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SIMPLIFIED CHARACTERISATION OF STRUCTURE-BORNE SOUND SOURCES

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1. INTRODUCTION

It is reasonable to assume that the adverse effect of placing a sound source in a given environment will be proportional in some way to the sound power it delivers into this environment. It is therefore important when characterising a source to do so in terms of quantities which govern the total delivered power. In the case of structure-borne sound sources the delivered power is that injected through any connections to the supporting structure, and for the case of contact at discreet multiple points this is expressed by the following matrix equation:

$$P = \frac{1}{2}\{\overline{v}_{fs}\}^{*T} \left\{\left[\left([\overline{Y}_R]+[\overline{Y}_S]\right)^{*T^{-1}} . \mathrm{Re}\left([\overline{Y}_R]\right) . \left([\overline{Y}_R]+[\overline{Y}_S]\right)^{-1}\right]\right\} \{\overline{v}_{fs}\} \qquad (1)$$

Here, the free velocity vector $\{\overline{v}_{fs}\}$ and mobility matrix $[\overline{Y}_S]$, are seen to be the properties of the source which govern the power. Also, the mobility matrix $[\overline{Y}_R]$ of the receiver plays an important role. All elements of $\{\overline{v}_{fs}\}$ and $[\overline{Y}_S]$ are therefore required for full characterisation of the source, all being complex and frequency dependent. However, whilst it is theoretically rigorous and inherently physical, the characterisation of a source by matrices and vectors containing many elements is conceptually difficult to handle. The problem considered in this paper is how to represent this data more simply, so as to provide more insight into the problem. In effect, this means collapsing the matrix equation 1 to a simpler form.

2. SOURCE ACTIVITY

From equation 1 it is seen that there are two independent elements to the characterisation of the source: the free velocity represents its 'activity', and the mobility matrix its passive structural dynamic properties. (Activity is here loosely defined as the strength of disturbances caused by the internal forces of operation).

Now, a simplification can be made by representing the free velocity vector by a single scalar quantity $|\overline{v}_{fs}| = \left(\sum_{i=1}^{n} |\overline{v}_{fsi}|^2 \right)^{1/2}$.(which is simply the length of the vector, or n times the spatially averaged free velocity where n is the number of contact points). It has been shown [1], [2] that although the power cannot be calculated exactly in terms of $|\overline{v}_{fs}|$ it can be bounded according to:

$$\frac{1}{2}\lambda_{c,min}|\overline{v}_{fs}|^2 \leq P \leq \frac{1}{2}\lambda_{c,max}|\overline{v}_{fs}|^2 \quad (2)$$

where $\lambda_{c,max}$ and $\lambda_{c,min}$ are respectively the maximum and minimum eigenvalues of the square matrix between double brackets in equation 1. These eigenvalues are real and positive, having units of mechanical impedance and are independent of the excitation, being functions of the passive structural dynamic properties only. They can be calculated exactly knowing all terms in the mobility matrices, and provide strict least upper and greatest lower bounds. Thus, the bounds in expression 2 are the narrowest that can be found when the source activity is represented by $|\overline{v}_{fs}|$ [2].

3. SOURCE AND RECEIVER MOBILITIES

The inequality 2 shows that bounds for the power can be written in a scalar form. This provides a significant simplification over equation 1, but the interpretation of the eigenvalues λ_c is still conceptually difficult. A clearer representation can be found by characterising the complex source and receiver mobility matrices $[\overline{Y}_S]$, $[\overline{Y}_R]$ by their singular values, y_{sj}, y_{rk}, j,k =1...n., and the real part of the receiver matrix $\mathrm{Re}([\overline{Y}_R])$ by its eigenvalues $y_{r(real)_i}$, i=1...n. The eigenvalues λ_c from expression 2 can then be bounded in terms of independent properties of the source and receiver as follows:

$$\min_{i,j,k}\left(\frac{y_{r(real)_i}}{[y_{rj} + y_{sk}]^2}\right) \leq \lambda_c \leq \max_{i,j,k}\left(\frac{y_{r(real)_i}}{[y_{rj} - y_{sk}]^2}\right) \quad (3)$$

In expression 3, $y_{r(real)}$, y_r, y_s are all real and positive, and span a range from the smallest to the largest eigenvalue or singular value. Clearly, when the ranges of y_r and y_s overlap the upper bound becomes infinite and the approximation is not useful in a practical sense (although it is still theoretically valid). For the case of a high mobility source, ie when y_r and y_s do not overlap, but $y_s > y_r$ the power is bounded by:

$$\frac{1}{2}|\overline{v}_{fs}|^2\left(\frac{y_{r(real)min}}{[y_{s_{max}} + y_{r_{max}}]^2}\right) \leq P \leq \frac{1}{2}|\overline{v}_{fs}|^2\left(\frac{y_{r(real)max}}{[y_{s_{min}} - y_{r_{max}}]^2}\right) \qquad (4)$$

(A similar expression can be found for the case of a low mobility source, but this will not be investigated here.) The disadvantage of this approximation is that the bounds may be wider than those in expression 2. The advantage is that the power is bounded in terms of scalar quantities which represent independent properties of the source and receiver.

