RANGE ESTIMATION USING CONTINUOUS CHAOTIC SIGNALS

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1. INTRODUCTION

Using a pulsed signal, target range is estimated by first finding the travel time to and from the target. Time is measured using the leading edge of the pulse if there is sufficient signal to noise ratio or from the peak in the output of a matched filter if not. Pulsed signals are not always the best option, for example if continuous coverage is to be maintained in surveillance and in communications.

With a continuous signal there is no leading edge to use as a marker. Range can be estimated in an FMCW system, where an FM glide is transmitted repeatedly, but the determination of range is ambiguous. This problem does not arise if an aperiodic signal is used, but a matched filter design is required. This and novel methods of processing are considered in section 3 of this paper. Chaotic signals are of interest because they have unique attributes useful in processing which are covered in the next section.

2. CHAOTIC SIGNALS

A chaotic signal is a time series which is generated by a non-linear process which is sensitive to initial conditions ([2], pp8-11). A consequence is that the oscillations appear noise like. One example is the position of the mass in a non-linear mass-spring system driven sinusoidally (see (1)). If the system is set oscillating from two slightly different rest positions, then for a while the observed positions and velocities of the two oscillations are indistinguishable, but very quickly they diverge until there is no obvious relationship between them. This is shown in fig 1(a). Although the time series is noise- like there is an underlying structure which is revealed in a plot of velocity against position. This is known as a phase portrait. An example is shown in fig 1(b) for the non-linear spring. The corresponding plot for a linear oscillator would give an ellipse.

In spite of sensitivity to initial conditions, it is possible to synchronise two chaotic systems with slightly different parameters ([3], pp299-302). It is also possible to control the oscillations of a chaotic system ([3], pp304-319). Another aspect is that the oscillation can change in character as a system parameter changes. The non-linear spring oscillates periodically for some input amplitudes. It is possible to generate a chaotic signal with specified higher statistics [4].

Analogue chaotic signals may generated by non-linear electric circuits, for example the piece-wise linear Chua circuit referred to in [4]. Digital signals may be generated by sampling an analogue oscillation but also directly using shift register systems with feedback [4], or by iterating a non-

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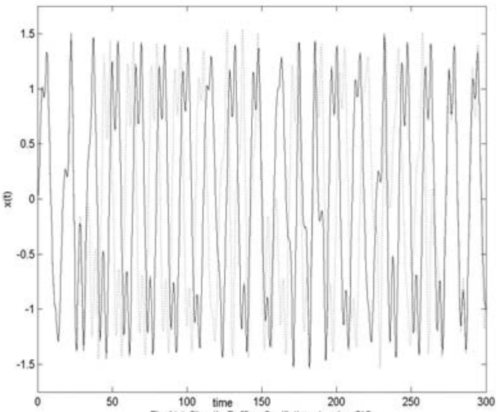

Fig 1(a) Chaotic Duffing Oscillation showing SIC

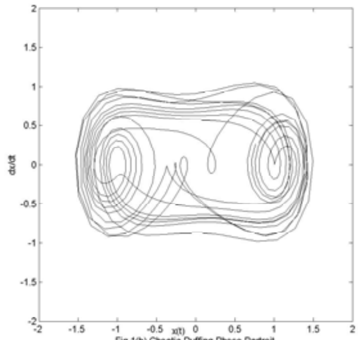

Fig 1(b) Chaotic Duffing Phase Portrait

linear function, for example $x_{n+1} = 4x_n(1 - x_n)$, $0 < x_0 < 1$. Under this definition, pseudo-random noise generated with a deterministic algorithm is chaotic, e.g. maximal M-sequences, or congruential generators, but the phase portraits are more truly noise-like. A chaotic signal may be transmitted directly or as a carrier modulation. It may be analysed in the usual ways. Bauer [4] gives the ambiguity functions for the Chua signal and two digital signals.

2.1 Nonlinear chaotic systems

The non-linear spring referred to is governed by the second order differential equation

$$\ddot{y} + b_d \dot{y} - y\left(1 - y^2\right) = F\cos(\Omega t) . \qquad (1)$$

which is known as a Duffing system. The restoring force is greater for larger displacements and thus (1) models a hardening spring [5]. For $b_d = 0.3$, $F = 0.5$, $\Omega = 1.2$ radians sec^{-1}, and many other combinations of parameters, the solution of (1) is chaotic, but for $F = 0.2$, it is periodic with period $2\pi/\Omega$. The period doubles as F increases. With suitable substitutions ([6], pp461), (1) can be expressed as a first order vector differential equation whose RHS is independent of time. A second chaotic signal generator is the Rossler system given by [2]

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{aligned} \qquad (2)$$

The phase portrait is a curve in three dimensions.

