NONWOVENS NEXT TOP MODEL?

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1 INTRODUCTION

Material parameter inversion is a very powerful tool for the acoustic characterisation of a range of materials. It allows for a wide collection of properties to be calculated from a single rapid acoustic measurement with a high degree of accuracy, and can be especially useful for determining the values of parameters that are otherwise difficult or time-consuming to measure. Above and beyond this, acoustic characterisation also allows for some scope into the design and optimization of materials for noise control. By allowing for the understanding of properties such as airflow resistivity, porosity, and tortuosity, one can tailor these properties to maximize the absorption efficiency without needing to spend time or money on the synthesis and prototyping of material samples. There are a wide range of models currently in use that deal with nonwoven media, including commonly utilized methods such as the Bies-Hansen model[1] and the Kozeny-Carman model[2,3], as well as the Miki model[4]. This paper studies the application of these models to the inversion of a variety of different nonwoven samples from acoustic impedance data, so that the key non-acoustical parameters can be related to the material properties.

Acoustic characterisation, performed via a two-microphone sound impedance tube, is a method for rapidly obtaining data on the acoustic absorbance and surface impedance of a material. There are numerous models, both old and new, that can utilise this data for the calculation of other material properties -such as porosity, tortuosity, airflow resistivity, and vice versa.

Airflow resistivity is a parameter that is known to have a considerable impact of the acoustic performance of a material, but in spite of this it can also be difficult to measure. Modelling the value of airflow resistivity can be favourable due to the time - where modelling takes seconds as opposed to minutes, and range of equipment required to measure airflow resistivity independently.

2 MODEL INTRODUCTION

2.1 Kozeny-Carman Model

The Kozeny-Carman equation[2,3] was developed in the 1930s and was used to relate porosity (typically of granular media), ϕ, particle size, d, and flow resistivity, σ, according to Equation 1 below:

$$\sigma = \frac{180\mu(1-\phi)^2}{d^2\phi^3} \tag{1}$$

where μ is the dynamic viscosity, derived from Poiseuille's equation for laminar flow of a liquid, and given a constant value of $1.81\text{x}10^{-5}$ for these calculations. In this experiment, d was set to the fibre diameter, d_f, assuming $d \equiv d_f$. Porosity was calculated from the ratio of bulk material density, ρ_m, to the fibre density, ρ_f, in accordance with the equation $\phi = 1 - \frac{\rho_m}{\rho_f}$.

This model has a physical basis and is hailed as being able to accurately estimate the flow resistivity – typically of polymer fibres, from the fibre density and diameter data.

2.2 Miki Model

The Miki model is an adaption of the Delany-Bazley model[5], modified by Y. Miki in 1989[4]. The modifications were proposed in order for the model to be more accurately applied in a wider frequency range than originally possible within the Delany-Bazley model[4]. In our work, a three parameter variant of the Miki model was used to invert the airflow resistivity; the three parameters are airflow resistivity, σ , tortuosity, α_{00}, and porosity, ϕ. For fibrous media with low densities, such as those used within this experiment, it is assumed that $\alpha_{00} \approx 1$, and that $\phi \approx 1$, meaning that for these samples the model is essentially single-parameter, considering only σ.

Flow resistivity is obtained from this model using a parameter inversion based on finding the minimum value of Equation 2, below:

$$F(x) = \sum_{n=1}^{N} \{z^{exp}(\phi, f_n) - z^{th}(f_n, \boldsymbol{x})\} \to min \tag{2}$$

where $z^{exp}(\phi, f_n)$ is the measured surface impedance spectrum, $z^{th}(f_n, \boldsymbol{x})$ is the predicted surface impedance spectrum, f_n is the frequency of sound between 200 and 1500Hz, $\boldsymbol{x}$ a design vector, which = $\{\sigma, \alpha_{00}\}$ when σ is the airflow resistivity and α_{00} is the tortuosity. As with the Miki model, the minimisation problem was solved using a standard Nelder-Mead algorithm as outlined in reference[6].

2.3 Padé Approximation

The Padé approximation model is a model which accounts for the non-uniformity of pores (NUP) in materials[7]. This NUP is a characteristic commonly found within samples of very high airflow resistivity or tortuosity, and can be a factor in difficulties found with measuring airflow resistivity in some materials.