Expression 4 is particularly helpful conceptually because it expresses the power from a multiple-point-mounted source in the same form as that from a single-point-mounted source. The well-known equivalent circuit for a single-point-mounted source as shown in figure 1(a) can therefore be generalised to apply to multiple-point-mounted sources as shown in figure 1(b): the only extension required is that the mobilities in the multiple point case are not single valued but have a range of possible values. The equivalent circuit in figure 1(b) thus allows insight from the single point case, which has been widely studied [3], to be applied directly to the multiple point case.

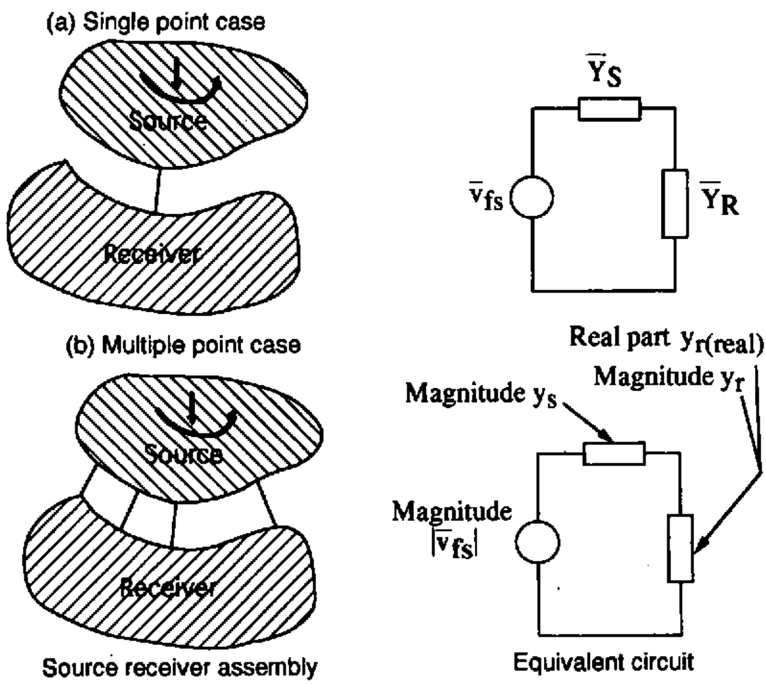

Figure 1. Equivalent circuits for single point and multiple point mounted sources

It can easily be confirmed that expression 4 collapses to the expression for the single point case when n=1, i.e.

$$\frac{1}{2}|\bar{V}_{fs}|^2\left(\frac{Re(Y_r)}{[|Y_r|+|Y_s|]^2}\right) \le P \le \frac{1}{2}|\bar{V}_{fs}|^2\left(\frac{Re(Y_r)}{[|Y_r|-|Y_s|]^2}\right) \qquad (5)$$

where Yr and Ys are the ordinary point mobilities.

4. EXAMPLE

As an example, consider the idealised model of a small pump attached via pipe work to a concrete wall as shown in figure 2. The source, which consists of the combination of the pump and pipe is represented by a free beam of 20mm diameter copper tube with a force applied close to the centre representing the internal forces in the pump. The receiver is modelled as an infinite plate of 100mm concrete. In figure 3 are shown the ranges of the singular values of the source and receiver mobility matrices which represent the 'magnitude' of the mobilities in the equivalent circuit, figure 1(b). It is seen that the ranges do not overlap, so the upper bound in expression 4 is finite. From figure 3 it is clear that the source approximates a force source because the 'magnitude' of the source mobility is always much greater than that of the receiver.

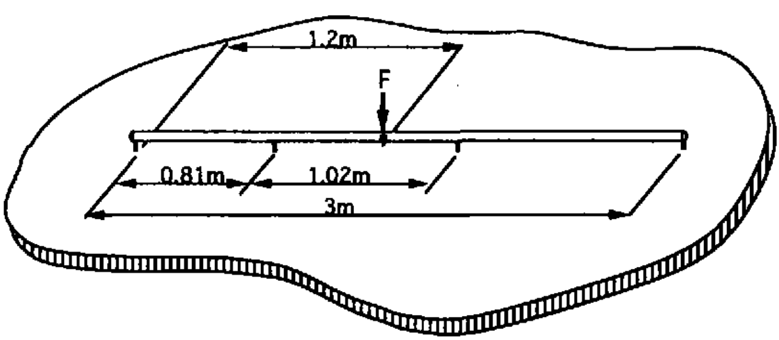

Figure 2. Model of source receiver assembly

In figure 4 are shown two sets of bounds: the inner set is the range of the eigenvalues λ_c (see expression 2), and the outer set are the bounds on these eigenvalues from inequality 3. In this case, the approximate bounds are reasonably close to the exact ones.

In figure 5 is shown the exact power, calculated from the full matrix equation 1, compared with the bounds from inequality 5. The exact power is seen to fall within the bounds. It is also evident that at certain frequencies the bounds are relatively wide, of the order of 10^6 in the worst case.