Equation (2) and the modified form of (1) are special cases of

$$\dot{\mathbf{x}} = \mathbf{f(x)} . \qquad (3)$$

The functions $x_i(t)$ are time varying coordinates along the phase portrait in the phase space of the dynamical system. If time is compressed or dilated so that $t' = \alpha t$, then defining $\mathbf{y}(t) = \mathbf{x}(t')$

$$\dot{\mathbf{y}} = \alpha \mathbf{f(y)} \qquad (4)$$

and the shape of the phase portrait is unaltered. The phase space can be reconstructed for most nonlinear dynamical systems using the method of delays ([3],pp35-39) . In signal processing terms, a shift register is filled with samples from an observation of one of the variables. With an appropriately chosen sample interval and shift register length, the contents of the shift register follow a curve which is equivalent to the original phase portrait. This representation can be used to estimate components in the data using linear and non-linear signal processing methods.

3. RANGE ESTIMATION METHODS

3.1 Matched Filtering

A matched filter for a pulse performs a cross-correlation of the pulse with the received data and is implemented digitally as a transversal filter. The signal samples are stored as the weights of a shift register into which the receiver samples are clocked. On each clock cycle, the received data and signal data are multiplied together and summed. If the signal is continuous and aperiodic, the difficulties in the routine application of this principle are clear. The signal must be captured as it is transmitted and stored as shift register weights which must then be frozen in order to provide the reference against which to correlate the received data.

One approach is to regard the signal as a set of pulses, transmitted one after the other, each stored in a separate shift register. At system initiation, when the transmitter is first switched on, the signal is fed into the weights of the first shift register until it is full. The weights are frozen and the delay line is enabled for input. The signal is switched to feed into the weights of the second shift register which fills up and is then enabled for receiver input. This sequence continues and when the last register is switched to receive data, the weights in the first register are replaced one by one with the latest signal samples (see fig 2). A demonstration of this system for three matched filters showing the direct arrival and one echo in noise limited conditions is given in fig 3. The time axis is normalised to the speed of sound.

The length of the individual filters is determined by the signal to noise ratio required for detection and the number of filters is determined by the expected maximum range $R_{\max}$. To demonstrate what is involved, consider noise limited operation. Assuming the signal is sufficiently broadband for the reverberation to be modelled as bandlimited Gaussian white noise, the signal to noise ratio is proportional to the number of taps. The length is limited by the stability of the channel and the speed of the target. The maximum range is the distance at which the signal to noise ratio after processing falls below that required to achieve the required probability of detection.

Detection is degraded by the contribution from the cross-correlation between the direct blast the echo. A method of removing the direct blast is to perform a delay embedding, carry out a principal component analysis and remove the contributions. This is under investigation. The number of processing operations required for the matched filter as presented is proportional to $R^2_{\max}$. Fewer are required if continuous coverage is sacrificed. One set of filters is required for each doppler bin. Other methods of detection and estimation are therefore of interest.

3.2 Higher Order Statistics

The use of higher order statistics is proposed by Bauer [1] for detection since a chaotic signal can be designed to have arbitrary higher order statistics. Specifically, in a truly Gaussian background the odd order moments are asymptotically zero. Skew and kurtosis have been used to demonstrate the effect of a resonant flat plate on a chaotic signal in an ultrasonics experiment [6].

