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ANALYSIS OF THE SQUEAL VIBRATION OF A DRUM BRAKE

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1. INTRODUCTION

Squeal noise of the automobile brake system has attracted much attention as the other parts become quieter. In case of the drum brake, friction induced vibration of coupled system, binary flutter, is the most prominent squeal mechanism instead of the stick-slip phenomenon.

While stick-slip occurs when the friction coefficient μ depends on the relative velocity, especially when μ-v curve has negative slope, binary flutter occurs even when μ is constant over some velocity range. In this model the lining material is considered as a soft elastic material while the drum and backplate of the shoe are considered to be rigid. Therefore the lining thickness is determined by the relative displacement between drum and backplate, and in turn the pressure, proportional to the thickness variation, also depends on the radial displacements of both structures. This change in radial force results in the friction force variation. If drum and shoes are modeled as a thin shell or its section, the dominant mode of vibration is the flexural vibration and the circumference of the neutral surface of each structure remains unchanged. Therefore coupling of the radial and circumferential displacement due to zero hoop strain and coupling of the normal and friction force result in a positive feedback and cause the unstable responses generating high intensity sound.

This model was first proposed by North[1] considering brake system as a lumped parameter system. Millner[2] considered the drum brake as a cylindrical shell and the shoes as curved beams. Okamura and Nishiwaki[3] developed an improved analytical model and studied the effect of friction coefficient on the stability. Lang et al.[4] developed a experimental modal analysis technique for rotational modes and showed that the noise generation is mainly due to the binary flutter. They also have investigated the complex modes resulting in the squeal vibration. Day and Kim[5] used FEM for modal analyses of S-cam drum brake.

In Okamura and Nishiwaki's model the coupling effect of drum vibration with the shoe through friction material requires the information about the mode shapes of the curved bar. In this study an improved eigenvalue approximation is proposed for a curved beam which is essential in estimating the coupled system's mode shapes. The calculated characteristic values showed good agreement with the exact solutions. Experimental verifications are also given to support the usefulness of the proposed method.

2. VIBRATION OF A DRUM BRAKE SYSTEM

Although a brake drum is an cylindrical shell of finite length with an end cap, it can be assumed to be one with both ends open, based on the experimental results of modal analysis especially at the lower modes. Also shoes are considered as sections of cylindrical shell or curved free-free beams. Figure 1 shows simplified geometrical configuration of the brake system. Here a shoe is named as leading shoe and the other as trailing shoe.

For the lining material between drum and shoes has smaller elastic modulus (usually less than 1/100 of steel) than the structure material, its thickness is determined almost by the relative displacement between drum and shoes. Therefore the pressure on the drum exerted by the leading shoe material can be represented as the product of Young's modulus and the normal strain of the lining material $R_{sp}=(E_l/h)(u_{sp}-u_d)$ and the shear stress on the contact surface is proportional to the normal pressure as $F_{sp}=\mu R_{sp}$.

Since the drum is considered as a thin shell the zero hoop strain condition gives the relation between the radial and circumferential displacements as $u_d=-\partial v_d/\partial\phi$. From the periodicity of 2π with respect to ϕ, the sth mode displacements can be represented as

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = \begin{bmatrix} -\partial\Phi_{d1}/\partial\phi & -\partial\Phi_{d2}/\partial\phi \\ \Phi_{d1} & \Phi_{d2} \end{bmatrix} \begin{bmatrix} q_{d1}(t) \\ q_{d2}(t) \end{bmatrix} \tag{1}$$

where $\Phi_{d1}=-\sin s\phi/s$, $\Phi_{d2}=\cos s\phi/s$, (s : integer) and $q_{d1}(t)$ and $q_{d2}(t)$ are generalised coordinates of the sth mode. Deformation energy of the drum, U_d, and overall kinetic energy, T_d, are represented as sums of modal contributions as

$$U_d=\frac{E_d I_z\pi}{2l_d^3}\sum_{s=1}^{\infty}(1-s^2)^2(q_{d1}^2+q_{d2}^2),\quad T_d=\frac{l_d\rho_d A_d\pi}{2}\sum_{s=1}^{\infty}(1+\frac{1}{s^2})(\dot{q}_{d1}{}^2+\dot{q}_{d21}{}^2) \tag{2}$$

The generalized force of the drum can be expressed as

$$Q_{di}=\int_{\gamma_1}^{\gamma_2}(\frac{\partial R_{sp}}{\partial\phi}\frac{\partial u_d}{\partial q_{di}}+\frac{\partial F_{sp}}{\partial\phi}\frac{\partial v_d}{\partial q_{di}})d\phi+\int_{\beta_1}^{\beta_2}(\frac{\partial R_{ss}}{\partial\theta}\frac{\partial u_d}{\partial q_{di}}+\frac{\partial F_{ss}}{\partial\theta}\frac{\partial v_d}{\partial q_{di}})d\theta \tag{3}$$

where subscripts d, sp, and ss represents drum, leading and trailing shoes respectively.

