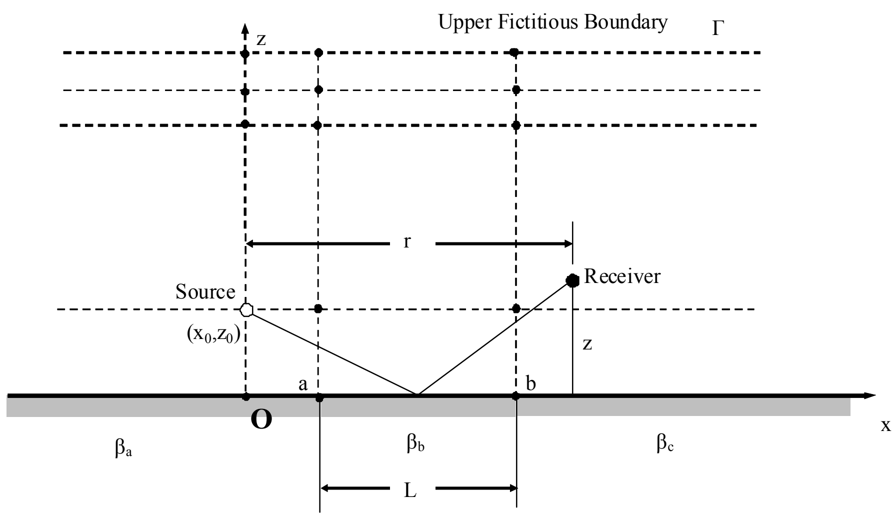

Figure 1: Geometry for numerical solution.

The analytical details of the solution including selection of right and left going waves are presented for a tanh-like profile. In the current note the spectral finite element method is used to predict the sound pressure for the tanh velocity profile, the geometry for the FE method is shown

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2. GREENS FUNCTION & MODEL

We study the propagation of sound waves in a half space $z \geq 0$ where the sound speed depends on the vertical co-ordinate z only. The sound source is a line source with amplitude 1 located at $(x,z)=(0,z_s), z_s \succ 0$ and the ground is modelled by an admittance boundary condition. A stationary time dependence $e^{-i\omega t}$ has been assumed and $k=\omega/c_0$ with c_0 being a fixed standardised sound speed. Let $n(z)=c_0/c(z)$ be the acoustic refractive index as the sound speed $c(z)$ varies with z. We express the Green's function as an inverse Fourier integral in the x-variable

$$G(x,z;z_s)=\frac{k}{2\pi}\int_{-\infty}^{+\infty} p(z,z_s,u)e^{ikux}dx \qquad (1)$$

Then p solves the inhomogeneous Helmholtz equation

$$\frac{d^2 p}{dz^2}+k^2\gamma^2(z)p=-\delta(z-z_s) \qquad (2)$$

where $\gamma=\sqrt{n^2-u^2}$ having branch cuts on the real u-axis from n to $+\infty$ and from $-n$ to $-\infty$. We find that exact solutions to Eq. (2) are known explicitly in terms of hypergeometric functions for special versions of the inhomogeneous term. Specifically it is a Rosen-Morse potential. Without going into too much detail the exact solution for Eq.(2) maybe written in terms of right-going (downward) and left-going (upward) waves :

$$\begin{cases} f_R(s)=s^{-\lambda_0/2}(1-s)^{\lambda_1/2}F(1-\lambda_0,-\lambda_0,1+\lambda_1;1-s) \\ f_L(s)=s^{-\lambda_0/2}(1-s)^{-\lambda_1/2}F(0,0,1-\lambda_1;1-s) \end{cases} \qquad (3)$$

Here

$$\begin{cases} \lambda_0=\sqrt{c_1(u^2-n_+^2)}, \operatorname{Re}\lambda_0 \geq 0 \\ \lambda_1=-i\kappa \\ \kappa=\sqrt{c_1(n_-^2-u^2)}, \operatorname{Im}\kappa \geq 0 \end{cases}$$

and F is a hypergeometric function. Hence the Green's function may be calculated through equations (3) and (2). Asymptotic expressions for the Green's functions and explicit comparisons with the numerical model have been presented in Peplow and Nilsson[1] and are a current subject for further study.

3. THE FINITE ELEMENT MODEL

Simulations over impedance ground will be presented in the current paper using the spectral finite element numerical solutions that previously have been derived by Peplow and Finnveden[2] the central theme of which is a generalized eigenvalue problem solved by a QZ algorithm in MATLAB. In order to make possible a coming comparison with analytical methods, the Rosen-Morse velocity profile is used. Figure 1 shows a typical spectral element geometry. In the usual finite element discretization the cross-section is divided so that material parameters are constant within each interval, here this is extended so a non-constant profile is possible. In the super-spectral method the velocity profile in the vertical direction may be specified arbitrarily. Here the velocity profile is defined through the potential function defined above where argument in the Rosen-Morse potential is az where the small constant, in the following examples, takes the value $a=0.1$.

