THE WAVE APPROACH TO ANALYSIS OF STRUCTURE-BORNE SOUND AND VIBRATION FOR ACOUSTICIANS

A T Moorhouse Acoustics Research Unit, School of Architecture and Building Engineering, University of Liverpool, Liverpool, L69 3BX

1. INTRODUCTION

Acousticians are generally familiar with waves, and most develop a 'feel' for wavelengths of sound in air. The wave approach is also valid for analysis of structure-borne sound and vibration, so in theory acousticians have a head start with such problems. In practice, very few have much facility with structure-borne sound and vibration. One reason is that most treatments of vibration and structural dynamics adopt a modal analysis approach in favour of the wave approach. A second reason is that, even if acousticians realise that their knowledge of waves also applies to structures, it is slightly more difficult to calculate wavelengths in solids than in air.

This paper addresses the second problem, and its objective is to present a simple method of calculating bending wavelengths which is suitable for practising acousticians. It is hoped that this will enable acousticians to develop a feel for structural wavelengths and thereby apply their existing understanding of waves to structures. No new ideas are offered, all the concepts presented are available in the scientific literature. However, we take up the conference theme 'Research into Practice' by presenting this information in a form which should prove more digestible than more commonly used, but less clear, formulations.

2. VIBRATIONS ARE WAVE MOTIONS

Initially, we make the broad statement that all vibrations are the results of wave motions; complex vibration fields are interference patterns between waves and structural resonances are simply standing waves. Usually only two wave types are important in the audible range, longitudinal and bending waves. Longitudinal waves have a constant speed, and wavelengths can be calculated in the same way as we calculate wavelengths of sound in air. Wave speeds are generally higher than in air, and longitudinal wavelengths for a given frequency are correspondingly longer (15 times longer in steel and 11 times longer in concrete). Wave speeds are found in many text books, and some are given in table 1.

Bending waves are more important in terms of subsequent sound radiation. Unlike longitudinal and pressure waves in air, their wave speed is dependent both on frequency and on the cross section of the structure. Hence, it is more difficult to obtain an appreciation of bending wavelengths and more difficult to develop a facility. However, if this minor difficulty can be overcome a practically useful understanding of structural dynamic behaviour immediately becomes possible.

BENDING WAVE SPEED	BENDING WAVELENGTH
$c_B = \sqrt{\kappa c_L \omega}$	$\lambda_B = 2\pi \sqrt{\kappa c_L / \omega}$

κ = radius of gyration, m (see below)
c_L = longitudinal wave speed, m/s
$\omega = 2\pi f$, rad/s

