SPATIAL CORRELATION OF NOISE NEAR MARINE HYDROCARBON SEEPS

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1 INTRODUCTION

Natural hydrocarbon seeps are found in varying intensity along most continental shelves[1]. These seeps emit gas, oil, or a mixture of both from seafloor vents. Free bubbles rise from the sea bed into the water columns and form a plume. Bubble plumes are registered by standard shipboard sonar as hydroacoustic anomalies in back scattering[2,3]. Few published quantitative observations of the bubble emission size distribution for hydrocarbon seeps exist. This research seeks to provide the necessary theoretical background to solve inverse problem of evaluation of parameters of gas vents. A passive acoustic technique for diagnostics of gas plume is suggested. A rising bubble plume forms an effective acoustic waveguide that possesses normal modes. The “birthing wails” of the bubbles as they depart from the vent is accompanied by generation of broadband noise[4]. The noise frequency spectrum has several peaks related to the lowest-mode frequencies of the bubble plume. We derive a general expression for the cross spectral density based on the Green’s function for the problem. This allows us to gain some physical insight into how the noise is spatially distributed near the seeps.

2 MODEL

As gas is entrained in liquid, sound is produced. The bubbles are transported into the host fluid upwards by buoyancy and laterally by turbulent diffusion (turbulence is generated by bubbles’ wakes) and currents. Geophysical bubble models have used the two-phase Euler/Lagrange approach with imposed fluid velocity structures unaffected by bubble motions[5]. The net effect of these transport mechanisms is to create a bubble plume which can demonstrate the great variability in the shape. On average it is shaped like a cylinder. The echogram of a typical plume registered by ship echo-sounder with working frequency 12 kHz is shown in figure 1. To model the shape of the plume we consider a circular column of bubbly liquid of radius R_c and length H shown in . While this model reduces the number of spatial variables to be considered and substantially simplifies the analysis, it clearly fails to match the actual geometry of the plume in details. The error introduced thereby is important in certain parameter ranges and will by analysed later.

Detection of acoustic signals in the ocean is always performed against a noise background. One of the major components of the ambient noise field is produced by the action of the wind at the surface. On the other hand the “birthing wails” of the bubbles[4] as they depart from the vent is accompanied by generation of broadband noise. While individual bubble resonance is usually responsible for the signals above 1 kHz, bubbles may also oscillate collectively to produce noise at much lower frequencies. These collective oscillations of the bubbles within the plume can be interpreted as that the two-phase bubbly medium behaves as a resonant, cylinder cavity. The dimensions of the column and the bubble void fraction control the resonant frequencies of the plume. The methane seeps thus create vertical acoustic waveguide that posses normal modes. The noise frequency spectrum has several peaks related to the lowest-mode frequencies of the bubble plume. A passive method for diagnostics of gas vents can be suggested. to predict seep intensity on the base of the measured sound spectral density and the solution of the Helmholtz equation.

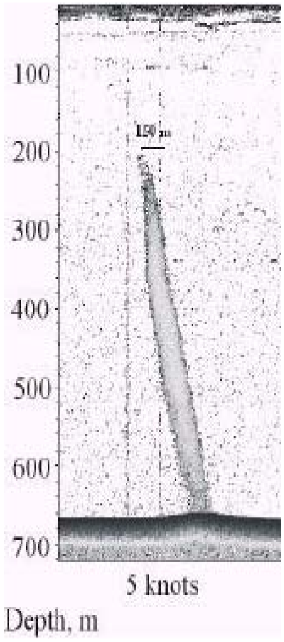

Figure 1. Echogram of gas plume on the move of RV, shelf of Sakhalin[3].

