INPUT ADMITTANCE AND SOUND FIELD MEASUREMENTS OF TEN CLASSICAL GUITARS

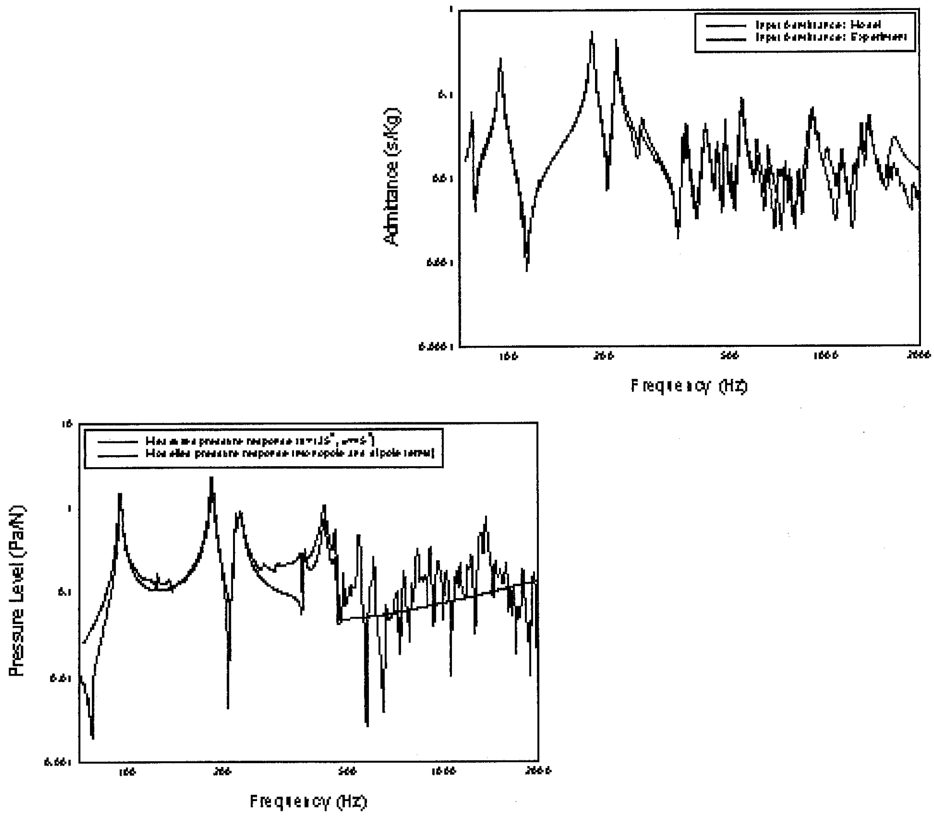
B E Richardson Department of Physics & Astronomy, Cardiff University, Cardiff UK
T J W Hill Department of Physics & Astronomy, Cardiff University, Cardiff UK
S J Richardson Department of Physics & Astronomy, Cardiff University, Cardiff UK

1. INTRODUCTION

Recent work at Cardiff has involved the measurement of acoustical parameters for the characterisation of the input admittance and radiated sound pressure response in the low to mid frequency range (70 Hz to 2 kHz) of classical guitars [1]. The parameters in question are the resonance frequencies, Q-values, effective masses and monopole and dipole "radiativities" of a number of low-order modes of vibration of the guitar body. Previous psychoacoustical investigations [2][3] have indicated which of these parameters might be expected to have the most influence on the musical quality of an instrument. The primary objective of this current project was to look at the variation in these acoustical parameters in a range of "high-quality" instruments, and to date, ten instruments have been studied. As well as giving an overview of the project, this paper will focus on three specific instruments in the set, showing how design variations may be used to produce acceptable though very different acoustical responses.

2. PARAMETER EXTRACTION

Input admittance and sound-pressure response curves have been common tools for investigating the mechanical and acoustical characteristics of stringed musical instruments. Figure 1 shows a typical set of measurements made on a classical guitar.



(a) (b)

Figure 1. (a) Measured and reconstructed input admittance at a guitar bridge (first-string position). (b) Measured and reconstructed sound-pressure response of a guitar. The fitted line includes only monopole and dipole contributions to the field (see text).

The peaks in the input admittance (velocity per unit driving force) are readily modelled using the following equation and the whole response curve is reconstructed by summing these response functions over the modes of interest:

$$y(\omega) = \frac{i\omega}{m\left(\omega^2 - \omega_0^{\,2} + i\omega\omega_0/Q\right)},$$

where $\omega_0 = 2\pi f_0$ is the resonance frequency, Q is the Q-value and m is the effective mass of the mode at the driving point. Because the damping is relatively low and the peaks tend to be reasonably well separated, curve-fitting techniques allow the accurate determination of modal parameters up to about 1 kHz [4]. Figure 1(a) clearly demonstrates that this sort of analysis can be used with some confidence.

