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COMPUTER COST OF A 3-D NUMERICAL MODEL FOR NOISE BARRIERS INSERTION LOSS

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1. INTRODUCTION

Some simulation models for noise barriers insertion loss are commonly used today. They can be divided mainly in two classes. The first one is constituted of the models based on the ray theory. This kind of modelling, widely used nowadays, provides accurate results only for the case of simple shaped or infinite barriers, flat ground, ... The second class is constituted of models based on integral formulations. These latter models provide good accuracy of the results, and any kind of shape and absorption of barriers and ground can be theoretically described. Models based on integral formulations have been applied to different profile of infinite barriers and some other cases of interest [1], but they require high computer facilities. This is one of the reasons why they have been applied only to 2-D configurations or for very low frequencies. We decided to apply one of these models to 3-D configurations which can deal with barriers of finite length and various shapes of ground. The first step of this study is to precise what are the limits in terms of computer costs, which are most often supposed. The computer code has been implemented using simple numerical schemes. Without improvements, this model leads to high calculation times and it is not possible to deal with the whole traffic noise spectrum on a workstation. We try here to examine the possibility to introduce some improvements in the code, which can be performed by engineers and searchers non specialised in mathematics or computer science. It could be a way to transform the original heavy model to provide informations within the low frequency range of traffic noise. Moreover, some conclusions could be incorporated in a simplified predictive method.

2. THE MODEL

We consider a point source S in the semi-infinite propagation domain Ω (Fig. 1.). The ground and a barrier constitute the boundary σ of the domain Ω. We consider the case of harmonic signals with temporal dependency $\exp(-i\omega t)$, where ω is the circular frequency.

Fig. 1. Notations

We consider also linear acoustics assumptions. The total acoustic pressure p_t in Ω can thus be expressed as the sum of the pressure due to the source S as if it where alone in an infinite medium (p_i) and the pressure (p_a) scattered by the boundary : $p_t = p_i + p_a$.

The problem can be modelled by the system :

$$\begin{cases} \left(\Delta + k^2\right) p_i \left(M\right) = \delta_S \quad , \quad M \in \Omega & (1) \\ \left(\Delta + k^2\right) p_a \left(M\right) = 0 \quad , \quad M \in \Omega & (2) \\ \lim\limits_{M \in \Omega \to P' \in \sigma} \left[\partial_n^r(P') p_t(P) - ik \dfrac{\rho c}{Z_n} p_t(P) \right] = 0 \quad , \quad P' \in \sigma & (3) \\ \text{Sommerfeld conditions for } p_i \text{ and } p_a & (4) \end{cases}$$

where ρ is the density of air, $k = \omega / c$ is the wavenumber, c the speed of sound in the air, Z_n is the normal acoustic impedance of σ, supposed to be locally reacting.

Eq. (1) and (4) for p_i yield to :

$$p_i = -\frac{\exp\left(ikr(M,S)\right)}{4\pi r(M,S)},$$

where M is the receiver point in Ω.

The pressure p_a,solution of eq. (2), (3) and (4), can be expressed in terms of a linear combination of simple and double layer potentials [2]. It is simply written here as a simple layer potential :

$$p_a(M) = \int_\sigma \mu(P) G(M,P) d\Gamma(P) \quad , \quad M \in \Omega \tag{5}$$

where G has the same form than p_i, μ is the layer density, which is an intermediate unknown. The introduction of p_t in the boundary condition (3) leads to a boundary integral equation :

$$\frac{\exp\left(ikr(P',S)\right)}{4\pi r(P',S)}\left[ik\frac{\rho c}{Z_n}-\left(ik-\frac{1}{r(P',S)}\vec{n}(P')\nabla_{P'}r(P',S)\right)\right]$$

$$-\frac{\mu(P')}{2}+\int_\sigma \mu(P)\partial\vec{n}(P')G(P',P)d\Gamma(P)-ik\frac{\rho c}{Z_n}\int_\sigma \mu(P)G(P',P)d\Gamma(P)=0 \quad (6)$$

where $\vec{n}(P')$ is the outward normal at $P'\in\sigma$, and ∇ is the gradient operator taken with respect to P' coordinates .

The resolution of this equation to find μ is performed using a simple classical discrete formulation, based on subdividing the boundary σ into plane rectangular elements [3]. The complex layer density μ is approximated by a piecewise constant function upon these elements, on the center of which a collocation point is located. The boundary equation (6) is written for each collocation point. This leads to a square linear system whose dimension is the number of subdivisions of σ. The resolution of this system provides the complex values defining the function μ for each subdivision of σ. Finally, a discretised form of eq. (5) provides the scattered acoustic pressure, then the total acoustic pressure in Ω.

The algorithm used here starts with inputing datas : in particular, the frequencies of interest, and the locations of source(s) and receiver(s) are to be known. The boundary σ is described by the 3-D geometrical coordinates and the acoustic impedance of each of its subdivisions. Then every element of the matrix corresponding to the linear system is calculated by surface integrations, and the system is solved. Finally the acoustic pressure is calculated performing a new serie of surface integrations.