Since the problem is linear, we can separate the solution in two parts. First, we assume that the system is forced by an imposed point source of pressure field of unit amplitude and time dependence $\exp(-i\omega t)$. When column of bubbles exist in fluids, there are two regions of interest; the bubbly mixture and the pure liquid. Upon separation of the time variable, the disturbance pressure field in the pure liquids satisfies the Helmholtz equation with the wave number

$$k^2(\rho) = \omega^2/c^2, \; R_c \le \rho < \infty \,.$$

The disturbance of pressure field in the bubbly mixture satisfies the Helmholtz equation with the wave number

$$k^2(\rho) = \omega^2/c_m^2, \qquad 0 \le \rho \le R_c \,,$$

$$\omega^2/c_m^2 = \omega^2/c^2 + 4\pi\omega^2 \int \frac{g(R)RdR}{\Omega_0^2(R) - \omega^2 + 2i\delta\omega} \quad ,$$

where c is the speed of sound in the fluid, R is the bubble radius, $\Omega_0(R)$ and δ are the natural frequency and damping parameter of the individual bubbles, $g(R)$ is the bubble size distribution.

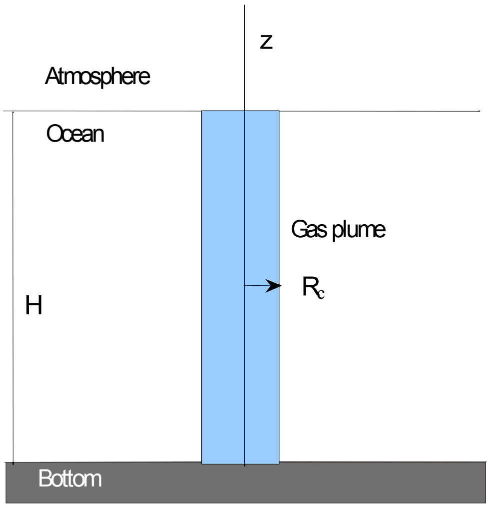

Figure 2. Geometry of a model gas plume: a circular column of bubbly liquid of radius R_c and length *H*.

Assuming that the resonance frequencies of individual bubbles are well above the eigenfrequencies of the plume, the bubbly medium may be treated as a two-phase continuum, in which case the sound speed c_m is related to the void fraction, β, through Wood’s equation

$$\frac{1}{c_m^2} = \frac{1}{c^2} + \frac{\beta\rho_0}{\gamma P}, \quad \beta = \frac{4\pi}{3}\int g(R)R^3 dR \,,$$

here *P* is the hydrostatic pressure and ρ_0 is the water density, γ is the polytropic index of the gas in the bubble. For the acoustic analysis the sound speed distribution in the plume is required. The hydrostatic pressure $P = P_0 + \rho_0 gz = P_0(1 + z/h)$ growths linear with the depth, here g is the gravitational constant, z is the water depth (positive with increasing depth) and h is the characteristic depth where hydrostatic pressure is doubled ($h \approx 10$ m). As the bubble rise, its radius changes due to mass flux and decrease in the hydrostatic pressure. As a result, variation in void fraction has a complicated character[5,6] , but in general, it decreases with approach to the surface. For depths smaller than one hundred meters the model of constant sound speed in the plume is crude but a reasonable one as it accounts for the compensation of decrease in the hydrostatic pressure from the bottom to the surface by decreasing in the void fraction.

If the void fraction near the surface exceeds $\beta > 10^{-4}$ than the effective sound speed in the plume is sufficiently smaller than sound speed in pure water and the expression for c_m reduces to the approximate form $c_m \approx (\gamma P / \beta\rho_0)^{1/2}$. For $\beta \approx 0.01$ the sound speed ($c_m \approx 100$ m/s) is one of the order of magnitude smaller than sound speed in water. This is a realistic case as for the world's most spectacular marine hydrocarbon seeps (Coal Oil Point, Santa Barbara Channel, California) where seep gas is continually collected and piped onshore, $\beta \approx 0.05$ [7].