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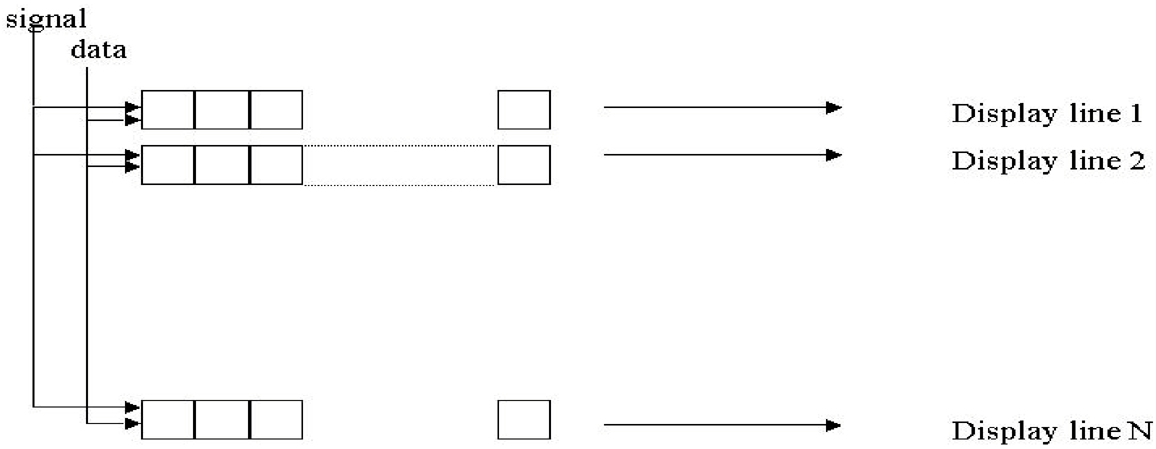

Fig 2(a) Matched Filter for a continuous signal

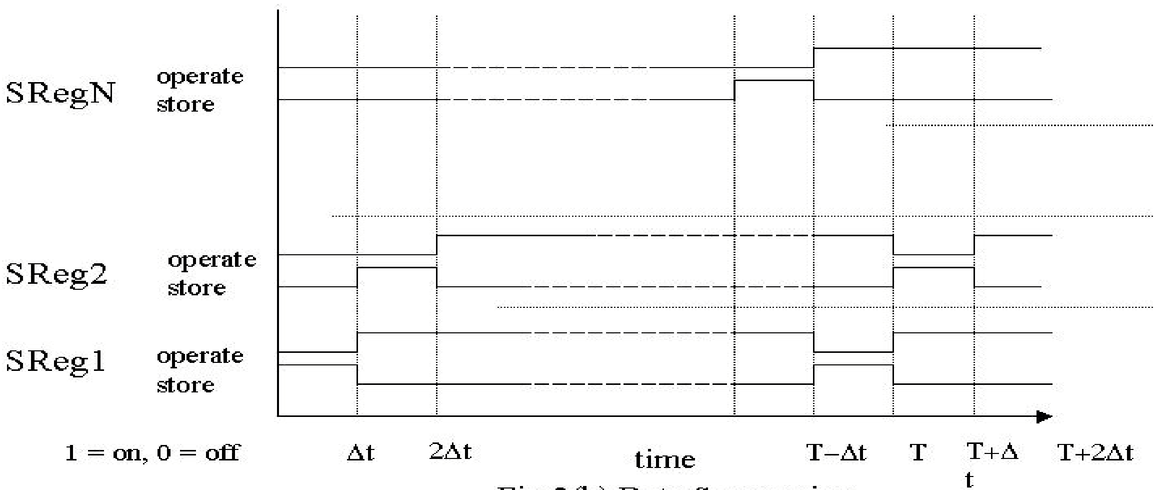

Fig 2(b) Data Sequencing

3.3 Synchronisation

A method discussed in [7], uses the first component of the Rossler system in a processing system in which range is estimated by bringing a prediction for the received signal into synchronism with the actual one. The estimator comprises two identical Rossler systems and a channel model. The signal is the first component of one of the Rossler systems which is used to modulate a carrier on transmission. On reception the signal is demodulated. The output of the second system is filtered using a channel model and the difference between the predicted and actual received signals is found. This difference is added to the input of the second system. The

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outputs of the two systems are compared while the parameters of the channel model are adjusted. When the two are synchronised, the parameters of the channel model are read off.