Applying Hamilton's principle, the equations of motion of the coupled system for the sth mode is derived as below. The details of coefficient matrices are given in ref.[3].

$$M\ddot{X}+KX=LAX+\mu LBX \tag{4}$$

where M : modal mass matrix, K : modal stiffness matrix, of uncoupled system,
A, B : mode coupling matrices , $X^T=[\ q_{ss}(t)\ \ q_{sp}(t)\ \ q_{d1}(t)\ \ q_{d2}(t)\ ]$

3. DERIVATION OF THE MODE SHAPE FUNCTIONS OF A CURVED BEAM.

In calculating the elements of the matrices in eq.(4) the mode shape functions of circularly curve beam is required. The equation of flexural vibration of a circularly curved beam is

$$\frac{1}{K^2}\ddot{u}+(1+\frac{\partial^2}{\partial\phi^2})(u+\frac{\partial^2 u}{\partial\phi^2})=0 \qquad \text{where} \qquad \frac{1}{K}=\sqrt{\frac{l^3\rho a}{EI}} \tag{5}$$

Assuming time-harmonic motion with frequency ω, this reduces to an ordinary differential equation with constant coefficients and the corresponding wavenumber-frequency relation is

$$\lambda^4+2\lambda^2+1-\alpha^4=0\ ,\ \text{where}\ \ \omega^2/K^2=\alpha^4 \quad (\alpha^4>1). \tag{6}$$

General form of the mode shape function Φ_p is given as a linear combination of trigonometric and hyperbolic functions. If the solution is grouped as a symmetric part including cosine and cosh terms and an anti-symmetric part with remaining terms, from the boundary conditions of a free-free beam, the characteristic equation is given as

$$\frac{\lambda_1\gamma}{2}\tan\frac{\lambda_1\gamma}{2} = -\frac{\lambda_2\gamma}{2}\tanh\frac{\lambda_2\gamma}{2}\text{, for symmetric parts, where } \lambda_{1,2}^2=\alpha^2\pm1. \qquad (7)$$

Although this equation may be solved numerically, a closed form approximation can make it easy to see the parametric dependence of the system behavior. For $\alpha \gg 1$, λ's can be approximated as $\lambda_{1,2} \simeq \alpha\pm1/2\alpha$. In reality α for drum brake is usually of the order of 10^3 that this is quite reasonable. Denoting m_n as the nth solution of $\tanh m=-\tan m$, α can be expressed as $\alpha\gamma/2=m_n+x$ where x is the perturbation variable. Substituting this, λ_1 and λ_2 are approximated as $\lambda_{1,2}=m_n+x\pm\gamma^2/4m_n$. Since m_n is very near to $(4n-1)\pi/4$, $(n=1,2,\ldots)$, facilitating the smallness of x, the solution is obtained as $x = -\gamma^2(m_n-1)/4m_n^2$. Therefore the symmetric mode shape function is given as

$$\Phi_s=\cosh\lambda_2(\phi-\frac{\gamma}{2})+\frac{\lambda_2^2+1}{\lambda_1^2-1}\frac{\cosh\frac{\lambda_2\gamma}{2}}{\cos\frac{\lambda_1\gamma}{2}}\cos\lambda_1(\phi-\frac{\gamma}{2}),\quad \lambda_{1,2}=\frac{2}{\gamma}\left(\frac{m}{2}+x\pm\frac{\gamma^2}{4m}\right) \qquad (8)$$

The shape functions for anti-symmetric mode can be obtained in a similar fashion.

4. NUMERICAL RESULTS AND DISCUSSIONS.

The approximate solution of the characteristic equation for a curved beam is compared with those given by Okamura and Nishiwaki[3]. Table 1 shows the results, where the exact value is obtained through numerical analysis of the eq(7). The proposed approximation gives much better results especially in the lower modes which is of more significance in squeal vibration.