3 THEORY

The Green's function of the problem, satisfies the Helmholtz equation

$$\Delta G(\omega;\mathbf{r}\,|\,\mathbf{r}') + k^2(\rho)G(\omega;\mathbf{r}\,|\,\mathbf{r}') = -4\pi\delta(\mathbf{r}-\mathbf{r}') \tag{1}$$

and the appropriate boundary conditions. For the free surface (pressure release boundary) and acoustically rigid bottom we have

$$G(\omega;\mathbf{r}\,|\,\mathbf{r}') = \sum_{n=1}^{\infty}\sum_{m=-\infty}^{+\infty} G_{nm}(\rho\,|\,\rho')\frac{\sin\left[\frac{\pi z}{H}(n-1/2)\right]\sin\left[\frac{\pi z'}{H}(n-1/2)\right]}{\pi H}e^{im(\varphi-\varphi')},$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}G_{nm}(\omega;\rho\,|\,\rho')\right) + \left[k^2(\rho) - \frac{m^2}{\rho^2} - \left(\frac{\pi}{H}\right)^2(n-1/2)^2\right]G_{nm}(\omega;\rho\,|\,\rho') = -\frac{4\pi}{\rho}\delta(\rho-\rho'), \tag{2}$$

here we used polar coordinates (ρ,φ,z). The explicit form of the radial part of the Green's function $G_{nm}(\omega;\rho\,|\,\rho')$ is given by the following formulae

$$G_{nm}(\rho\,|\,\rho') = 2\pi\begin{cases} J_m(\kappa_n\rho)\left[i\pi H_m^{(1)}(\kappa_n\rho') - d_1(n,m)J_m(\kappa_n\rho')\right] & \rho < \rho' < R_c, \\ \left[i\pi H_m^{(1)}(\kappa_n\rho) - d_1(n,m)J_m(\kappa_n\rho)\right]J_m(\kappa_n\rho') & \rho' < \rho < R_c, \\ (-\pi / d_2(n,m))J_m(\kappa_n\rho)K_m(k_n\rho') & \rho < R_c < \rho', \\ (-\pi / d_2(n,m))K_m(k_n\rho)J_m(\kappa_n\rho') & \rho' < R_c < \rho, \\ 2\left[I_m(k_n\rho) - c_1(n,m)K_m(k_n\rho)\right]K_m(k_n\rho') & R_c < \rho < \rho', \\ K_m(k_n\rho)2\left[I_m(k_n\rho') - c_1(n,m)K_m(k_n\rho')\right] & R_c < \rho' < \rho; \end{cases}$$

$$d_1 = \frac{\left[K_m(k_nR_c)i\pi H_m^{(1)\prime}(\kappa_nR_c) - K_m'(k_nR_c)i\pi H_m^{(1)}(\kappa_nR_c)\right]}{\left[K_m(k_nR_c)J_m'(\kappa_nR_c) - K_m'(k_nR_c)J_m'(\kappa_nR_c)\right]},$$

$$d_2 = -(\pi R_c/2)\Big[K_m(k_nR_c)J_m{}'(\kappa_nR_c) - K_m{}'(k_nR_c)J_m(\kappa_nR_c)\Big] \quad ,$$

$$c_1 = \frac{\Big[I_m(k_nR_c)J_m{}'(\kappa_nR_c) - I_m{}'(k_nR_c)J_m(\kappa_nR_c)\Big]}{\Big[K_m(k_nR_c)J_m{}'(\kappa_nR_c) - K_m{}'(k_nR_c)J_m(\kappa_nR_c)\Big]} \quad ,$$

$$k_n^2 = \left(\frac{\pi}{H}\right)^2 (n-1/2)^2 - \frac{\omega^2}{c^2}, \quad \kappa_n^2 = \frac{\omega^2}{c_m^2} - \left(\frac{\pi}{H}\right)^2 (n-1/2)^2, \tag{3}$$

where $J_m, H_m^{(1)}, I_m, K_m$ are the Bessel functions of order *m*, the primes on the Bessel functions denote differentiation with respect to the argument.

At this point, it may be anticipated that due to the smallness of sound speed in the plume it can be considered as an effective acoustic waveguide and one can make use of the similarity between a waveguide laser and the plume[8]. The presence of natural modes trapped by this waveguide is manifested in the existence of simple poles in Green's function (2)-(3). In the absence of the plume all the normal modes $K_m(k_n\rho)\sin\left[\pi z(n-1/2)H^{-1}\right]$ attenuate exponentially in the horizontal direction for frequencies below critical one $\omega < \omega_0 = c(\pi/2H)$ and Green's function has no poles located near the real axis. But at the presence of the plume the Green's function exhibits poles at the eigenfrequencies of the cylindrical cavity. The poles occurs at the zeros of the function in square brackets $d_2(0,n) = (\pi R_c/2)\left[K_0(k_nR_c)\kappa_nJ_1(\kappa_nR_c) - k_nK_1(k_nR_c)J_0(\kappa_nR_c)\right]$ in the denominator of axisymmetric ($m = 0$) Green's function $G_{0n}(\omega;\rho\,|\,\rho')$. These localised eigenoscillations are described by $J_0(\kappa_n\rho)$ within the plume and attenuate exponentially into the pure water $K_0(k_n\rho)$.

