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UNCERTAINTY MODELLING FOR THE STRUCTURAL VIBRATION CONTROL USING H_∞ CONTROLLERS

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1. INTRODUCTION

In the modern control period, a controller K are usually designed to work with a single nominal model G_0. But G_0 cannot always exactly match the true plant. The actual plant can be expressed by $G=G_0+\Delta G$, where ΔG is the difference between the true plant and the nominal plant model. Plant uncertainty, which is represented by ΔG, is one of major factors that results in controller K working 'improperly' in real applications.

The basic motivation for this paper are to investigate uncertainty models, which represent the plant transfer function perturbations using the modal analysis techniques and the highly structured parametric variations, for the implementation of H_∞ control theory and to verify the uncertainty models through experiments using a real structure.

2. BASIC CONCEPTS OF H_∞ CONTROL THEORY

In general, the basic goal of H_∞ control theory is to find a controller K which minimises the worst-case response of system to any disturbances including uncertainties and stabilises the closed-loop system as shown in fig. 1.

Plant uncertainties

There are three types of plant uncertainties used in conventional H_∞ control theory : additive, multiplicative, parametric.

$$G(s) = G_0(s) + \Delta G_A(s)\ ,\ \Delta G_A(s) = \text{additive uncertainty} \qquad (1)$$

$$G(s) = G_0(s)\,(\,1 + \Delta G_M(s)\,)\ ,\ \Delta G_M(s) = \text{multiplicative uncertainty} \qquad (2)$$

$$G(s) = \left\{ \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}} : \zeta_{min} < \zeta < \zeta_{max} \right\} \qquad (3)$$

Robustness for stability and performance

The concept of robustness has been raised from the presence of plant model uncertainties since the control system can give very poor performance or, worse, destabilisation if the plant model does not match the actual plant in certain respects. A control system is said to be *robust* to a given set of plant uncertainties if it provides satisfactory performance or stability for all plant models in this set.

3. UNCERTAINTY MODELS FOR STRUCTURAL SYSTEMS

The nominal model used for structural control design generally shows the input/output relationships between actuators and sensors. It is essential to set up tight lower and upper bounds of uncertainty model for achieving robust stability and high performance simultaneously when implemented on the actual system.

System modelling

For theoretical analysis and control design, a structural system can be modelled to be linear, multi-degree of freedom with viscous damping that is assumed not to couple with the normal modes of vibration.

$$M_n q_n + C_n q_n + K_n q_n = L_n(t) \tag{4}$$

Parametric uncertainty and corresponding transfer function variables

The uncertainty of the system can be classified in two ways such as parametric(structured) uncertainty and nonparametric(unstructured) uncertainty. The parametric uncertainty represents parametric variations in the system dynamics while the nonparametric uncertainty usually represents frequency-dependent components such as the unmodelled system dynamics in the high frequency range. As seen in eq. 4, there are several parameters(i.e. mass, stiffness, damping coefficient) and the mass term is selected as a varying parameter since it is easy to implement. By a simple derivation, the relationships between the mass variations, and the damping ratio and the natural frequency variations are expressed as

$$-\frac{\Delta\zeta}{\zeta} = \frac{1}{2}\cdot\frac{\Delta M_n}{M_n + \Delta M_n} \quad \text{and} \quad -\frac{\Delta\omega_n}{\omega_n} = \frac{1}{2}\cdot\frac{\Delta M_n}{M_n + \Delta M_n} \tag{5}$$

Next the transfer function uncertainties, which are based on the mobility of the structural system dynamics, due to the damping ratio variations and natural frequency variations are investigated with a multiplicative form.

$$\Delta G_M(j\omega) \cong \Delta\zeta / \zeta \qquad \text{due to damping ratio variations} \tag{6}$$

$$|\ \Delta G_M(j\omega)\ | \cong \frac{1}{\zeta_n}\cdot\frac{\Delta\omega_n}{\omega_n} \qquad \text{due to natural frequency variations} \tag{7}$$

It means that if the mass variation is 10%, transfer function multiplicative uncertainty due to the damping ratio variations is 5% but multiplicative uncertainty is extremely sensitive to the natural frequency variations since most structural systems have damping ratio ζ_n normally between 0.1 - 0.01.

