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ADAPTIVE INTERNAL MODEL CONTROLLER - STABILITY ANALYSIS

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1. INTRODUCTION

Recently, there has been a growing interest in active control of sound and vibration using feedback systems. Some of the studies use Internal Model Control (IMC) [1], where a model of the plant is used to cancel the contribution of the controller output back to its input, turning the system feedforward when the plant model is perfect, and enabling the use of feedforward adaptive control methods [2-5]. However, the practical situation where the plant model is not perfect due to modelling errors and variability in time, is not addressed in these works. In this case the closed-loop system might go unstable, or the adaptive process might diverge, or converge to a non-optimal solution. An adaptive IMC system is presented here, which takes into account robustness in the face of plant changes. The stability of the closed-loop and the adaptive process in an Active Headsets implementation is analysed.

2. INTERNAL MODEL CONTROLLER

Figure 1 shows the adaptive IMC system used in this work, with C the feedback controller (outlined in the dotted line) and P the plant. The feedback controller C consists of the control filter Q and plant model $\hat{P}$, and can be written in the frequency domain as $C = Q/(1-Q\hat{P})$. It is designed to reject the disturbance d by minimising the variance of the residual control error e. The exact method used to design the controller is described in [6], and involves measures adopted from H_∞ control to insure stability in face of plant changes. The final cost function, which includes a performance term of residual control error variance, and a robustness term of uncorrelated stabilising noise z, was obtained:

$$J = E\left[e(n)^2\right] + E\left[z(n)^2\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\left|S(e^{j\omega})W_1(e^{j\omega})\right|^2 + \beta\left|Q(e^{j\omega})\hat{P}(e^{j\omega})W_2(e^{j\omega})\right|^2\right]\cdot\partial\omega \qquad (1)$$
$$= \left\|SW_1\right\|_2^2 + \beta\left\|Q\hat{P}W_2\right\|_2^2$$

where $S = 1/(1+PC) = \left(1-Q\hat{P}\right)/\left(1+Q\left(P-\hat{P}\right)\right)$ is the sensitivity function, $|W_1|^2$ is the disturbance power spectral density, and $\|\cdot\|_2$ is the 2-norm operator. The power spectrum of the stabilising noise z, which is equal to $\beta\left|Q\hat{P}W_2\right|^2$, was shown to increase the controller robustness at frequencies where the plant uncertainty is large [6]. The term W_2 is the bound of the plant multiplicative uncertainty about a nominal value P_0 [7], i.e. $|\Delta_P| < W_2$ for $P = P_0 \cdot (1+\Delta_P)$. A large enough value of β should be chosen to achieve the desired robustness.

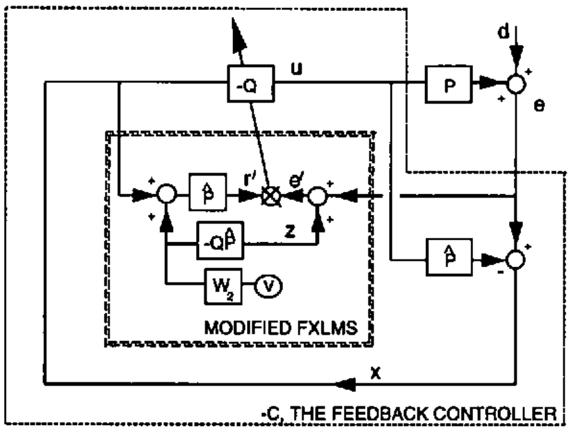

Figure 1 The adaptive IMC controller.

When the plant model is perfect the IMC system become purely feedforward, with x=d, and the controller Q can be designed using feedforward methods [2-5,8-10]. In this work, a modified Filtered-X LMS algorithm was used to adapt Q to minimise equation (1) in the steady-state, as shown in Figure 1. A pseudo-random noise v is filtered by W_2 and $Q\hat{P}$ and is added to signals x and e, to form r′ and e′ which are then used for the adaptation of the control filter coefficients vector **q**:

$$\mathbf{q}_{n+1} = \mathbf{q}_n + \mu\, \mathbf{r}'_n\, e'_n \qquad (2)$$

where μ is the convergence coefficient, $\mathbf{r}'_n$ is the vector of current and previous samples of r′, and e'_n is the current sample. Since the disturbance d is not correlated with the pseudo-random noise v, it can be shown that the expectation of the error minimised by the adaptive algorithm, $E\left[e'(n)^2\right]$, can be written in the same frequency domain form as equation (1), where $\beta = \sigma_v^2$. Another way to increase robustness is by using the Leaky-LMS [11] with the cost function:

$$J = \|SW_1\|_2^2 + \beta\|Q\|_2^2 \tag{3}$$

which can be implemented using the update equation $\mathbf{q}_{n+1} = \gamma \cdot \mathbf{q}_n + \mu\, \mathbf{r}_n'\, e_n'$, where $\gamma = 1 - \mu\beta$ and v=0. It is shown [12], however, that minimising the cost function given by equation (1) is superior to using the Leaky-LMS for plants with non-uniform response, i.e. $|P| \neq \text{constant}$.

3. STABILITY ROBUSTNESS

Closed-loop stability

Robust closed-loop stability, which is necessary in feedback controller design, is maintained if the closed-loop response remains stable in the face of plant changes. Using multiplicative plant uncertainty, the condition for robust closed-loop stability, as used in H_∞ control [7], becomes $|TW_2| < 1$ for all ω, were $T = PC/(1+PC)$ is the complementary sensitivity function. As described earlier, the controller in this work can be designed to be robust to plant changes by an appropriate level of stabilising noise. It is important to note that if robustness is ignored, even a very small difference between the plant and the plant model, caused by modelling errors, for example, can drive the system to instability.

Adaptive Process Stability

Robust adaptation will be maintained if the control filter converges to the optimal controller even when the plant changes. The adaptation process in this work is based on the Filtered-X LMS algorithm, and is performed by conducting a gradient-based search for the minimum value on the error surface, using an estimate of the gradient at any point. When the plant model differs from the real plant, the error surface becomes non-quadratic, and the gradient estimate becomes less accurate, which can cause the search to fail. Stability analysis for the *feedforward* Filtered-X LMS [13-15] shows that the adaptive process will convergence if the cross correlation matrix between R and $\hat{R}$ (where $\hat{R} = X \cdot \hat{P}$, $R = X \cdot P$, and X is the input to the control filter) has positive eigenvalues. For tonal disturbances, a phase error of less then 90° in the plant model was found to be sufficient for convergence. An analytical analysis of the Filtered-X LMS stability in an IMC system, however, is complicated by the existence of the feedback path and a non-quadratic error surface. Nevertheless, the effect of the various system parameters on the adaptation stability is studied by means of examples, using both an LMS-based search and a search with the true gradient, in numerical simulations. The simulations used a plant response and a 500Hz tonal disturbance both measured from implementation on an active headset. An FIR plant model filter with 32 coefficients that modelled the plant response corresponding to a normal headset fit was used in all the examples, together with an FIR control filter **q** with 2 coefficients. The true gradient of the error surface in equation (1) with respect to each of the control filter coefficients q_i was calculated [12], and is given by:

$$\nabla_i = \frac{\partial J}{\partial q_i} = -\frac{2}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{e^{-j\omega i\Delta}\left[\hat{P}S^* + \left(P-\hat{P}\right)|S|^2\right]}{\left[1+Q\left(P-\hat{P}\right)\right]}\right\}_R |W_1|^2 \cdot d\omega + \beta\cdot\frac{2}{2\pi}\int_{-\infty}^{\infty}\left|\hat{P}\cdot W_2\right|^2\left\{Q^* e^{-j\omega i\Delta}\right\}_R \cdot d\omega \quad (4)$$

where $\{\cdot\}_R$ denotes the real part and * denotes complex conjugate.

Example 1: A plant response corresponding to a normal headsets fit was used, resulting in a perfect plant model. Figure 2 shows the quadratic error surface, calculated with a robustness effort $\beta = 0.001$, and the course of the adaptation for the two searches, both starting from **q**=[0 0]. The perfect gradient search converged to the optimal value, while the LMS fluctuated around the optimal value due to the residual control error [11].