For our case, where $R_c/H \ll 1$, one can find eigenfrequencies in the explicit form. Expanding Bessel functions as far as the terms $(R_c\kappa_n)^2$, $(R_ck_n)^2$ gives

$$J_0(R_c\kappa_n) \simeq 1 - \frac{(R_c\kappa_n)^2}{4}, \quad J_1(R_c\kappa_n) \simeq \frac{R_c\kappa_n}{2}\left(1 - \frac{(R_c\kappa_n)^2}{8}\right),$$

$$K_0(R_ck_n) \simeq -\left[\ln\frac{(R_ck_n)}{2} - \psi(1)\right], \quad K_1(R_ck_n) \simeq \frac{1}{(R_ck_n)} + \frac{(R_ck_n)}{2}\ln(R_ck_n),$$

$$d_2(0,n) \approx -(\pi/2)\left(1 + \frac{(R_c\omega/c_m)^2\left(1 - c_m^2/c^2\right)}{2}\ln(R_ck_n)\right). \tag{4}$$

It follows from Eq. (4) that the eigenfreqitncy ω_{Rn} of n^{th} mode is obtained from the condition

$$(R_c\omega/c_m)^2\left(1 - c_m^2/c^2\right) = -2/\ln(R_ck_n),$$

which for $(cR_c/c_mH) \geq 1$ leads to the following simple expression

$$\omega_{Rn}^2 = \frac{2c_m^2}{R_c^2}\left|\ln\left(\frac{R_c\pi(n-1/2)}{H}\right)\right|^{-1}. \tag{5}$$

It follows from (5) that the eigenfrequencies are not equidistant.

One can evaluate the lowest eigenfrequency by use very simple physical arguments. Really, our model can be interpreted as the eigenvalue problem of cylindrical cavity with compressibility

$$K = -\frac{1}{V}\frac{\partial V}{\partial P} = \frac{\beta}{P} = \frac{1}{\rho_0 c_m^2} .$$

To determine the inertial mass of this cavity we can ignore water compressibility for the distances shorter than wavelength $\lambda = 2\pi / k \geq H$ and the velocity potential, denoted by φ, is governed by

$$\Delta\varphi = 0 , \quad \varphi = -\varphi(t, R_c)\ln(\rho/H)$$

We express variations in pressure ΔP_{out} through the velocity potential $\Delta P_{out} = -\rho_0(\partial\varphi/\partial t)$ and write down the kinematic and dynamic boundary conditions on the cavity wall in a form

$$\Delta\dot{R} = -\frac{\varphi(t,R_c)}{R_c} ; \ \Delta P_{out} = \Delta P_{in} , \ \Delta P_{in} = -\frac{1}{K}\frac{2\Delta R}{R_c} = -\rho_0\frac{2c_m^2}{R_c}\Delta R ,$$

$$\Delta P_{out} = \rho_0\frac{\partial\varphi(t,R_c)}{\partial t}\ln(R_c/H) = -\rho_0\ln(R_c/H)R_c\Delta\ddot{R} ;$$

$$-\ln(R_c/H)R_c\Delta\ddot{R} + \frac{2c_m^2}{R_c}\Delta R = 0 . \qquad (6)$$

This is an analogue of the Rayleigh equation for the cylindrical bubble and the natural frequency of this bubble is determined by Eq. (5).

