Proceedings of the Institute of Acoustics

TRANSMISSION OF VIBRATIONAL POWER THROUGH A BEAM JUNCTION

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1. INTRODUCTION

Machinery systems have the potential to generate undesirable noise and vibration to the surrounding environment. When considering the isolation of a vibrating machine it is vital to be able to identify the possible paths for vibration transmission from the structure. Previous investigations [1] have shown that it is important to determine wave types likely to be induced by the source in order to minimise the transmitted vibrational energy. In this paper by using the same concept it is assumed that the system allows vibrational power flow in three different wave types - compressional, torsional and flexural. Power transmission is studied in various configurations and a frequency independent loss factor η is introduced to the model to allow for energy dissipation in the flexible structure.

2. POWER FLOW EXPRESSIONS FOR A BEAM

Consider a section of a uniform beam with longitudinal, torsional and flexural waves in both horizontal and vertical planes, propagating through the beam. It is assumed that the propagating flexural waves can be described by Euler - Bernoulli beam theory. The displacements of these wave types can be represented as for longitudinal, flexural and torsional wave motion respectively.

$$U(x,t) = A_L \sin(\omega t - k_L x)$$
$$W(x,t) = A_f \sin(\omega t - k_f x)$$
$$T(x,t) = A_t \sin(\omega t - k_t x)$$

The shear forces acting on a section of the beam in the horizontal and vertical planes can be expressed as:

$$S_1 = EI\frac{\partial^3 W_{=}}{\partial x^3} \quad ; \quad S_2 = EI\frac{\partial^3 W_{\perp}}{\partial x^3}$$

and the bending moments as

$$B_1 = EI\frac{\partial^2 W_{=}}{\partial x^2} \quad ; \quad B_2 = EI\frac{\partial^2 W_{\perp}}{\partial x^2}$$

For sinusoidal wave motion if the time-averaged power transmission is developed as defined by [1], it is possible to determine the vibrational power in a beam as a function of the travelling wave amplitude due to each wave type. For longitudinal power flow;

$$< P >_L = \frac{1}{2} E A \omega k_L e^{-k_L \eta x} A_L^2 \tag{1}$$

For torsional power flow;

$$< P >_t = \frac{1}{2} G J \omega k_t e^{-k_t \eta x} A_t^2 \tag{2}$$

For flexural power flow;

$$< P >_f = E I \omega k_f^3 e^{-k_f \eta \frac{x}{2}} A_f^2 \tag{3}$$

Here k_L, k_t, k_f, represent the wave numbers for the longitudinal, torsional and flexural waves respectively.

3. VIBRATIONAL POWER TRANSMISSION IN A JOINTED BEAM

Before calculating the transmitted power it is necessary to predict the reflected and transmitted wave amplitudes from a discontinuity. Figure 1 shows a bent beam through an angle φ. The loads at the end of each beam consist of two transverse forces P_1, P_2, an axial force Q_1, a torsional moment T_1 and two bending moments M_1 and M_2. The force P_1 and the moment M_1 are associated with the horizontal plane of the structure. Similarly the transverse force P_2 and the bending moment M_2 relate to the vertical plane. The amplitudes of the reflected and transmitted waves are found by solving sets of simultaneous equations derived from the continuity and equilibrium conditions at the joint. The displacements for the wave types considered for the first arm are:

$$U_1(x,t) = (A_a e^{ik_{L_1}x} + A_b e^{-ik_{L_1}x}) e^{i\omega t} \tag{4}$$

$$T_1(x,t) = (A_c e^{ik_{t_1}x} + A_d e^{-ik_{t_1}x}) e^{i\omega t} \tag{5}$$

$$W_{\parallel 1}(x,t) = (A_1 e^{k_{f_1}x} + A_2 e^{-k_{f_1}x} + A_3 e^{ik_{f_1}x} + A_4 e^{-ik_{f_1}x}) e^{i\omega t} \tag{6}$$

$$W_{\perp 1}(x,t) = (A_5 e^{k_{f_1}x} + A_6 e^{-k_{f_1}x} + A_7 e^{ik_{f_1}x} + A_8 e^{-ik_{f_1}x}) e^{i\omega t} \tag{7}$$