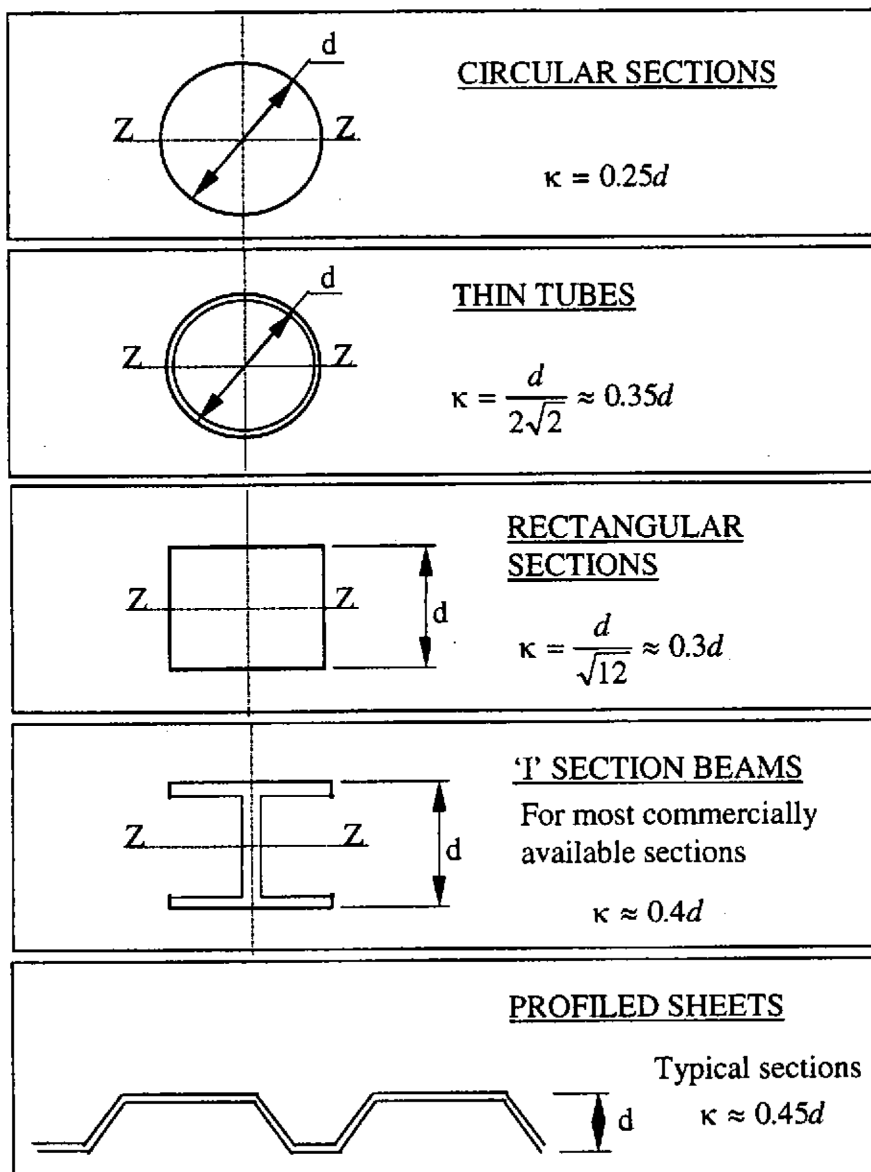

Figure 1 Radius of gyration for common sections

3. CALCULATION OF BENDING WAVELENGTHS

Bending wave speed and wavelength can be calculated from the simple formulae given below (see reference [1] and [2]). These are not the formulations found in most text books, but are considerably easier to use.

$$c_B = \sqrt{\kappa c_L \omega} \qquad \lambda_B = 2\pi\sqrt{\kappa c_L / \omega} \qquad 1, 2$$

where c_B is the bending wave speed in m/s, λ_B is bending wavelength in m. c_L is the longitudinal wave speed in m/s as given in table 1 which is constant for a given material[1]. $\omega = 2\pi f$ is the radian frequency in rad/s where f is frequency in Hz. κ is the radius of gyration in m, which will be a new concept to many acousticians and is explained below.

3.1 Radius of gyration

Radius of gyration, κ is defined as the ratio

$$\kappa = \sqrt{I / A} \qquad 3$$

where I is the moment of inertia (units of m^4), and A the cross sectional area.

Radius of gyration, κ, has units of length. For beams and plates of any cross section it is between 1/4 and 1/2 the overall depth of the section taken around the bending neutral axis marked Z-Z in figure 1[2].

Most books use an approach where it is required to know I to calculate wave speeds. However, many acousticians are not familiar with this concept, and so are unable to perform the calculation with confidence. Furthermore, slightly different equations are required for beams and plates using this conventional approach. Yet it is not necessary to know I explicitly to calculate wavelengths and wave speeds, since the geometric properties of the section are adequately described by κ. The advantages of using κ are:

- its physical significance as a fraction of the section depth is easily appreciated
- the same equations apply for beams and plates
- it can be estimated with reasonable accuracy from the section depth without performing any detailed calculations

Given these advantages it is surprising that the approach presented here is not in common use. It is believed that this is simply because the conventional method, which is based on the language of structural engineers, has gradually become established.

Values of κ for various common sections are given in figure 1. More insight can be gained by noting that the ratio given in equation 3 is proportional to the square root of stiffness to weight ratio. Therefore, more structurally efficient shapes have a higher value of κ for a given overall thickness than inefficient ones. For example, hollow sections such as the tube have a higher value of κ than

[1] Strictly speaking the longitudinal wavespeed in beams is slightly lower than that in plates although the difference is so small that it is usually neglected. See section 6.

[2], This is valid for structures of a single material, although there may be exceptions for structures composed of different materials

a solid section circular section of the same diameter as seen in figure 1. A useful reference point is the solid rectangular section for which $\kappa \approx 0.3d$. The I section and hollow circular section are more efficient structurally and hence κ is a higher proportion of the overall section depth. Appreciation of this principle means that the value of κ for a given section may often be guessed with sufficient accuracy without a need for detailed calculations. Even without making an estimate of κ, the limits $0.25d \le \kappa \le 0.5d$ are sufficiently narrow as to provide a first estimate of the bending wavelength which will never be more than 40% in error, and usually much less than this. This relative insensitivity to errors in κ is because the wavelength and wave speed are proportional to its square root as seen in equations 1 and 2.