Now we can describe the spatial properties of noise near the seep. The pressure near the seep is obtained by summing over all source contributions

$$P_T(\mathbf{r},\omega) = \int G(\omega;\mathbf{r}\,|\,\mathbf{r}')q(\omega;\mathbf{r}')d\mathbf{r}' , \qquad (7)$$

here $P_T(\mathbf{r},\omega)$ and $q(\omega;\mathbf{r})$ are the Fourier transform of the pressure and the density of monopole sources. The function $q(\omega;\mathbf{r})$ is a random variable. The cross-spectral density is a measure of the spatial coherence of the noise field. To obtain the cross-spectral density we form the product of $P_T(\mathbf{r},\omega)$ and $P_T^*(\mathbf{r},\omega)$ and take the ensemble average ($P_T^*(\mathbf{r},\omega)$ is the complex conjugate of $P_T(\mathbf{r},\omega)$). Thus

$$< P_T(\mathbf{r},\omega)P_T^*(\mathbf{r}_1,\omega) = \int d\mathbf{r}'d\mathbf{r}'' < q(\omega;\mathbf{r}')q^*(\omega;\mathbf{r}') > G(\omega;\mathbf{r}\,|\,\mathbf{r}')G^*(\omega;\mathbf{r}_1\,|\,\mathbf{r}'') ,$$

where the angle brackets indicate an average taken over the random function $q(\omega;\mathbf{r})$.

We use a model of surface generated noise developed by Kuperman *et al.*[9,10] The random noise sources are represented by correlated monopoles distributed over an infinite plane located below the surface at depth $\tilde{z}$. Assume that at each point in the plane there is a monopole source of the strength $s(\boldsymbol{\rho},t)$. Therefore, the source function is $q(\mathbf{r},t) = s(\boldsymbol{\rho},t)\delta(z-\tilde{z})$ and the cross-spectral density takes the form[9,10]

$$< P_T(\boldsymbol{\rho},z;\omega)P_T^*(\boldsymbol{\rho}_1,z_1;\omega) = \int d^2\boldsymbol{\rho}'d^2\boldsymbol{\rho}'' < s(\omega;\boldsymbol{\rho}')s^*(\omega;\boldsymbol{\rho}'') > G(\omega;\boldsymbol{\rho},z\,|\,\boldsymbol{\rho}',\tilde{z})G^*(\omega;\boldsymbol{\rho}_1,z_1\,|\,\boldsymbol{\rho}'',\tilde{z}) ,$$

Let $\boldsymbol{\eta} = \boldsymbol{\rho}' - \boldsymbol{\rho}''$, and assume that the spatial coherence of the noise sources, $< s(\omega;\boldsymbol{\rho}')s^*(\omega;\boldsymbol{\rho}'') >$, depends only on $\boldsymbol{\eta}$. Using the notations of previous works[9,10] we denote $< s(\omega;\boldsymbol{\rho}')s^*(\omega;\boldsymbol{\rho}'') >$ as $q(\omega)^2 N(\boldsymbol{\eta})$. An important special case is that of uncorrected noise sources. For uncorrected noise sources[10] $N(\eta) = 2\delta(\eta)(\omega^2\eta / c^2)^{-1}$. In this case we have

$$C_\omega(\rho, z; \rho_1, z_1; \varphi - \varphi_1) \equiv < P_T(\boldsymbol{\rho}, z; \omega) P_T^*(\boldsymbol{\rho}_1, z_1; \omega) =$$

$$= 4\pi \left(\frac{q(\omega)c}{\omega} \right)^2 \int d^2\boldsymbol{\rho}' > G(\omega; \boldsymbol{\rho}, z \mid \boldsymbol{\rho}', \tilde{z}) G^*(\omega; \boldsymbol{\rho}_1, z_1 \mid \boldsymbol{\rho}', \tilde{z}) =$$

$$= 8\left(\frac{q(\omega)c}{\omega H} \right)^2 \sum_{n=1}^{\infty}\sum_{l=1}^{\infty} \sin\left[\frac{\pi z}{H}(n-1/2)\right] \sin\left[\frac{\pi \tilde{z}}{H}(n-1/2)\right] \sin\left[\frac{\pi z_1}{H}(l-1/2)\right] \sin\left[\frac{\pi \tilde{z}}{H}(l-1/2)\right] \times$$

$$\times \sum_{m=-\infty}^{+\infty} \exp[im(\varphi - \varphi_1)] \int_0^\infty G_{nm}(\rho \mid \rho') G_{lm}^*(\rho_1 \mid \rho') \rho' d\rho' . \quad (8)$$