Similarly for the second arm

$$U_2(x,t) = (B_a e^{ik_{L_2}y} + B_b e^{-ik_{L_2}y}) e^{i\omega t} \tag{8}$$

$$T_2(x,t) = (B_c e^{ik_{t_2}y} + B_d e^{-ik_{t_2}y}) e^{i\omega t} \tag{9}$$

$$W_{=2}(x,t) = (B_1 e^{k_{f2}\psi} + B_2 e^{-k_{f2}\psi} + B_3 e^{ik_{f2}\psi} + B_4 e^{-ik_{f2}\psi}) e^{i\omega t} \tag{10}$$

$$W_{\perp 2}(x,t) = (B_5 e^{k_{f2}\psi} + B_6 e^{k_{f2}\psi} + B_7 e^{k_{f2}\psi} + B_8 e^{k_{f2}\psi}) e^{i\omega t} \tag{11}$$

where $\psi = x\cos\varphi$.

In the above expressions A_3, A_4, B_3, B_4, are travelling flexural wave amplitudes and A_a, A_b, B_a, B_b are travelling longitudinal wave amplitudes, both in the plane of the structure. A_7, A_8, B_7, B_8 are flexural travelling wave amplitudes in the vertical plane and A_c, A_d, B_c, B_d are the torsional travelling components. The near field variables denoted by A_1, A_2, B_1, B_2, for the flexural waves in the horizontal plane and A_5, A_6, B_5, B_6, for the flexural waves in the vertical plane. For the joint, the approach in [2] has been adopted which represents the joint by a rigid mass. In this investigation the rigid mass is assumed to be a section of a cylinder, the mass of which is denoted by M_j and its moment of inertia by I_j. In [2] has also been shown that the effect of the joint mass on the reflected and transmitted power is insignificant for the frequency range used in this work. The boundary conditions at the joint consist of continuity of the displacements and equilibrium of moments and forces along the three coordinate axes.

The coupling of the wave motions is separated in two planes and a compressional wave can only couple to flexural waves in the horizontal plane. Similarly a torsional wave can couple only to flexural waves in the vertical plane. By considering the boundary conditions of the beam, the following expressions may be written:

at $x = -l$

$$E_1 I_1 \frac{\partial^2 W_{=1}}{\partial x^2} = M_1 e^{i\omega t} \quad ; \quad E_1 I_1 \frac{\partial^2 W_{\perp 1}}{\partial x^2} = M_1' e^{i\omega t} \quad ; \quad E_1 I_1 \frac{\partial^3 W_{=1}}{\partial x^3} = P_1 e^{i\omega t} \tag{12}$$

$$E_1 I_1 \frac{\partial^3 W_{\perp 1}}{\partial x^3} = P_1' e^{i\omega t} \quad ; \quad E_1 I_1 \frac{\partial U_1}{\partial x} = -Q_1 e^{i\omega t} \quad ; \quad G_1 J_1 \frac{\partial T_1}{\partial x} = T_1 e^{i\omega t} \tag{13}$$

at $x = 0$

$$U_1 = -W_{=2} \sin\varphi + U_2 \cos\varphi + \frac{L}{2} \frac{\partial W_{=2}}{\partial \psi} \sin\varphi \tag{14}$$

$$W_{=1} = W_{=2} . \cos\varphi + U_2 \sin\varphi - \frac{L}{2} \frac{\partial W_{=2}}{\partial \psi} (\cos + 1) \tag{15}$$

$$W_{\perp 1} = W_{\perp 2} \tag{16}$$

$$E_1 I_1 \frac{\partial^2 W_{=1}}{\partial x^2} + \frac{L}{2} E_1 I_1 \frac{\partial^3 W_{=1}}{\partial x^3} = E_2 I_2 \left(\frac{\partial^2 W_{=2}}{\partial \psi^2} - \frac{L}{2} \frac{\partial^3 W_{=2}}{\partial \psi^3} \right) - I_j \frac{\partial^2 W_{=1}}{\partial x^2} \quad (17)$$

$$E_1 I_1 \frac{\partial^2 W_{\perp 1}}{\partial x^2} + \frac{L}{2} E_1 I_1 \frac{\partial^3 W_{\perp 1}}{\partial x^3} = E_2 I_2 \frac{\partial^2 W_{\perp 2}}{\partial \psi^2} \cos\varphi \, \frac{L}{2} E_2 I_2 \frac{\partial^3 W_{\perp 1}}{\partial \psi^3} \cos\varphi +$$