Note that κ is independent of width of the section, so the value for rectangular beams, $\kappa = d/\sqrt{12}$ is also valid for plates of constant thickness. In this case the plate can be considered as a beam with a very wide section.

Note also that for most sections there are two different bending wave speeds for bending about the two principle axes. The same procedures can be used to calculate both.

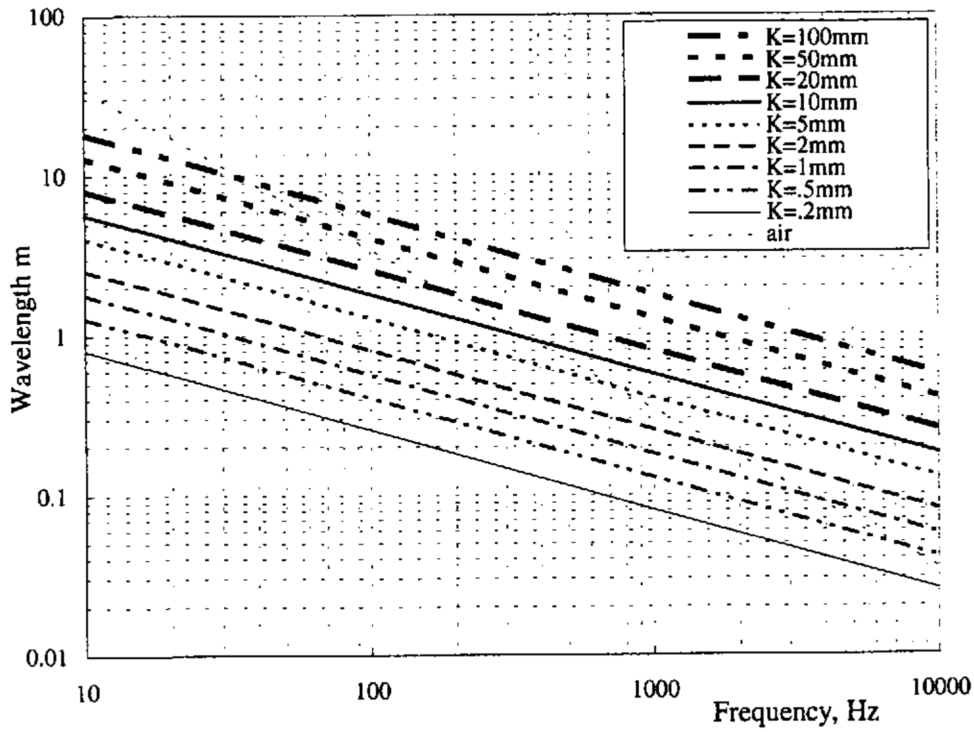

Figure 2 Bending wavelength in steel, aluminium or glass for various values of κ

3.2 Procedure for calculation of wavelengths

To calculate bending wavelengths read κ from fig 1 and substitute the value into equation 2. Alternatively, for structures made of steel, aluminium or glass (which all have very similar longitudinal wave speeds) the values can be read directly from figure 2.

Figure 2 can also be used for other materials by applying corrections found in table 2. A little care is needed to apply the correction in the correct way:

a) to estimate wavelengths for a known frequency, read off the wavelength and multiply by the factor given in table 2a
b) to estimate frequencies for a known wavelength, multiply the wavelength by the factor in table 2b to obtain the corresponding wavelength in an equivalent steel structure before entering the chart.