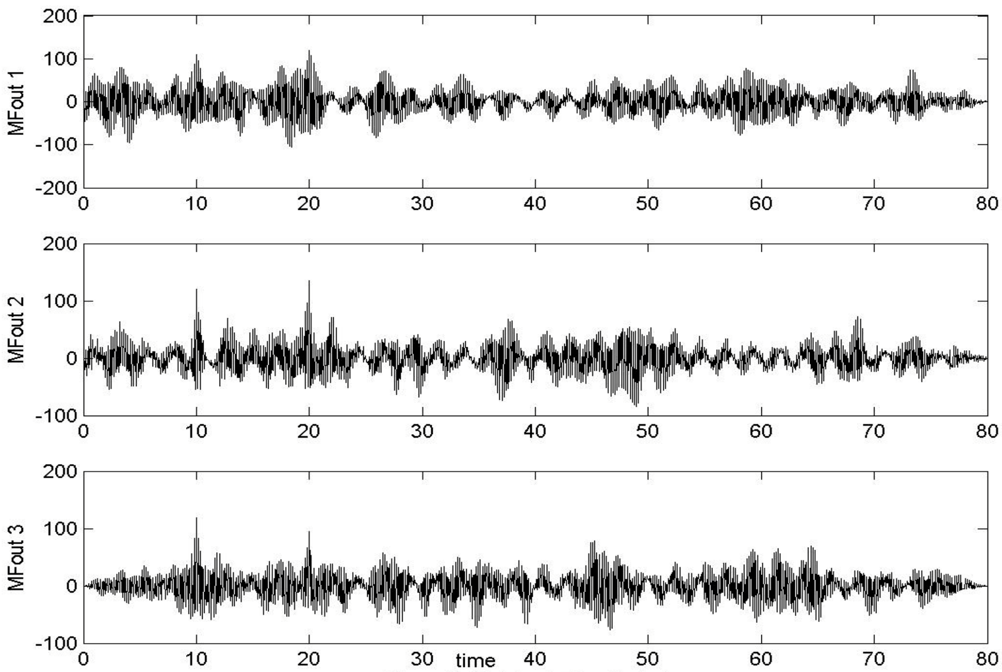

Figure 3: Matched Filter Outputs

Let the signal generator be given by $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$, and let the second be $\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y})$, where ***f*** is given by the RHS of (2). The signal is $x_1(t)$. Let *r(t)* be the received signal and the predicted signal be $\hat{r}(t) = \sum \alpha_i x_1(t-\tau_i)$. When the difference is added to the shadowing system, it becomes $\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}) + \boldsymbol{h}$, where ***h*** is a vector whose only non-zero component is the first term which is given by $\hat{r}(t) - r(t)$. Synchronisation is achieved when $\hat{r} = r$. This is detected from the behaviour of the phase portrait as may seen in fig 4 which shows the behaviour for a source in free space. When synchronised the phase portrait if a single point at the origin, othewise it is chaotic. Ref [8] shows that the method is capable of estimating model parameters when there are multiple arrivals. The invariance of the phase portrait under Doppler may be exploitable.

The effect of noise has not so far been studied. There is no general theory for non-linear systems and it is largely the case that each case must be treated separately. An approach which has had great success is to study a reduced model retaining the essential features of the problem. Often this is a discrete time model which here simplifies the study of the effect of noise.

4. CONCLUSIONS

Chaotic signals may used to estimate range unambiguously using a matched filter scheme. Novel methods using the properties unique to chaotic signals have been proposed but are at an early stage in their development.

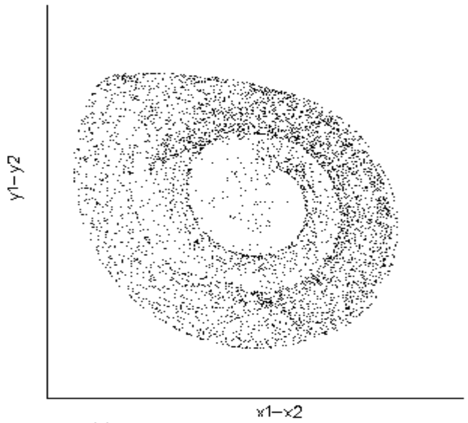

Fig 4(a) Phase Portrait of difference out of synch

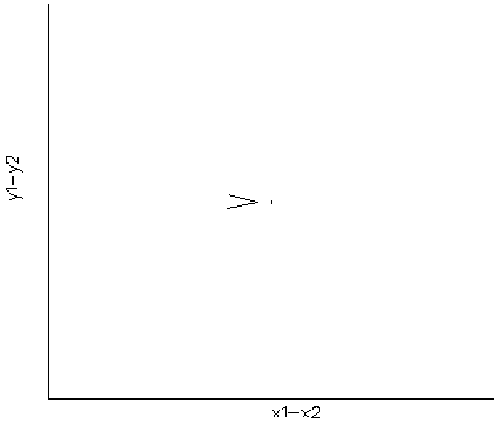

Fig 4(b) Phase Portrait of difference in synch

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ACKNOWLEDGEMENTS

This work was supported by the Sensors & Electronic Warfare Domain of UK MOD's Corporate Research Programme.

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