$$- G_2 J_2 \frac{\partial T_2}{\partial \psi} \sin\varphi - I_j \frac{\partial W_{\perp 1}}{\partial x^2} \quad (18)$$

$$G_1 J_1 \frac{\partial T_1}{\partial x} = E_2 I_2 \frac{\partial^2 W_{\perp 2}}{\partial \psi^2} \sin\varphi + \frac{L}{2} E_2 I_2 \frac{\partial^3 W_{\perp 2}}{\partial \psi^3} \sin\varphi - G_2 J_2 \frac{\partial T_2}{\partial \psi} \cos\varphi - I_j \frac{\partial T_1}{\partial x} \quad (19)$$

$$E_1 S_1 \frac{\partial U_1}{\partial x} = E_2 S_2 \frac{\partial U_2}{\partial \psi} \cos\varphi + E_2 I_2 \frac{\partial^3 W_{=2}}{\partial \psi^3} \sin\varphi - M_j \frac{\partial^2 U_1}{\partial t^2} \quad (20)$$

$$E_1 I_1 \frac{\partial^3 W_{=1}}{\partial x^3} = E_2 S_2 \frac{\partial U_2}{\partial \psi} \sin\varphi - E_2 I_2 \frac{\partial^3 W_{=2}}{\partial \psi^3} \cos\varphi - M_j \frac{\partial^2}{\partial t^2} \left[W_{=1} - \frac{L}{2} \frac{\partial W_{=1}}{\partial x} \right] \quad (21)$$

$$E_1 I_1 \frac{\partial^3 W_{\perp 1}}{\partial x^3} = E_2 I_2 \frac{\partial^3 W_{\perp 2}}{\partial \psi^3} - M_j \frac{L}{2} \frac{\partial^2 W}{\partial x^2} \quad (22)$$

$$T_1 = T_2 \cos\varphi - \frac{\partial W_{\perp 2}}{\partial \psi} \sin\varphi \quad (23)$$

$$\frac{\partial W_{\perp 1}}{\partial x} = T_2 \sin\varphi - \frac{\partial W_{\perp 2}}{\partial \psi} \cos\varphi \quad (24)$$

$$\frac{\partial W_{=1}}{\partial x} = \frac{\partial W_{=2}}{\partial \psi} \quad (25)$$

at x = l

$$E_2I_2\frac{\partial^2 W_{=2}}{\partial\psi^2} = M_2 e^{i\omega t} \quad ; \quad E_2I_2\frac{\partial^2 W_{\perp 2}}{\partial\psi^2} = M_2' e^{i\omega t} \quad ; \quad E_2I_2\frac{\partial^3 W_{=2}}{\partial\psi^3} = P_2 e^{i\omega t} \tag{26}$$

$$E_2I_2\frac{\partial^3 W_{\perp 2}}{\partial\psi^3} = P_2' e^{i\omega t} \quad ; \quad E_2S_2\frac{\partial U_2}{\partial\psi} = Q_2 e^{i\omega t} \quad ; \quad G_2J_2\frac{\partial T_2}{\partial\psi} = T_2 e^{i\omega t} \tag{27}$$

where the joint mass $M_j = p_j\pi L^2 J_w/4$ and the moment of inertia of the joint is $I_j=M_jL^2/8$. Substituting the wave equations (4) - (11) into the above system of equations determines a set of twenty four simultaneous equations. The desirable wave amplitudes can be found by solving the set of simultaneous equations. Figures 2-9 show the results in the form of normalised nett power due to each wave type.

Nett power is defined as the power flowing from the left to right minus the power flowing from right to left. Results in figures 2-9 are presented for beams 1m long with a cross sectional area of 50x6mm constructed from mild steel. The system can be excited by an incoming wave of any of the four types described so far. Two harmonic transverse forces P_1 and P_2 of 1N each at 500Hz are applied at the left arm of the system. The angle of attachment varies between 0 and 90 degrees and the results are normalised with respect to the input power.

