Proceedings of the Institute of Acoustics

VIBRATION OF COMPLEX FLUID-LOADED STRUCTURES

A J Cooper, D G Crighton

DAMTP, University of Cambridge, Cambridge

1. INTRODUCTION

This paper investigates the response to excitation of fluid-loaded elastic structures supported by a finite array of thin ribs. Numerical calculations have been undertaken when the ribs of the array are arranged in two ways : (1) strictly periodically, (2) randomly displaced from these periodic sites. The response of the structure to excitation at a single rib is characterised by the vertical velocity of the structure along the array. This can be related to the force on the structure at each rib and the Green's function for the unribbed structure. Numerical investigations concentrate on the effect of the form of the Green's function on the membrane response. In general the full Green's function comprises a subsonic surface wave component (G_s) which provides local bay-to-bay coupling, and an acoustic or hydrodynamic component (G_a) which provides more long-range coupling between the source and distant bays. Most studies to date have concentrated on the regime in which G_a can be neglected. This paper addresses the question of what effect the inclusion of the acoustic component, G_a, has on the characteristic response of the structure.

2. PERIODIC STRUCTURES

Consider an elastic membrane supported by a finite array of thin ribs at locations x_m above which lies static compressible fluid. A vacuum is assumed to occupy the region below the structure. The central rib (x_0) is driven by a time-harmonic line force with all other ribs assumed to have infinite mechanical impedance, so that fluid loading provides the only mechanism for the transmission of energy along the array.

The general structural response of the membrane at each rib is expressed as

$$V(x_m) = V_m = \sum_{n=-N}^{N} F_n G(x_m - x_n), \qquad m = -N, \ldots, N. \tag{1}$$

The velocity $V(x_0)$ at the driven rib may be taken as known, and $V(x_m) = 0$ at the other ribs. This defines a system of complex linear equations, or a matrix equation, which can be solved numerically

VIBRATION OF COMPLEX FLUID-LOADED STRUCTURES

to determine the unknown forces, F_n.
The subsonic part of the Green's function is expressed as :

$$G_s(x \neq 0) = A_\infty \exp(i\kappa|x|), \tag{2}$$

where κ is the wavenumber and A_∞ is the transfer admittance.
For the present investigation asymptotic solutions of Crighton [1] are used to describe the acoustic part of the Green's function. Under various regimes of fluid loading G_a can take one of the following forms :

$$G_a(x \neq 0) = \frac{-3iA_\infty}{\pi\kappa^2 x^2}, \tag{3}$$

$$G_a(x \neq 0) \sim \varepsilon|x|^{-\frac{1}{2}} \exp(i\kappa|x|) \qquad 1 \ll x_o \ll \varepsilon^{-2}, \tag{4}$$

$$G_a(x \neq 0) \sim \frac{1}{\varepsilon}|x|^{-\frac{3}{2}} \exp(i\kappa|x|) \qquad x_o \gg \varepsilon^{-2}. \tag{5}$$

where ε is a fluid loading parameter which depends on the material constants of the plate and the loading fluid. $x_o = k_m|x|$ with k_m being the free wavenumber on the membrane in the absence of fluid loading.
In all cases

$$G(x = 0) = A_0 \tag{6}$$

where A_0 is the drive admittance.

When the Green's function takes the form of plane waves (G_s only) it has been shown for infinite arrays (Crighton [1]) that the response of the structure depends critically on the scaled driving frequency ϕ ($\phi = \kappa h$ where h is the rib spacing) and that local coupling induces a frequency pass/stop band structure. For frequencies in pass bands energy is found to propagate along the array without attenuation, whereas at frequencies in the stop-bands the energy becomes exponentially localised around the source.

Crighton [1] obtained, using asymptotic methods for an infinite array of ribs, expressions for the $|F_n|$ as $|n| \to \infty$ in the different regimes of long-range acoustic coupling only. When the Green's function consists of G_a alone there exists no banding structure and algebraic decay of some form is predicted for all frequencies. When $G_a(x) \sim 1/x^2$ it is predicted that $|F_n| \sim (n\phi)^{-2}$ as $|n| \to \infty$. For the other two cases the analytical prediction is that $|F_n| \sim |n|^{-\frac{3}{2}}$ as $|n| \to \infty$.

VIBRATION OF COMPLEX FLUID-LOADED STRUCTURES

Numerical simulations, under the influence of long-range acoustic coupling only, have been able to verify these results including the detailed predictions for the coefficients as well as the decay law. In order to highlight the behaviour at manageable values of N a scaling parameter, λ, was inserted into the Green's function so that $G(x) = \lambda G_a(x)$. In the simulation for G_a given by Eq.(4) a wide range of λ values, both above and below unity, were considered. This can be justified as more than a numerical scaling since λ effectively scales the fluid loading parameter ε which can takes a wide range of values depending on the material of the structure and the surrounding fluid. For large $|n|$ the solution was found to be in agreement with the analytical prediction. However, for small values of λ and/or n numerical simulations indicate that $|F_n| \sim |n|^{-\frac{1}{2}}$ which eventually gives way to the $|n|^{-\frac{3}{2}}$ decay if $|n|$ becomes large enough. The $|n|^{-\frac{1}{2}}$ decay has not yet been identified analytically.