Table 1 Longitudinal wave speeds c_L	
Steel	5200 m/s
Aluminium	5150 m/s
Concrete	3700 m/s
Brick	3000 m/s
Plywood	3000 m/s
Glass	5200 m/s
Gypsumboard	1600 m/s

Table 2a Corrections to wavelengths for material	
To correct bending wavelengths obtained from figure 2 for materials other than steel, aluminium or glass	
For material	multiply λ by
Concrete	0.84
Brick	0.76
Plywood	0.76
Gypsumboard	0.55

Table 2b Corrections to wavelengths for materials	
To obtain the frequency for a known wavelength in other than steel, aluminium or glass, correct the wavelength before entering figure 2 as below.	
For material	multiply λ by
Concrete	1.19
Brick	1.32
Plywood	1.32
Gypsumboard	1.82

Table 2c Corrections to critical frequencies for graphical method	
To correct critical frequencies obtained from figure 2 for materials other than steel, aluminium or glass	
For material	multiply f_c by
Concrete	0.71
Brick	0.58
Plywood	0.58
Gypsumboard	0.31

The next two sections give examples of how an appreciation of structural wavelength can be used to gain insight into the behaviour of structures.

4. GENERIC FORM OF STRUCTURAL RESPONSE CURVE

Most acousticians are familiar with the generic form of the sound reduction index (SRI) curve, with stiffness, resonance, mass regions and the critical frequency. It is much less well known that the velocity response of any structure to an applied force can be described by a similar generic curve. A knowledge of the governing wavelength is the key to understanding. Distinct regions of behaviour can be identified as illustrated in figure 3:

a) when the structure is much larger than a wavelength the behaviour will be mass-like if the structure is freely suspended, (such as a resiliently mounted machine), or stiffness-like if it is built-in (such as a floor or wall)
b) when the wavelength is around twice the longest structural dimension resonant behaviour will start and will continue as the frequency increases until typically 5-10 wavelengths fit across the structure
c) at higher frequencies the structure appears large, and will behave as if it was infinite.

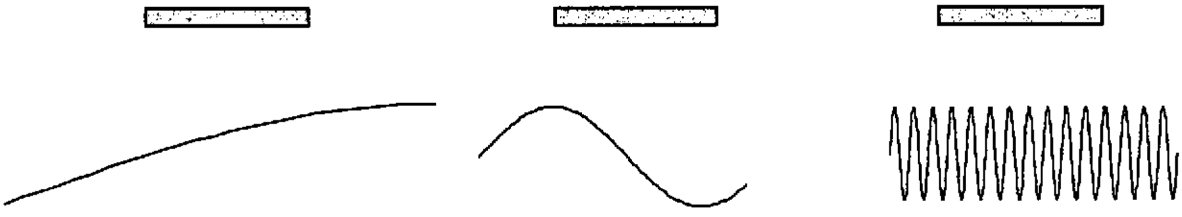

Figure 3 Regions of behaviour determined by wavelength compared with structure size

The above regions a, b and c correspond to the stiffness, resonant and mass law regions in the SRI curve of walls and floors. The critical frequency is also determined by the structural wavelength. Thus, knowledge of the bending wavelength is sufficient to identify all the important regions of the structural response curve. It applies to building elements and hence also to the SRI curve.

An example is now given. Figure 4 shows the velocity response of a 1.05m x 0.8m x 20mm steel plate, simply supported at its edges. The applied force is a point force, but an essentially similar curve would be obtained in response to a distributed pressure excitation. Using the procedures from section 2 the bending wavelengths is calculated and is marked on the x axis in figure 4 as a proportion of the diagonal distance across the plate. The lowest resonance is seen to occur when about one wavelength fits across the diagonal. The analogous situation in acoustics is room modes or organ pipe resonances which are simply standing waves in the same way as the plate resonances. For beams the lowest resonance occurs when its length is between 1/4 and 3/4 of a wavelength. For plates, as a fairly crude although useful rule of thumb one can say a probable lower bound to the lowest natural frequency is that frequency at which at a quarter wavelength can just fit inside the structure; a probable upper bound is the frequency at which a single wavelength just fits across the shortest dimension. This lowest resonance frequency marks the change in behaviour from stiffness-like behaviour (characterised by a straight line) to resonant behaviour (characterised by strong fluctuations in response). When more than 5 wavelengths fit across the structure it behaves as infinite so that individual resonances are not pronounced enough to show up on a third octave plot. This region, as long as the frequencies are below the critical frequency, corresponds to the region of mass law behaviour in the SRI.