When we set $\rho = \rho_1$, $z = z_1$ and $\varphi = \varphi_1$ in the expression for C_ω, we obtain the intensity of the noise field at a point. Equation (8) then reduced to

$$C_\omega(\rho, z) = I_\omega(\rho, z) = 8\left(\frac{q(\omega)c}{\omega H} \right)^2 \sum_{n=1}^{\infty}\sum_{l=1}^{\infty} \sin\left[\frac{\pi z}{H}(n-1/2)\right] \sin\left[\frac{\pi z}{H}(l-1/2)\right] \times$$

$$\times \sin\left[\frac{\pi \tilde{z}}{H}(n-1/2)\right] \sin\left[\frac{\pi \tilde{z}}{H}(l-1/2)\right] \sum_{m=-\infty}^{+\infty} \int_0^\infty G_{nm}(\rho \mid \rho') G_{lm}^*(\rho_1 \mid \rho') \rho' d\rho' . \quad (9)$$

In order to eliminate the dependence on the unknown source depth $\tilde{z}$, the monopole source strength q is normalised the way described previously[10] to yield the pressure level Q in an infinitely deep ocean, independent of the actual value $\tilde{z}$. This is accomplished to first order in $\tilde{z}$ if q^2 is assigned the value $q^2(\tilde{z}) = Q^2 / (16\pi \tilde{z}^2)$, provided $\tilde{z}$ is chosen small compared to the characteristic vertical wavelengths involved in the sum $H/(\pi n)$, $H/(\pi l)$. Evaluation of Eq. (9) will be approximately independent of $\tilde{z}$ for the adopted normalisation.

In this section we consider the distribution of noise produced by near bottom bubbles and will follow to the approach of Nicolas *et al.*[12] As gas is pushed out of the vent, the surface separating it from the surrounding liquid deforms until, at a certain moment, opposing points come together and a closed surface is formed. The cavity thus produced is not in equilibrium, and it is this initial energy that gives rise to oscillation of the bubble after it detached from the vent. We assume that $q(\mathbf{r}, t)$ is due to the process of bubble formation at the base of the plume (bubbly column). With the assumption that each such process is independent from the others, the associated acoustic emissions are incoherent[11] and we have

$$< q^*(\omega; \mathbf{r}') q(\omega; \mathbf{r}'') >= 4\rho_0^2 \dot{n} T (\bar{R}_0^4 / R_c^2) \left| (\ddot{R})_\omega \right|^2 \left[1 - \theta(R_c - \rho') \right] \delta(z' - H) \times$$

$$\times \left[1 - \theta(R_c - \rho'') \right] \delta(z'' - H) \rho'^{-1} \delta(\rho' - \rho''),$$

in which $\dot{n}$ is the number of bubbles generated per unit time, $\dot{n} = \dot{V}(4\pi \bar{R}_0^3/3)^{-1}$, $\bar{R}_0$ is the characteristic (mean) bubble radius, $\dot{V}$ is the total volume flow rate of the gas out of vents; if the system consists of *N* vents, the initial radial expansion velocity $\dot{R}(0)$ of the bubble wall is given by $4\pi N\bar{R}_0^2\dot{R}(0) = \dot{V}$; *T* is the duration of each sampling interval (we have assumed that the acoustic emission of each bubble is virtually completed by the end of the sampling time), $(\ddot{R})_\omega$ is the Fourier transform of the acceleration of the characteristic bubble.