Example 2: After the controller converged in example 1, the plant response is changed to that corresponding to a tight headsets fit, resulting in an increase of 5dB in the magnitude, and a change of 20° in the phase of the plant response at the disturbance frequency. The closed-loop system remained stable after the change, but will the adaptive process maintain stability? Figure 3 shows that now the error surface is no longer quadratic, and has regions of instability. However it still has a single minimum, although it is different from the minimum in example 1. The adaptation with the true gradient converged to the new minimum, and improved both stability and performance, while the LMS converged to a point near the minimum, due to the error in the gradient estimate.

Example 3: The plant response of example 1, but with 2 extra samples of delay was used here, resulting in a change of 36° in the phase of the plant response at the disturbance frequency. Figure 4 shows that the stable area of the error surface is small, and no distinct minimum exists. Instead, the minimum appears to be very close to the instability border. μ=0.0005 was used here, which is about 50000 times smaller than that allowed by the Filtered-X LMS. Both searches did not stop at the instability border, but diverged into instability, because the slope of the error surface changed instantly from descent to ascent near instability. Factors such as increasing β (β=0.001 was used here), or increasing disturbance power at the "unstable" frequencies, could "round-off" the error surface near instability and improve convergence. For the LMS, detecting the instability border was even more difficult due to its instantaneous estimate. Both searches may converge using a smaller value of μ, but the resulting controller will probably not be practical due to the very low μ and the proximity to instability.

Example 4: The plant response of example 1, but with 6 extra samples of delay was used here, resulting in a change of more then 90° in the phase of the plant response at the disturbance frequency, so the stability condition for the feedforward Filtered-X LMS was not satisfied. Figure 5 shows that the LMS diverged immediately, while the search with the true gradient converged to the optimal controller, since a large robustness effort β=0.08 was used.

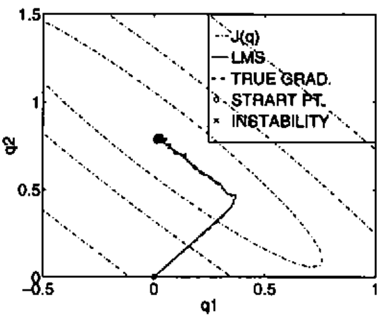

Figure 2 (Ex1) Perfect plant model.

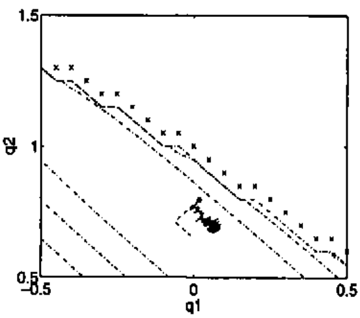

Figure 3 (Ex2) Plant response changes to a tight headsets fit.

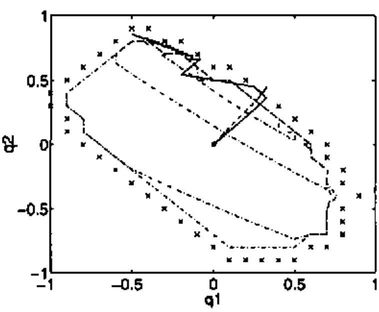

Figure 4 (Ex3) Plant delay increased by 2 samples.

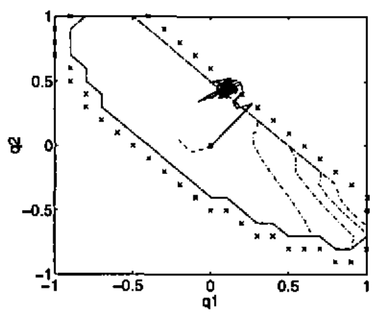

Figure 5 (Ex4) Plant delay increased by 6 samples.

5. CONCLUSIONS

An adaptive Internal Model Controller was studied in this work. Such a controller is attractive since it enables the use of adaptation methods developed for feedforward systems. However, when the plant model is not perfect, the adaptation process can fail. In this work the stability of a modified Filtered-X LMS based adaptive process was examined. It was found that when the differences between the plant response and that of the plant model are not too large, the search will still converge near the optimum. Factors such as plant model accuracy, the convergence coefficient μ, the robustness effort β, the residual control error and the disturbance spectrum, all affected the success of the search, and no simple criterion for stability was found. When implemented in active headsets, the adaptive controller was found to be robust when a high enough stability effort was used. Generally, the adaptive IMC system will give better performance when the plant response only changes slightly. However, when the changes are large but slow, they can be tracked by a separate identification system [16].

6. REFERENCES

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