In the fluid-loading regime where $G_a(x) \sim 1/x^2$ the combination of G_s and G_a still gives rise to some sort of banding structure where the behaviour of frequencies in the previously defined pass and stop bands clearly differs. In the stop bands a period of initial exponential decay occurs which is found to follow the decay curve of G_s alone. As progression is made along the array away from the source the acoustic part of the Green's function starts to become dominant. Exponential decay eventually gives way to the slower algebraic decay associated with G_a only, i.e. $|F_n| \sim |n|^{-2}$ at large n and the coefficient of the $|n|^{-2}$ decay is close to that found when G_s is entirely absent.

The behaviour in the pass-bands produces some distinctive results when G_a is included. For central pass-band frequencies where, for G_s alone, energy is transmitted without attenuation, the inclusion of G_a generates a periodic response in the forces along the array. Although there are large fluctuations in $|F_n|$, substantial transmission of energy remains, and in contrast to the stop-band results the algebraic decay associated with G_a never dominates. At off-centre frequencies similar behaviour is observed. The period of the response is found to be independent of the number of ribs but increases as the order of the pass band increases.

The inclusion of G_a in the other fluid loading regimes appears to completely destroy any pass/stop band frequency structure. With G_a given by the expression in Eq.(5) results show a period of exponential decay at *all* frequencies which then develops into $|n|^{-\frac{3}{2}}$ algebraic decay if $|n|$ is large enough.

At present there is no analytical treatment for the pass-band behaviour with G_s and G_a combined.

VIBRATION OF COMPLEX FLUID-LOADED STRUCTURES

3. IRREGULAR STRUCTURES

Although some structures can be defined as having strict periodicity in rib alignment it is often likely that some irregularity occurs as a result of faults in manufacture or because of constraints imposed in the design of structures. When the structure consists of ribs which are displaced randomly from a strict periodic arrangement the response of the structure, if only the subsonic component of the Green's function is included, becomes exponentially localised (Sobnack & Crighton [2]). This phenomenon, known as Anderson localisation, destroys the strict banding structure associated with periodic arrays.

The basic construction of this problem is the same as that described for the periodic array of ribs except that each rib is now displaced by a small random amount, u_m. Since the rib locations are chosen randomly it is usual to gauge the effect of irregularity by averaging over a large number of realisations for the u_m. Thus the matrix equation is solved numerically for a large number of rib configurations and the average of those results is taken to represent the general structural response for a given degree of irregularity. The logarithmic (or geometric) average is used in preference to the arithmetic average because the latter can be heavily weighted by certain realisations which exhibit degrees of symmetry but which only occur very rarely (Hodges & Woodhouse [3],[4]).

The effect of extended disorder on the long-range coupling mechanism (where the Green's function is taken to consist of G_a) only was studied and numerical results indicate that the long-range acoustic coupling mechanism is remarkably insensitive to any degree of irregularity, both on average and for individual realisations.

In combining G_s and G_a for the irregular array two competing mechanisms exist : (i) exponential localisation in pass bands due to irregularity, (ii) slow algebraic decay at all frequencies due to long-range coupling which is insensitive to irregularity.

In the pass band frequency range, calculations show that near to the source of excitation irregularity induces exponential decay as it would for a strictly periodic structure. This, however, ultimately gives way to algebraic decay following the decay law which would occur for G_a alone with or without irregularity. Increasing the order of the pass band generally results in the exponential decay extending over a shorter number of ribs. This kind of behaviour is similar to the stop-band characteristics for the regular array. Stop-band behaviour is found to be only slightly modified by the presence of irregularity. Any modulation arises because extended disorder tends to slightly delocalise the exponential response associated with the G_s component in the stop band.

4. REFERENCES

[1] D G CRIGHTON, 'Transmission of energy down periodically ribbed elastic structures under fluid loading', *Proc. R. Soc. Lond.* A, **394** p405 (1984)

[2] M B SOBNACK & D G CRIGHTON, 'Anderson localization effects in the transmission of energy down an irregularly ribbed fluid-loaded structure', *Proc. R. Soc. Lond.* A, **444** p185 (1994)

[3] C H HODGES & J WOODHOUSE, 'Confinement of vibration by one-dimensional disorder. I. Theory of ensemble averaging', *J. Sound Vib.*, **130** p237 (1989)

[4] C H HODGES & J WOODHOUSE, 'Confinement of vibration by one-dimensional disorder. II. A numerical experiment on different ensemble averages', *J. Sound Vib.*, **130** p253 (1989)