The basic quantity measured experimentally is the sound spectral density SD defined by

$$\mathrm{SD} = 10\lg\left(\frac{4\pi\,|P_T(\omega;\mathbf{r})|^2}{T\left(P_{ref}^2/Hz\right)}\right),\; P_T(\mathbf{r},\omega) = \int G(\omega;\mathbf{r}\,|\,\mathbf{r}')q(\omega;\mathbf{r}')d\mathbf{r}', \tag{10}$$

here P_{ref} is the reference pressure taken as 1 μ Pa. Assuming ergodicity and replacing average over the time by average over an ensemble we get

$$|P_T(\mathbf{r},\omega)|^2 = 4\rho_0^2\dot{n}T(\bar{R}_0/R_c)^4\left|(\ddot{R})_\omega\right|^2\iint d\boldsymbol{\rho}'G(\omega;\mathbf{r}\,|\,\boldsymbol{\rho}',H)G^*(\omega;\mathbf{r}\,|\,\boldsymbol{\rho}',H)\left[1-\theta(R_c-\rho')\right] =$$

$$= 16\frac{\rho_0^2\dot{n}T(\bar{R}_0^4/R_c^2)\left|(\ddot{R})_\omega\right|^2}{H^2}\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\sin\left[\frac{\pi z}{H}(n-1/2)\right]\sin\left[\frac{\pi z}{H}(l-1/2)\right]\times$$

$$\times\sum_{m=-\infty}^{+\infty}\int_0^{R_c}G_{nm}(\rho\,|\,\rho')G_{lm}^*(\rho\,|\,\rho')\rho'd\rho \tag{11}$$

The physical interpretation of this result is that the base of the column has been divided into *N* incoherent «pistons», each of which acts on the column with the same pressure over the area $\pi R_c^2/N$.

Now we insert the radial Green's function of Eq. (3) into the expression (10). We see that near the frequencies of the normal (collective) modes the denominator of Green's function $d_2(n,0)$ vanishes in the absence of attenuation. Although the full solution containing all the angular modes, that is for $m = 0, \pm 1, \pm 2 \pm ....$ appears in Eq. (11), it may be anticipated that only the lowest-order angular mode, for which $m = 0$, is required for the low-frequency resonance of the bubble plume. The higher-order angular modes also yield spectral peaks, but at frequencies well above those presented an interest. It is evident that the main contribution to the spectral peak, corresponding to the n^{th} eigenvalue comes from the terms with $n = l$: $\int_0^{R_c}G_{n0}(\rho\,|\,\rho')G_{n0}^*(\rho\,|\,\rho')\rho'd\rho'$. Substituting the explicit form of the Green's function into Eq. (11) we obtain the simple result

$$|P_T(\mathbf{r},\omega)|^2 \approx \frac{64\pi^2\rho_0^2\dot{n}T\bar{R}_0^4\left|(\ddot{R})_\omega\right|^2H^{-2}K_0^2(k_n\rho)\sin^2\left[\frac{\pi z}{H}(n-1/2)\right]}{\left\{1-\frac{\omega^2}{\omega_{Rn}^2}\right\}^2+\left\{4\pi\omega_{nR}^3R_c^2\ln\left[\frac{R_c\pi(n-1/2)}{H}\right]\int\frac{\delta g(R)RdR}{(\Omega_0^2(R)-\omega_{nR}^2)^2+4\delta^2\omega_{nR}^2}\right\}^2}. \tag{12}$$

This expression describes the noise intensity near the bubble plume ($\rho > R_c$). From Eq.(12) it is obvious that the intensity of the noise field is highly dependent on attenuation. In writing Eq. (12) we assume that attenuation in bubbly column dominates attenuation in surrounding water, but this is true for not so great difference between R_c and *H*.

The spatial properties of surface generated noise are given by Eq. (9). Noise intensity as a function of depth and horizontal range from the plume has the form

$$\left|P_T(\mathbf{r},\omega)\right|^2 = \frac{\pi Q^2 (n-1/2)^2}{2(\omega/c)^2 (H)^4}\sin^2\left[\frac{\pi z}{H}(n-1/2)\right]\int_0^\infty d\rho'\rho' G^*_{n0}(\omega;\rho\,|\,\rho')G_{n0}(\omega;\rho\,|\,\rho'). \qquad (13)$$