In this model, the real and imaginary parts of the normalized surface impedance data are utilized in to invert the flow resistivity of the sample. This is achieved by minimizing the difference described by Equation 2.

3 METHODOLOGY

All measurements were performed in the Acoustic Materials Laboratory (Sir Frederick Mappin Building, The University of Sheffield) using a 100mm impedance tube manufactured by Materiacustica[8], which was used in accordance with ISO10534-2[9] to measure both the normalized surface impedance and the acoustic absorption of the sample. Each sample was prepared in triplicate and cut to 100mm diameter circles using an in-house manufactured hole-cutter attached to a Pollard pillar drill. Each specimen was then run in a two microphone set-up, a schematic for which is provided in Figure 1, to generate data on the surface impedance for use in the Miki and Padé approximation models.

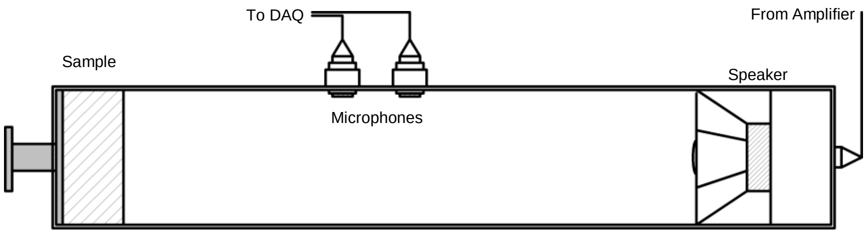

Figure 1: Two-microphone sound impedance tube

The modelling for both Miki and the Padé approximation were completed using MatLab (R.2016), whereas the results for Kozeny-Carman model were completed in Excel (Office 2013). Data regression was completed using a combination of both MatLab (R.2016) and Excel (Office 2013).

Fibre diameter was calculated for the samples by the conversion of denier into µm and then into m; this was done using the following conversion, where d_f is the fibre diameter, d is the denier, and ρ_f is the fibre density.

$$d_f = 11.89 \times \sqrt{\frac{d}{\rho_f}} \tag{3}$$

For samples which had a combination of different fibres with different diameters, the composition was factored in to yield an average fibre diameter for each sample. This value was then substituted into the Kozeny-Carman equation to generate the airflow resistivity data.

The samples used within this experiment were all provided by John Cotton Group Ltd. Their compositions can be seen in Table 1, and Table 2 shows the measured and known properties of the materials used in this experiment. Fibre density values have been provided by John Cotton Group Ltd. A value could not be obtained for the fibre diameter of the samples Rebound Felt or RPC Denim; this is due to the fact that John Cotton Ltd was unable to provide data for the rag and cotton constituents of those samples as they are recovered from recycled materials. Attempts were made to obtain a value for diameter by measuring but the spread of diameters was found to be too great and so they were omitted from this part of the experimental. In the table below, “PE” is polyester, “binder” is 4 denier polyester binder fibre, and “d” equates to denier, “rags” is pulled recycled clothing (primarily cotton).

Material Sample	Composition
Autobloc	25% 4d PE, 55% 6d PE, 20% 6d PE
Memory Fibre 8	28% 4d PE binder, 52% 4d, 20% 1.7d PE
Rebound Felt	10% 4d PE, 75% rags, 12% 15d, 3% 4d
RPC Denim	15% 4d PE, 50% cotton, 17.5% 6d PE, 17.5% binder
PE Sample 3	75% 6.7d PE, 25% binder
PE Sample 8	75% 1.5d PE, 25% binder
PE Sample 10	75% 1.5d PE, 25% binder
WT3950b	40% 1.5d PE, 35% 15d PE, 25% binder

Table 1: Compositions of the samples used; supplied by John Cotton Ltd.