For the numerical results the admittance for the absorbing ground is calculated using an admittance formula which gives the normalised admittance. A value appropriate for grassland at 300 Hz $\beta = 0.34 - 0.29i$. Values taken outside the grass strip are for rigid ground; in addition all the examples here represent numerical simulations for a point source 1.0 m above the ground at a single frequency, 300 Hz and the grass-strip is between 10 and 45 m from the origin.

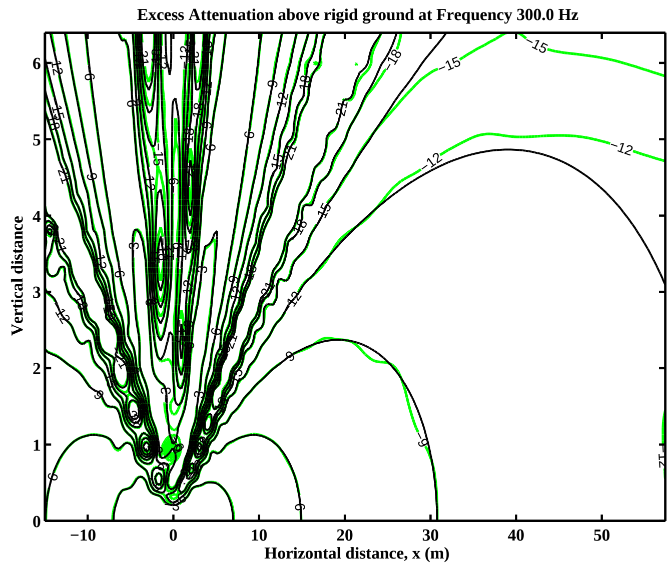

Figure 2: Excess attenuation above rigid ground at 300Hz for homogeneous medium. Solid black isolines represent exact solution and green isolines approximations by the finite element method.

The calculation shown in Figure 2 was carried out in terms of excess attenuation over free-field. In free-field conditions, the whole of space is filled with the medium of propagation and there are no boundaries.

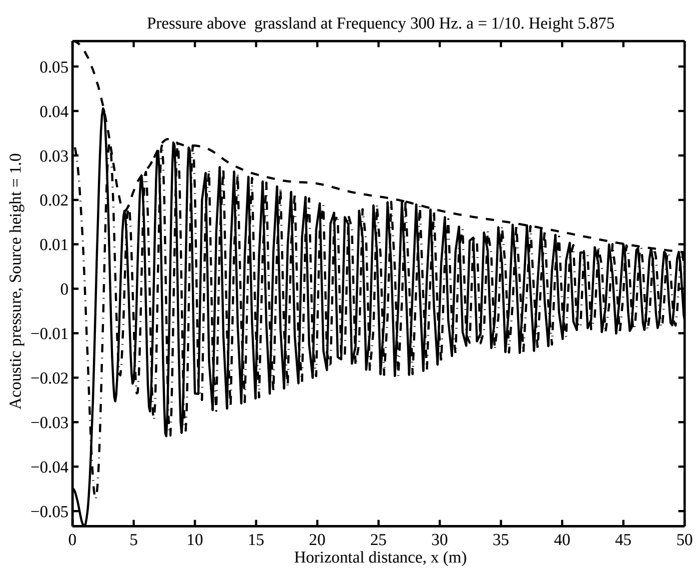

Figure 3: Acoustic pressure in media with profile *V*(z) with *a* = 0*.0*, 5*:*875m above rigid ground surface at 300Hz. Real part pressure (solid line), imaginary part (dash–dot), and magnitude (dashed line).

Figure 3 shows the numerical solutions at constant height 5.875 m above the ground surface and may be compared to Figure 4 due to the sound velocity profile. An increasing sound speed profile when a positive gradient is common. This is caused by the rapid cooling of air at the surface as heat is now absorbed by the ground. This is called an inversion or negative lapse and the sound waves are bent downward.

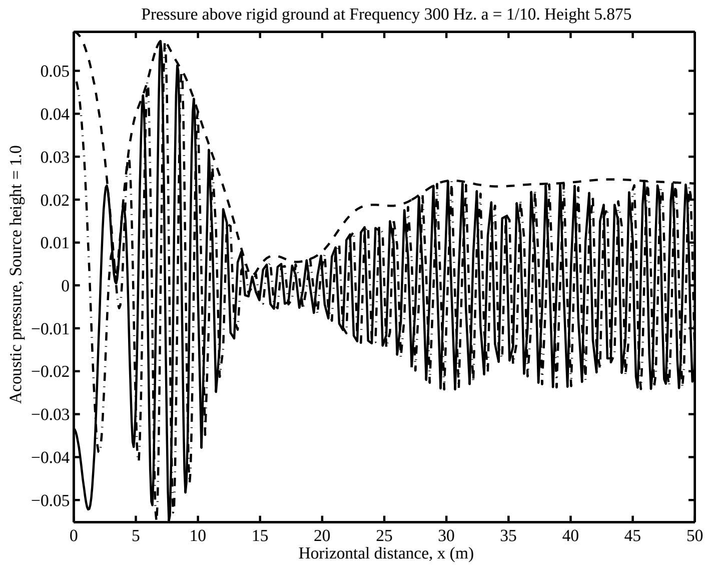

Figure 4: Acoustic pressure in media with profile *V*(z) with *a* = 0*:*1, 5*:*875m above total grassland surface at 300Hz. Real part pressure (solid line), imaginary part (dash–dot), and magnitude (dashed line).