New frequency shift uncertainty model

As discussed in previous paragraph, transfer function multiplicative uncertainty is too sensitive to the natural frequency variations, the frequency shift fotm of the perturbed plant transfer function, $G(j\omega)$, due to the natural frequency variations and the damping ratio variations is proposed as follows;

$$G(j\omega) \approx G_0(j(\omega + p)) \tag{8}$$

where $G_0(j\omega)$ is the unperturbed plant transfer function , p is a shifted amount of the perturbed natural frequency to the lower frequency direction.

4. EXPERIMENTAL VERIFICATION OF STRUCTURAL UNCERTAINTY MODELS

The natural frequency variations due to the mass variations were investigated experimentally as seen in fig. 2. As we had derived the mathematical model and the relationships between the mass variations and the natural frequency variations, and damping ratio variations, the mass variations could cause ideally natural frequency variations and damping ratio variations. The measured frequency response function curves are shown in figure 3. The frequency shift uncertainty model for the representation of the natural frequency variations and the damping ratio variations was simulated for the real structural system using the measured frequency response function curve. The purpose of this simulation was to verify how well the frequency shift uncertainty model can match the natural frequency variations and the damping ratio variations in the actual structural system. The simulated results are given in fig. 4 and the results show that the differences between before and after the frequency shift is less than -20dB.

5. CONCLUSIONS

For the implementation of H_∞ control theory to robust structural vibration control, the focus has been placed on plant uncertainties. The plant transfer function perturbations due to the natural frequency variations were too sensitive to be a standard form such as a multiplicative form of parametric uncertainty while the plant transfer function perturbations due to the damping ratio variations were well represented by the standard form. Thus a *new frequency shift uncertainty model*, which represents the plant transfer function perturbations due to natural frequency variations and damping ratio variations, was prepared. The frequency shift uncertainty model was simulated to demonstrate its effectiveness for real structural systems and the experimental results showed good consistency between the theoretical prediction and the measured transfer function curves.

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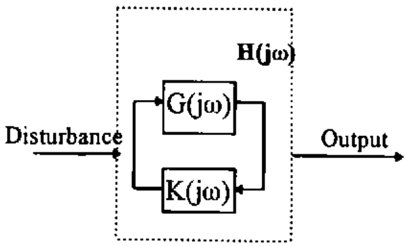

Fig. 1. Schematic block diagram of general closed-loop control system

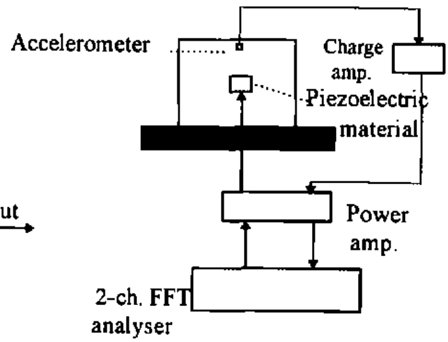

Fig. 2. Layout of Experiment

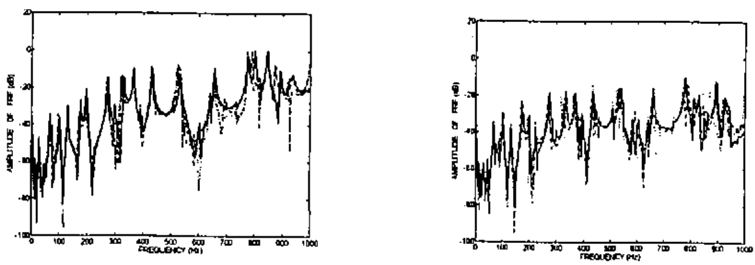

Fig. 3. Measured frequency response function curves with two added masses

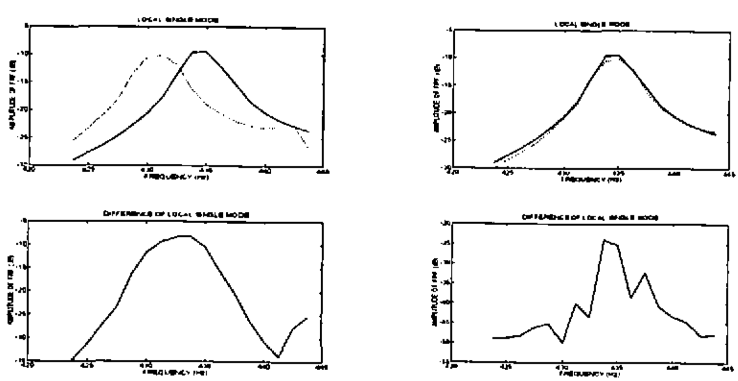

Fig. 4. Simulation of shifted plant transfer function perturbation