Material Sample	Fibre Diameter (m)	Fibre Density (kg/m^3)	Bulk Density (kg/m^3)	Porosity (ϕ)	Thickness (mm)
Autobloc	2.366×10^{-5}	1380	60.35	0.9563	14.98
Memory Fibre 8	1.883×10^{-5}	1380	32.68	0.9763	21.43
Rebound Felt	/	1380	43.82	0.9682	21.21
RPC Denim	/	1380	27.94	0.9798	20.96
PE Sample 3	2.471×10^{-5}	1380	21.71	0.9843	22.87
PE Sample 8	1.436×10^{-5}	1380	24.68	0.9821	21.15
PE Sample 10	1.436×10^{-5}	1380	38.47	0.9721	21.18
WT3950b	2.374×10^{-5}	1380	17.57	0.9873	26.81

Table 2: Material properties of the John Cotton Ltd samples

4 RESULTS

For the Kozeny-Carman model, the values of fibre diameter in Table 2 were substituted into Equation 1 in Section 2.1, generating the airflow resistivity value. Due to being unable to calculate fibre diameter for the Rebound Felt and RPC Denim samples, no airflow resistivity value could be obtained for those two samples. This is to be considered a downside of the Kozeny-Carman model, as unless properties such as fibre diameter – or any variant of – or fibre density are either known or readily measureable then the Kozeny-Carman model cannot be applied. It is also especially hard to complete any sort of data regression or evaluation on results, to assess accuracy or fit from this model.

Both the Miki and Padé models can be applied without knowledge of material properties such as the fibre diameter or density, and instead only utilize properties which are more simple to obtain, such as thickness. In the case of the both these models, properties just as porosity and tortuosity can be measured or set to a constant value. This makes either of these models an attractive prospect, due to the simplicity involved regarding material characterization in advance of these measurements.

Table 3 below shows the airflow resistivity values obtained for the all three models in this experiment.

	Kozeny-Carman	Miki Model	Padé Model
Material Sample	Flow Resistivity (Pa s/m^2)		
Autobloc	1.27x10^{+4}	3.00x10^{+4}	2.92x10^{+4}
Memory Fibre 8	5.53x10^{+3}	1.44x10^{+4}	1.27x10^{+4}
Rebound Felt	/	1.47x10^{+4}	1.90x10^{+4}
RPC Denim	/	1.96x10^{+4}	1.31x10^{+4}
PE Sample 3	1.38x10^{+3}	4.92x10^{+3}	3.50x10^{+3}
PE Sample 8	5.34x10^{+3}	1.70x10^{+4}	1.59x10^{+4}
PE Sample 10	1.34x10^{+4}	2.47x10^{+4}	2.76x10^{+4}
WT3950b	9.74x10^{+2}	8.79x10^{+3}	5.03x10^{+3}

Table 3: Results from the Kozeny-Carman, Miki and Padé approximation models.

As can be seen from the table, the two models offer values which appear to be somewhat consistent with each other, yet both differ quite substantially from some of the values offered by the Kozeny-Carman equation. For example, Autobloc has only a 2.54% difference between values, but comparing either of these to the Kozeny-Carman result shows a difference of 57.49% and 56.38% for Miki and Padé respectively.

On average, the percentage difference between Miki and Padé approximation models is 19.92%, but the difference between Miki v Kozeny-Carman and Padé v Kozeny-Carman models are 65.73% and 61.91%. Figure 2 shows a visual comparison of the models, highlighting where they agree with each other and where the results most significantly differ.