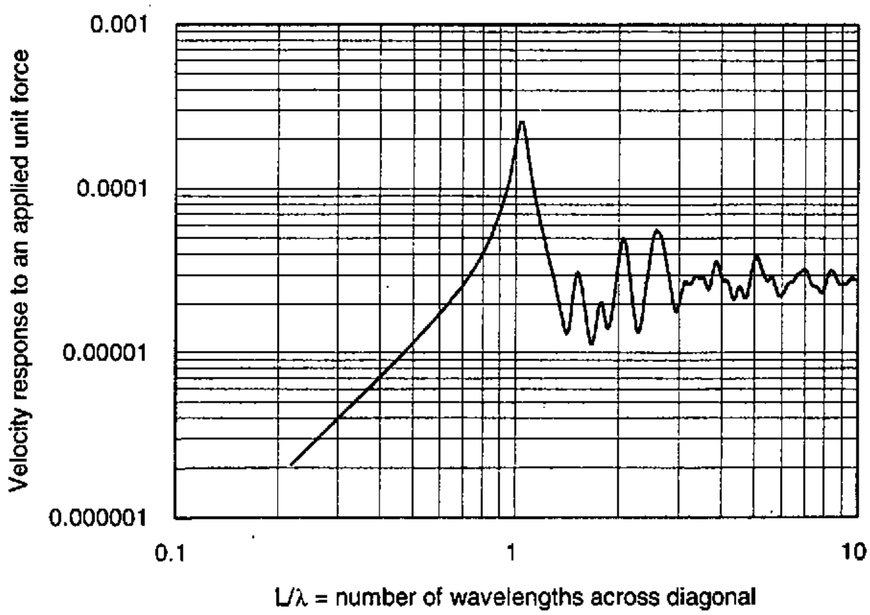

Figure 4 Typical response curve for a plate

5. EXAMPLE ESTIMATION OF CRITICAL FREQUENCIES

The critical frequency is that at which the wavelength of sound in air is equal to the bending wavelength. Critical frequencies for flat plates are found in many reference books. However, the usual techniques can be cumbersome when dealing with stiffened or profiled plates, and it is more convenient to use the radius of gyration as described above.

To calculate critical frequency, read κ from fig 1 and substitute the value into the equation

$$f_c = \frac{c^2}{2\pi\kappa c_L} \qquad 4$$

where c is the speed of sound in air and f_c the critical frequency.

Alternatively, f_c can be read from figure 2 by finding the point of intersection of the line for the relevant κ with the line for air. For materials other than steel, aluminium or glass, a correction according to the values in table 2c is also required.

As an example, consider a profiled steel sheet, currently popular for claddings . It is well known that such plates do not display a single coincidence dip in their SRI curve, but a reduced plateau between two frequencies corresponding to matching of the air wavelengths with those of the bending waves in the x and y directions on the plate. For bending waves propagating parallel with the corrugations the plate appears 'thick' and the relevant κ is obtained from figure 1 using the overall section depth (it is not required to know the sheet thickness for waves in this direction). For bending waves propagating perpendicular to the corrugations, the relevant κ is obtained from the sheet thickness. The two critical frequencies are then calculated from equation 4. For example, the lowest critical frequency for 25mm, 35mm and 45mm sections are calculated as 327, 234, and 182Hz respectively. Such estimates can be made rapidly, and agree to within a few percent with those obtained from more lengthy calculations (see for example reference [3]). Moreover, estimations based on κ are more easily verified than those based on the more abstract concept of I, and hence tend to engender more confidence. Even if more detailed calculations are performed, the quick method serves as a useful cross check.

6. LIMITS OF APPLICABILITY

Equations 1 and 2 are based on the same assumptions as the more conventional forms and are therefore subject to the same regions of validity. They are as accurate as the conventional calculations except in so far as approximations to κ have been made in figure 1.

The main assumption is that the bending behaviour occurs in thin sections. Thus, these equations are not accurate for wavelengths less than 6 times the section thickness. In practice this limitation is only usually reached in building structures composed of concrete or brick and for mid and high frequencies.

Strictly speaking the longitudinal wave speed is different in beams and plates, because plates appear slightly more stiff than beams of the same thickness by restricting lateral (Poisson) expansions. Therefore, the appropriate value should be substituted into equations 1, 2 and 4. Since the difference for most materials is less than 5% this correction can usually be safely ignored.

7. CONCLUSION

It is hoped that the above approach will enable acousticians to develop a 'feel' for structural dynamics and vibration simply by applying what they already know about waves to solid structures.

8. REFERENCES

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[1] LAM Y W & WINDLE R M 'Noise transmission through profiled metal cladding', Journal of Building Acoustics, p357, (1995).