3. COMPUTING COST

This model is characterised by two kinds of computing cost. In the first part of the code, every element of the linear system matrix has to be calculated, performing surface integrations. This leads to large calculation time. Then the system has to be solved. In the general case, the matrix is not symmetric nor multi-diagonal. The most classical numerical schemes involve keeping the entire matrix in the computer memory.

Memory Size

The first problem comes from the size of the linear system matrix. This size is directly related to the number of subdivisions of the boundary. When the function μ is approximated by a piecewise constant or linear one, the size of each subdivision has to be no larger than the sixth of the acoustic signal wavelength. This numerical convergence criteria has been the result of ancient numerical experiments and is widely used. It means for example that for a harmonic signal of frequency 1 kHz, each dimension of the subdivision has to be no larger than 0.06 m. The table 1 gives some ideas of the size of the matrix we would have to keep in memory space for low frequencies of traffic noise spectrum and some dimensions of ground.

Each complex element need 8 octets of memory when calculations are performed using single precision.

frequency \ dimensions	10 m * 10 m	20 m * 20 m
125 Hz	230 000 / 1.8 Mo	4 000 000 / 32 Mo
250 Hz	3 700 000 / 29.6 Mo	68 000 000 / 544 Mo

Table 1. Minimum number of elements of the matrix / memory size (Mo)

The memory size of a computer can be virtually extended using the hard disc. In this case, a part of the disc is definitely devoted to this task (swap space), no file can be written there, the quantity of files to be saved on the disc is reduced. Moreover, this task consists in often reading and writing on the disc : it is very costly regarding to the calculation time. Finally, it seems that it is not possible to model realistic configurations using a workstation, without improvements of the technique.

Calculation Time

One part of the calculation time corresponds to the calculation of each element of the matrix. The major numerical schemes involved in this step are the numerical integrations. The one used in a first step are combinations of Simpson's rules. The other part of the time is consumed by the linear system resolution. This step is performed by an optimised Gauss-Jordan rule. With these kinds of numerical schemes, it takes more than 30 hours CPU time on a HP 715/75 MHz workstation (RAM : 128 Mo), to calculate the acoustic level at some receiver points with one source upon a 10*12 m ground at a frequency of 250 Hz (3000 subdivisions). In this example, the integration rules are performed with double precision and the memory size is 72 Mo.

4. IMPROVEMENTS POSSIBILITIES

The first version of the computer code can be improved using better numerical schemes. For example, as the function to integrate is regular, a Gauss Quadrature (9 points rule for instance) should be much more efficient than the simple Simpson - combination used. It could also be worth using more appropriate basis functions to obtain sparse matrix for which linear system solving is much easier, with e. g. iterative solutions like conjugate gradient, generalised conjugate residuals ...[4], [5].

These improvements reduce greatly the total calculating time, but it is still not possible to reach the high frequencies of the traffic range because of the size of the linear system matrix. The more important problem is then related to the memory size. For this purpose, a few solutions can be considered. One of them is widely used : the function μ can be approximated by at least a quadratic function. In this case, each dimension of the boundary subdivisions can be extended to the quarter of the wavelength λ of the harmonic signal. The values involved in the Table 1. are then greatly modified, see Table 2.

frequency \ dimensions	10 m * 10 m	20 m * 20 m
125 Hz	1.8 Mo / 0.3 Mo	32 Mo / 4.9 Mo
250 Hz	29.6 Mo / 5.6 Mo	544 Mo / 90 Mo

Table 2. Size of the minimum memory with the $\lambda/6$ criteria / $\lambda/4$ criteria

One can see that it is then possible to apply the model to a realistic part of the traffic noise spectrum with a workstation.

The other solutions are related to approximations of the model itself. On a first step, it could be possible to try to extend the '$\lambda/4$ criteria' for large ground dimensions. It seems physically reasonable to try to use larger subdivisions when it is situated far from the source, the receiver or the barrier. This idea has to be quantified using numerical experiments performed on a large number of well chosen geometrical configurations and frequencies.

Following the same idea, we could go back to the method itself : the acoustic pressure could be modelled by a hybrid formulation. An integral one could be applied to a propagation subdomain close to the source, the receiver(s) and the barrier. The field corresponding to the other infinite subdomain could be evaluated by an asymptotic development for which a set of virtual sources would be located on the boundary between the two subdomains.

5. CONCLUSION

The improvements related to the first version of the computer code are in progress. A large number of simulations are to be performed to validate correctly the results. We can already say that it is possible to consider the low range of the traffic noise spectrum, within which the most important acoustic power is radiated. Moreover, this range of spectrum is the one for which it is difficult to obtain precise results by the ray theory models. The next steps of the study may be worked out in parallel. On the one hand, the advanced improvements of the technique discussed above have to be tested. On the other hand, it is possible to work on some problems of interest : diffraction by the vertical edges of a barrier, array of incoherent sources, and other applications at frequencies beyond 1 kHz. The results could be compared to others models for which only 2-D calculations or low frequencies has been taken into accont.

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