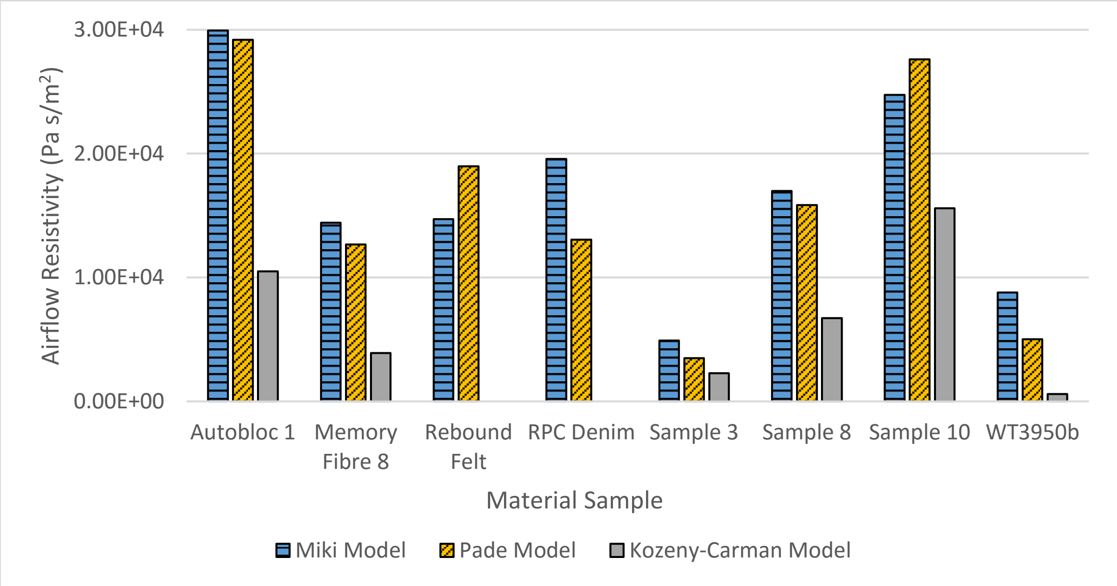

Figure 2: Comparison of the three models results for airflow resistivity.

Table 4 shows the percentage difference between the airflow resistivity values for the two computational models versus the Kozeny-Carman model. The average difference for the between the Kozeny-Carman model and the Miki model is 65.73%, whilst it is 61.91% for Kozeny-Carman and Padé approximation models.

	Miki vs K-C	Padé vs K-C
Sample	Difference (%)	Difference (%)
Autobloc	57.49	56.38
Memory Fibre 8	61.63	56.30
PE Sample 3	71.84	60.24
PE Sample 8	68.57	66.36
PE Sample 10	45.96	51.56
WT3950b	88.92	80.63

Table 4: Percentage difference between the computational models and the Kozeny-Carman (K-C) model for the samples used in this experiment.

For all materials the Padé approximation model has a better fit (lower error) than the Miki model. The average error value across all the samples for the Padé approximation is equal to 3.10%, which is lower than the Miki model, which has an average error of 6.50% for the same materials. The Padé approximation model also exhibits greater precision, with calculated errors ranging from 0.60% to 6.10%, whilst the Miki model's calculated errors ranging from 1.20% to 13.70%. Figures 3 and 4 illustrate the fit of the graphs and the error percentages from Miki and Padé approximation models, taking samples which represented the average error value best. This error is assessed by a root mean squared method, looking at the difference between measured and modelled values.

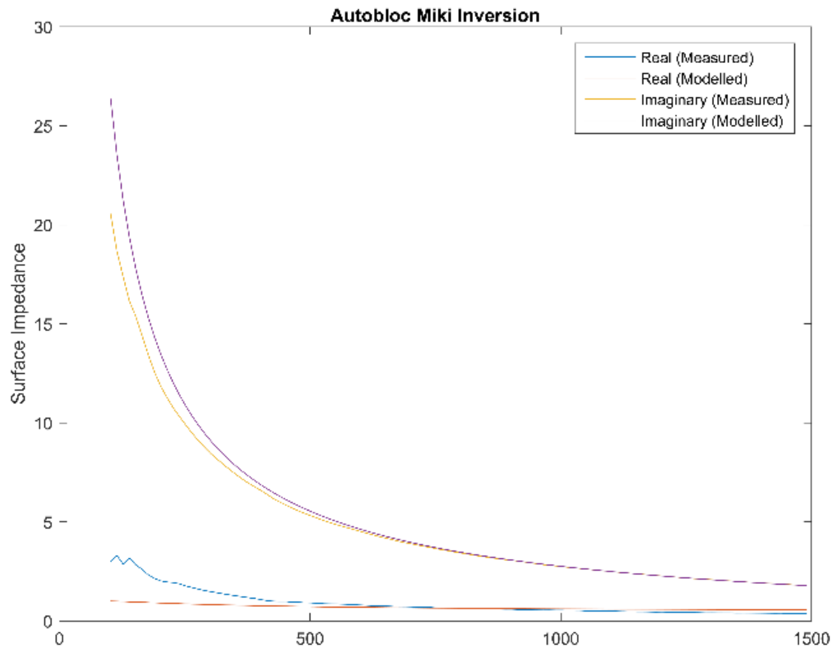

Figure 2: Graph representing average fit for Miki model.