Once the mode shape functions are determined, eq(4) gives the modal frequency ω where the eigenvectors give the mode shapes in terms of generalized coordinates. Table.2 shows modal frequencies calculated by the present method and those measured by impact tests. The first two frequencies coincide with the measured ones though the 3rd and 4th components show discrepancy of about 5% and 15%. Also frequencies predicted by finite element method shows similar but a little higher than those measured. However, this result looks quite satisfactory because dominant squeal frequencies lie in the range from 1 to 6 kHz. Fig.2 shows the mode shapes corresponding to each modal frequency. For a modal frequency a pair of complex conjugate mode shapes are possible as shown in these figures. This kind of degeneracy is called doublet modes[5] and has no preferred angular positions. Fig.3 shows 4 mode shapes of brake system measured on the drum by impact test.

While the drum is modelled as an open cylinder of finite length and deformation of the hub is neglected in this study, the hub motion becomes significant at higher order modes and more elaborate model is required to take this into account.

5. CONCLUSIONS

Using an analytical dynamics model and normal mode functions, a equation of motion of the drum brake is derived. In order to get a closed form expression of coefficient matrix in that equation a better approximation to the characteristic values for the vibration of a curved beam is proposed. Comparing with the numerical analysis results the proposed approximation gives better results. Calculated natural frequencies showed good agreement with measured ones at the 1st and 2nd modes but some discrepancy at the 3rd and 4th modes. Calculated mode shapes look almost the same as those measured up to 4th mode.

References

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Table 1. Solutions of eq.(7) by various methods

	numerical analysis		proposed method		Okamura et al. [3]	
order	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
1	2.18	1.66	2.15	1.66	1.89	1.16
2	4.79	4.58	4.79	4.58	4.74	4.43
3	7.51	7.37	7.51	7.37	7.49	7.29
4	10.23	10.13	10.23	10.13	10.22	10.07
5	12.96	12.88	12.96	12.88	12.95	12.85

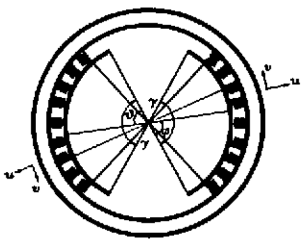

Fig.1 Geometry of drum brake system.

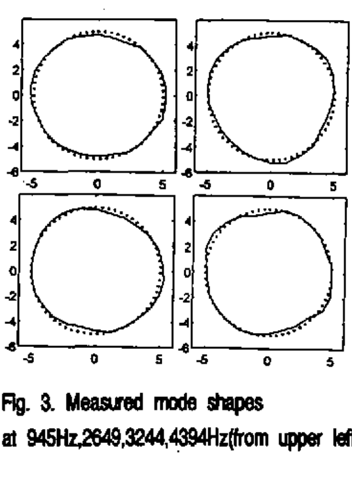

Fig. 3. Measured mode shapes at 945Hz,2649,3244,4394Hz(from upper left)

Table 2. Modal frequencies calculated by the proposed method, by FEM and measured

[unit : Hz]

order	1	2	3	4
Calculation	938.83	2645.75	3088.47	5070.56
Measurement	945.35	2648.80	3244.00	4394.50
F E M	1001.7	2783.0	3438.2	4756.0

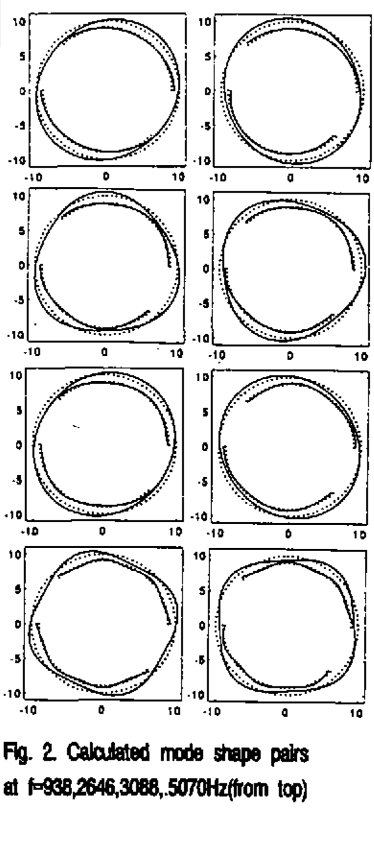

Fig. 2. Calculated mode shape pairs at f=938,2646,3088,.5070Hz(from top)