From the figures it may be observed that the power in the second beam contains the highest values in the system. The angle of attachment has a small effect on the nett power generated by flexural waves in both the horizontal and vertical planes. The position of interest influences less the longitudinal than the flexural power. Coupling of flexural waves with displacements normal to the system plane with the torsional waves proved a significant source of time averaged transmitted power in the system. The set of the twenty four simultaneous equations has been solved by using matrix inversion techniques. As the equations were complex the system matrix was rewritten in the form

$$a + ib = \begin{bmatrix} a & -b \\ b^T & a \end{bmatrix}$$

and inverted by using the Singular Value Decomposition algorithm. The singularity of the system matrix improved significantly by including the joint characteristics in the equations.

The power transmission through a beam junction has been investigated by considering the conditions of equilibrium and continuity at a joint. It is possible to apply translational, rotational and compressional loads to the system by introducing four wave types propagating along the beams. The contribution of the flexural power in both the horizontal and vertical planes is small at extreme angles, 0 and 90 degrees. The longitudinal and torsional contributions to the power transmission are minimised at angles approaching 90 degrees.

REFERENCES

[1] J L Horner, 'Prediction of vibrational power transmission in frameworks', *Proceedings of the Fourth International Congress on Intensity Techniques, Senlis, France,* p239-246 (1993).

[2] J F Doyle, S Kamle, 'An experimental study of the reflection and transmission of flexural waves at an arbitrary T-joint', *Journal of Applied Mechanics,* 54 p136-140 (1987).

[3] B M Gibbs & J D Tatttersall, 'Vibrational energy transmission and mode conversion at a corner junction of square section rods', *Journal of Vibration Acoustics Stress and Reliability in Design,* 109 p348-355 (1987).

APPENDIX 1 - NOTATION

S	- Cross sectional area	L	- Joint length
A_f	- Amplitude of flexural wave	M_j	- Joint mass
A_L	- Amplitude of Longitudinal wave	M	- Moment force in the horizontal plane
B	- Bending moment	M'	- Moment force in the vertical plane
E	- Young's modulus	P	- Transverse force in the horizontal plane
I	- Moment of inertia	P'	- Transverse force in the vertical plane
I_j	- Moment of inertia of joint	η	- Material loss factor
J_w	- Joint width	x	- Distance
k_f	- Flexural wave number	t	- Time
k_L	- Longitudinal wave number	S	- Shear force
k_t	- Torsional wave number	Q	- Axial force

$W_{=}$ - Displacement due to flexural wave motion in the horizontal plane
$W_{\perp}$ - Displacement due to flexural wave motion in the vertical plane
U - Displacement due to longitudinal wave motion
T - Displacement due to torsional wave motion
$\langle P \rangle_f$ - Time averaged flexural power
$\langle P \rangle_L$ - Time averaged longitudinal power
$\langle P \rangle_t$ - Time averaged torsional power
φ - Angle of arm 2
w - Frequency (rad/s)
ρ_j - Joint density