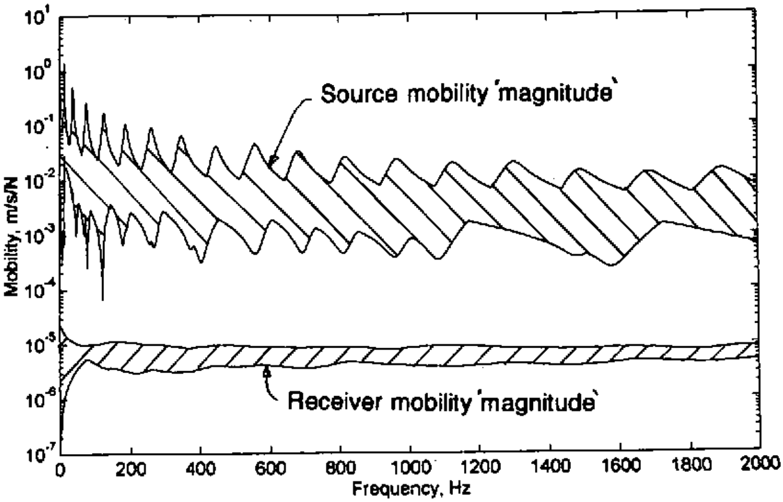

Figure 3. Magnitude of the mobility of source and receiver structures

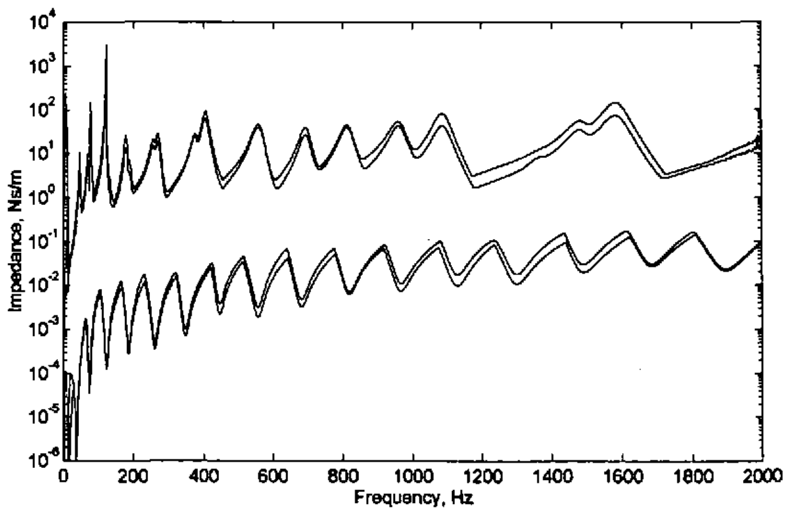

Figure 4. Range of eigenvalues λ_c and bounds from expression 3

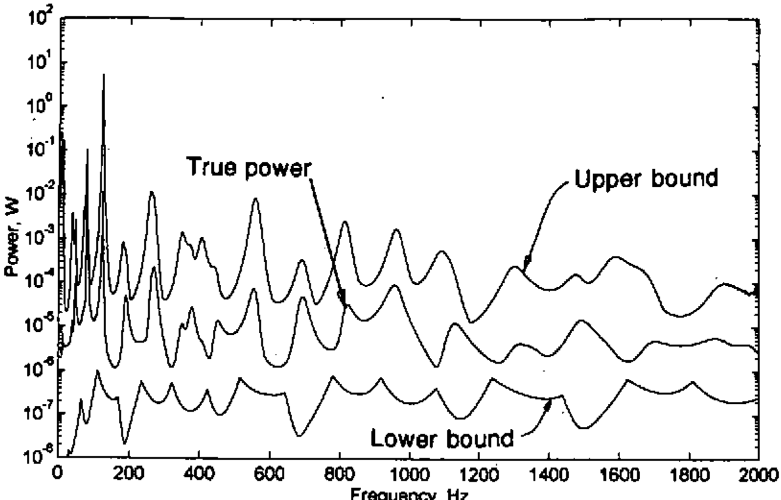

Figure 5. Exact power and bounds from expression 5

5. CONCLUDING REMARKS

It has been shown that a simplified characterisation of structure-borne sound sources exists, in which the free velocity vector and mobility matrix are replaced by scalar quantities. A multiple-point-mounted source can then be represented by essentially the same equivalent electrical circuit as for a single-point-mounted source. Such a representation has significant conceptual advantages, since understanding of the single-point case can be applied directly to the less well understood multiple-point case.

Inherent in the simplification is the fact that the power delivered by the source cannot be calculated exactly but can only be bounded. In the example presented here the bounds were probably too wide to be of practical use. Nevertheless, it is the simplicity of the representation which provides insight into the physics of the problem.

Research into alternative simplified source descriptors is continuing at Liverpool [2]. Signs are that, in certain restricted circumstances, a simple characterisation such as that above may lead to bounds on the delivered power which are likely to be acceptably narrow in a practical sense.

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