The Green's functions are dominated by high amplitude peaks corresponding to the plume normal modes. By the same reasoning, used to describe the noise produced by bubble's "birthing wails" the lowest-order angular mode $m=0$ and the diagonal in *n*, *l* indexes terms are required for the low-frequency resonance of the bubble plume. Evaluating integral for $R_c\kappa_n \ll 1$, $R_c k_n \ll 1$, we have for $\rho > R_c$

$$\int_0^\infty d\rho'\rho' G^*_{n0}(\omega;\rho\,|\,\rho')G_{n0}(\omega;\rho\,|\,\rho') \approx \frac{4\pi^4 K_0^2(k_n\rho)}{\left|d_2(n,0)\right|^2}\int_0^{R_c} J_0^2(\kappa_n\rho')\rho' d\rho' +$$

$$+16\pi^2\left|c_1(n,0)\right|^2 K_0^2(k_n\rho)\int_{R_c}^\infty K_0^2(k_n\rho')\rho' d\rho' \approx$$

$$\approx \frac{8\pi^2\left(R_c^2 + \dfrac{\omega^4 R_c^4}{c^4 k_n^2}\right)K_0^2(k_n\rho)}{\left\{1-\dfrac{\omega^2}{\omega_{Rn}^2}\right\}^2 + \left\{4\pi\omega_{nR}^3 R_c^2 \ln\left[\dfrac{R_c\pi(n-1/2)}{H}\right]\displaystyle\int\frac{\delta g(R)RdR}{(\Omega_0^2(R)-\omega_{nR}^2)^2+4\delta^2\omega_{nR}^2}\right\}^2}.$$

It is worth to point out that excitation of plume's normal modes by the surface noise sources located above the gas plume (which contribution is proportional to $\sim R_c^2$) is more effective than by the sources spread over more extended area ($\sim k_n^{-2}$) out of the body of plume, but due to Rayleigh's law scattering $(\omega^4 R_c^4/c^4) \ll 1$ giving smaller contribution to the mode amplitude. Substituting this expression into Eq. (13) gives

$$\left|P_T(\mathbf{r},\omega)\right|^2 \approx \frac{16\pi^3 Q^2(\omega)\dfrac{R_c^2(n-1/2)^2}{(\omega/c)^2 H^4}K_0^2(k_n\rho)\sin^2\left[\dfrac{\pi z}{H}(n-1/2)\right]}{\left\{1-\dfrac{\omega^2}{\omega_{Rn}^2}\right\}^2 + \left\{4\pi\omega_{nR}^3 R_c^2 \ln\left[\dfrac{R_c\pi(n-1/2)}{H}\right]\displaystyle\int\frac{\delta g(R)RdR}{(\Omega_0^2(R)-\omega_{nR}^2)^2+4\delta^2\omega_{nR}^2}\right\}^2}. \qquad (14)$$

Expressions (12), (14), the central result of our analysis, states that the noise spectral density near the gas seepage has peaks, corresponding to the existence of the normal modes in the gas plume.

Simple model used to obtain these results can be generalised to account some features of geometry of the bubble plume and variation of the void fraction with depth and laterally. Acoustical resonances in axi-symmetric, conical bubble plume formed by a plunging water jet, studied in details by Buckingham *et al.*[12], have an evident analogues in plume formed by intense seeps producing the streams of bubbles. As the conical coordinates belong to the eleven ones in which variables are separated in the Helmholtz equation one can obtain an analytical expressions for the noise spectral density and in this case. The variability in the shape of the plume can be described by introducing a random displacement. The peculiarities of normal modes propagation in waveguide with rough wall are rather good studied[13].

4 CONCLUSIONS

A rising bubble plume forms an effective acoustic waveguide that possesses normal modes Explicit expression for the frequencies of these modes have been derived for the bubbly medium in the form of cylinder with radius R_c and length *H*. It has been demonstrated that across the frequency band at the cut off of the normal modes propagation regime, the spatial distribution of low-frequency noise near the seeps is quite inhomogeneous and has a mode-like structure. This anomaly in noise intensity is localised in horizontal direction, moreover the radius of localisation is diminished with the growth of the mode number. The noise spectral density has high amplitude peaks corresponding to the plume normal modes – coherent, collective oscillations of the bubbles within the plume. The dimensions of the column and the bubble void fraction control the resonant frequencies of the plume.

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