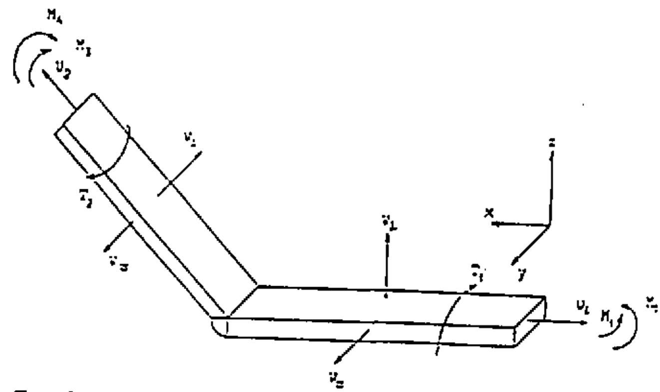

Fig.1 Coordinate system and modes of vibration in a bent beam

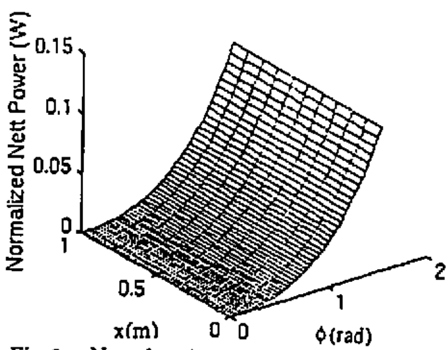

Fig.2 Nett longitudinal power Arm 1

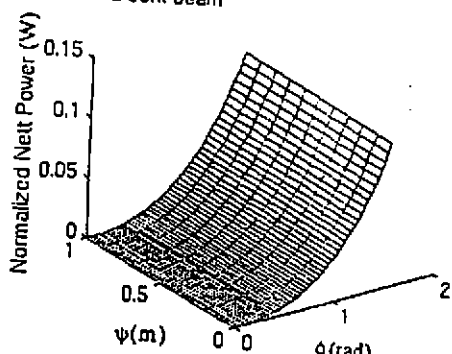

Fig.3 Nett longitudinal power Arm 2

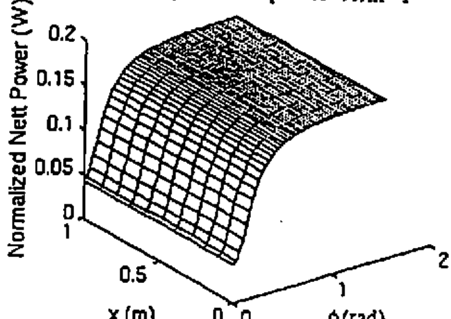

Fig.4 Nett flexural power in the horizontal plane Arm 1

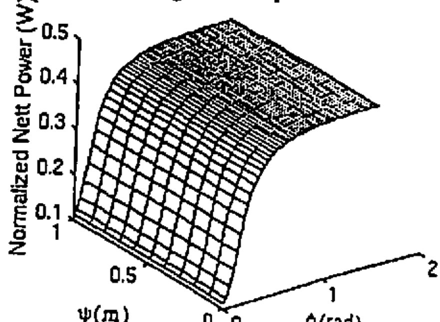

Fig.5 Nett flexural power in the horizontal plane Arm 2

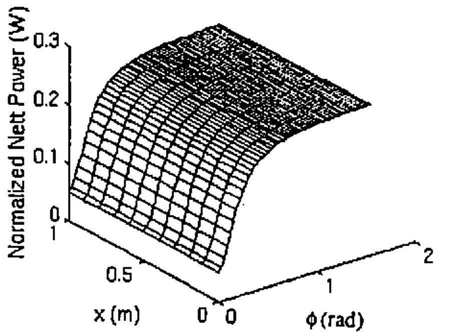

Fig. 6 Nett flexural power in the vertical plane Arm 1

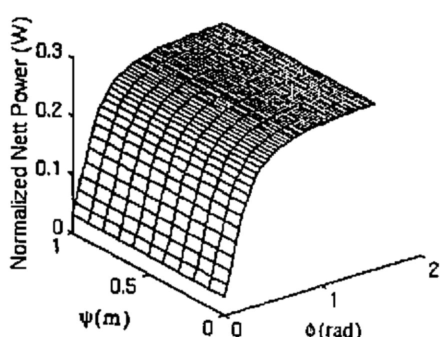

Fig. 7 Nett flexural power in the vertical plane Arm 2

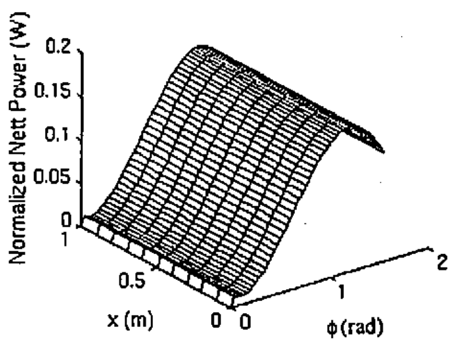

Fig. 8 Nett torsional power Arm 1

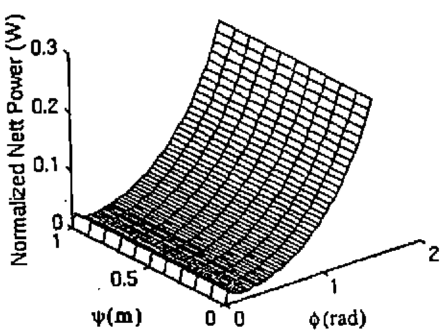

Fig. 9 Nett torsional power Arm 2