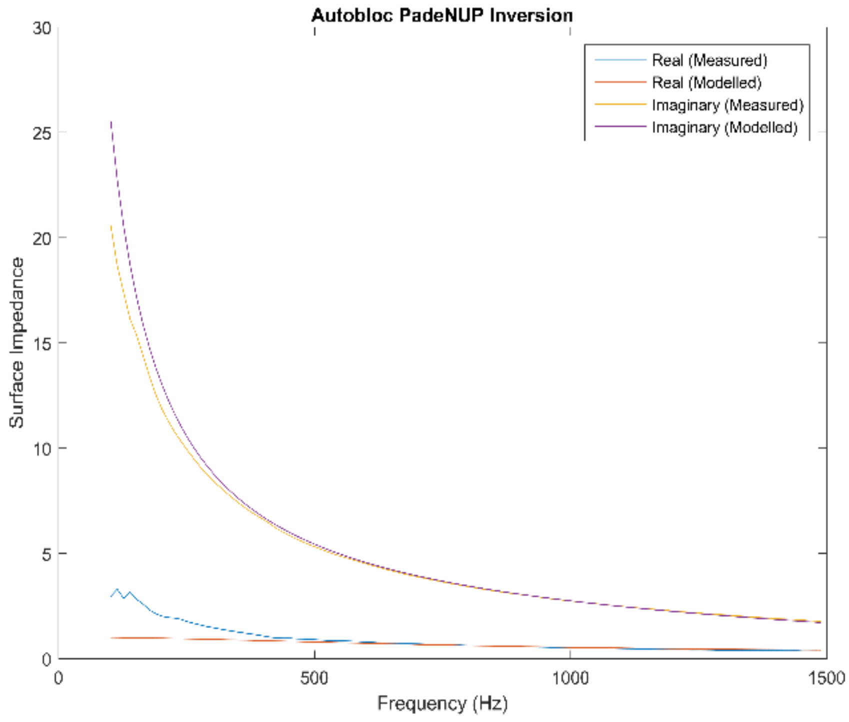

Figure 3: Graph representing average fit for Padé approximation model.

5 CONCLUSION

This experiment has looked at and contrasted the results obtained from two commonly used models within the nonwoven industry and introduced a third, newer, model with the aim of assessing which model is best suited to the prediction of airflow resistivity – and any other parameter attainable through the use of these models – from a combination of data readily obtainable through sound impedance tube testing and basic material properties. It has been found that the two models relying on sound impedance data, the Miki model and the Padé approximation model, generate airflow resistivity values that are more in agreeance with each other. As a result of this, the author believes that this suggests these two models are more reliable than the Kozeny-Carman equation for obtaining airflow resistivity values from polymeric fibrous samples such as those utilized within this experiment. On top of this, it has been observed that the Kozeny-Carman equation requires the knowledge of parameters that cannot always easily be obtained in order to work, which may not always be feasible. The combination of varying fibre diameters within the samples may also have contributed to the errors found within the Kozeny-Carman results, as the model as it stands cannot compensate for that. For samples with only a single diameter, it has been found by Pelegrinis *et al* that the values obtained from the Miki model and the Kozeny-Carman model are in agreement[10] and as such it could be argued that the Kozeny-Carman model is still reliable for that application. However, due to the agreement between the two, the author would still prefer to use either the Miki model or the Padé approximation due to time and ease of use.

When comparing the Miki model and the Padé approximation model, it was found that the Padé approximation model had better fit and generally better agreement between measured and modelled values and so is regarded to be superior for the calculation of airflow resistivity from samples with a range of fibre diameters present. The author surmises that the fact the Padé approximation accounts for a non-uniformity in pore size may be the reason for the lower errors, as a sample with a range of fibre diameters randomly intertwined will likely not have uniform pores present.

6 REFERENCES

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