Figure 4 clearly shows a small oscillation in sound pressure level, compared to Figure 3, in the immediate vicinity after destructive interference at 15.0 m due to the sound velocity profile. The inversion or negative lapse implies that the sound waves are bent downward causing oscillations in sound pressure against distance. Figures 4. 5 and 6 clearly show the effect of dissipation due to the the ground surface admittance. At 300Hz a value for the algebraic decay rate of the sound pressure along this level is not straightforward to estimate.

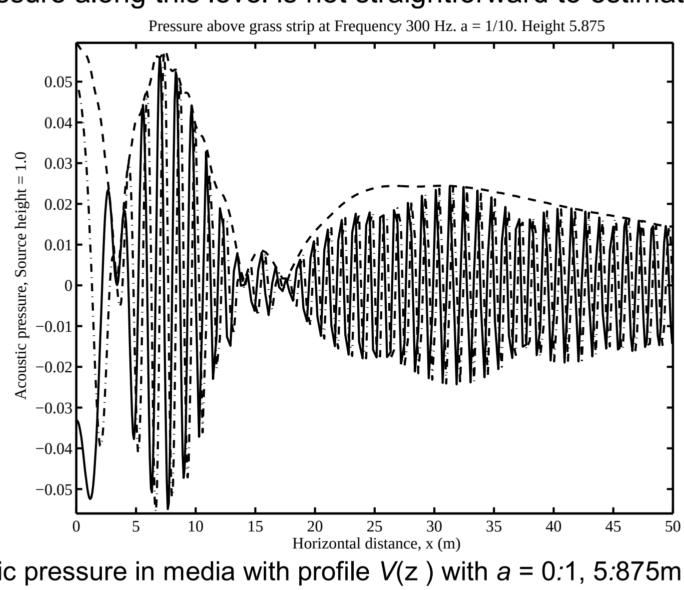

Figure 5: Acoustic pressure in media with profile *V*(z) with *a* = 0*:*1, 5*:*875m above grass–strip surface, 10m · *x* · 45m, at 300Hz. Real part pressure (solid line), imaginary part (dash–dot), and magnitude (dashed line).

However it is clear that the presence of refraction and dissipation are evident for both models which are a total grassland model, $\beta = 0.34 - 0.29i$,Figure 4, and a grass-strip model,

$\beta_b = 0.34 - 0.29i$, $\beta_a = \beta_c = 0$, Figure 5 and 6.. Interference effects due to the impedance discontinuity, as expected, are not observed over grassland Figure 4 but are present in the grass-strip model Figures 5 and 6.

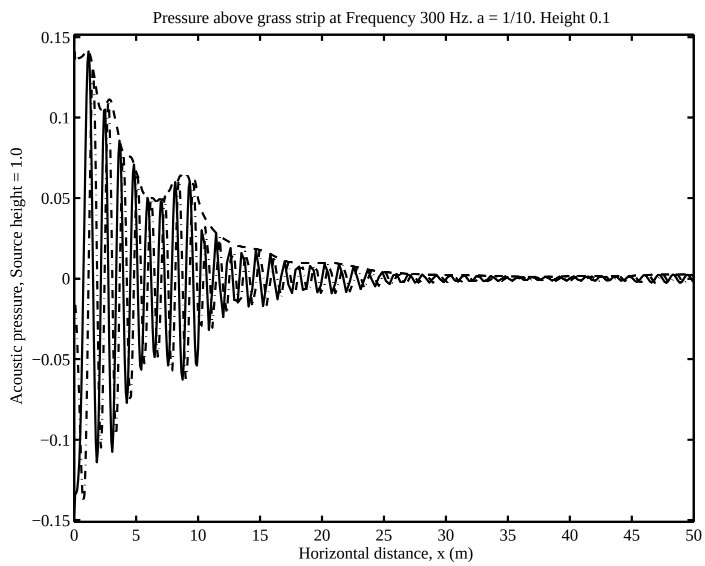

Figure 6: Acoustic pressure in media with profile *V*(z) with *a* = 0*:*1, 0*.1*m above grass–strip surface, 10m- 45m, at 300Hz. Real part pressure (solid line), imaginary part (dash–dot), and magnitude (dashed line).

4. CONCLUSIONS

A general discussion has been given of a Greens function model and an FE model including radiating boundary conditions for the wave equation within a variational framework. The method is demonstrated with a classic example from sound propagation over an embedded grass–strip in an atmospheric profile. Results show some promise in this direction in terms of accuracy in the proximity of the grass–strip discontinuities and asymptotic analysis between the two methods in the future.

ACKNOWLEDGMENTS

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