Extracting information about the radiativity of the instrument is more difficult because of the directional nature of the sound radiation. Christensen [5] developed a simple model for the sound radiation from guitars, which involved determining the "effective piston area" of each mode and hence estimating the monopole radiation from the instrument. Very few attempts have been made to study in detail the sound fields from stringed instruments. For various experimental reasons, we adopted a similar scheme to that described by Arnold and Weinreich [6] for the spherical-harmonic decomposition of sound fields. The important distinction, however, is that we determine the monopole and dipole components of the fields associated with *individual* modes of the instrument when driven at resonance. In practice, it is not usually possible to isolate and excite an individual mode of a complex system, but holographic or speckle interferometric studies of the mode shapes of each instrument allow us to choose single or multiple driving positions which strongly excite one mode with minimal excitation of neighbouring modes. The driving positions are then cross-referenced back to the various string positions at the bridge by means of transfer admittance measurements. These procedures are explained in detail elsewhere [1][4][7]. Sound fields were measured in a small semi-anechoic chamber by a mobile pair of microphones at 324 field points from which the field components ($G_{\ell m}$) for each mode were determined. Although the system is capable of resolving octopoles, only the monopole and dipole components were deemed significant in the frequency range investigated.

Consideration of the theory for reconstructing sound fields gives some insight into the physical significance of $G_{\ell m}$. The sound pressure per unit driving force F radiated to an arbitrary point in the field by an individual mode is given by

$$\frac{p_{\ell m}(r,\theta,\phi)}{F} = \frac{\rho_0 c}{4\pi} k^{(\ell+2)} y(\omega) G_{\ell m} Y_{\ell m}(\theta,\phi) h_\ell(kr),$$

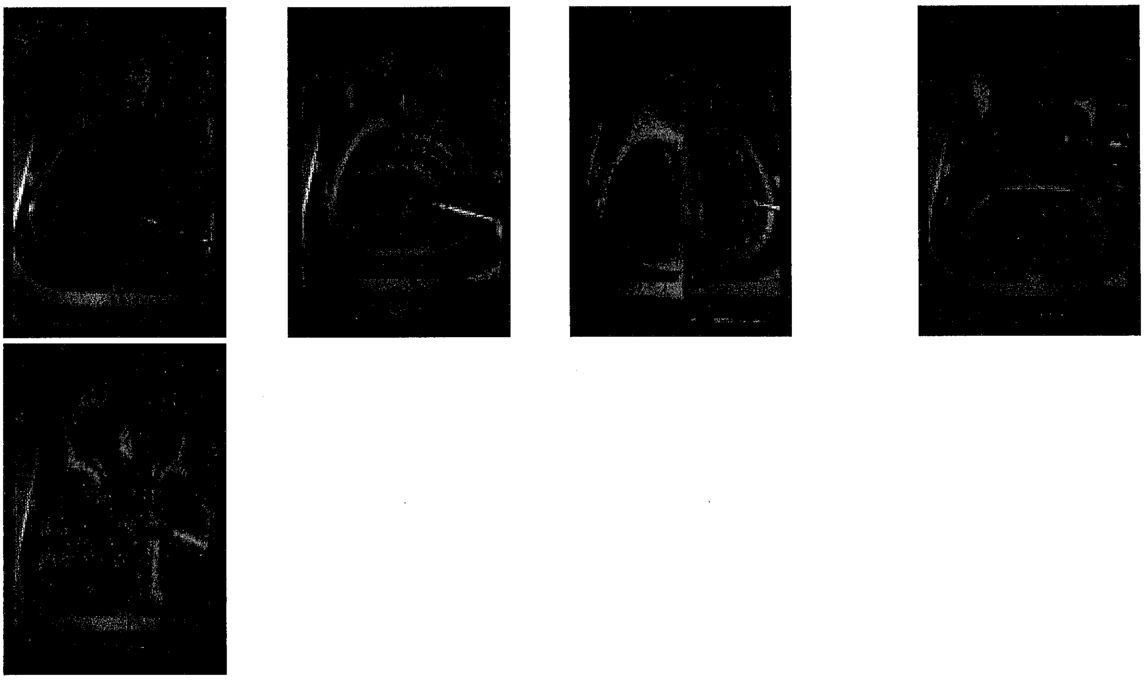
where $k = \omega/c$ is the wavenumber, c is the speed of sound in air, ρ_0 is the density of the air, and $Y_{\ell m}$ is the spherical harmonic and h_ℓ the spherical Hankel function associated with state ℓ, m of the field. $G_{\ell m}$ are the generalised weights of the orthogonal radiation components. We make the assumption that $G_{\ell m}$ is constant across a relatively broad frequency range (though this would not be the case at higher frequencies when the wavelength in air approaches the effective size of the source). G_{00} has the units of area and is equivalent to the "effective piston area" introduced by Christensen [5] and is a measure of the monopole radiativity of the source. The three components of G_{1m} have the units of volume and represent a measure of the dipole radiativity of the source. In passing, it is important to note that $G_{\ell m}$ can be positive or negative – this is simply a consequence of the mode shapes. It has the effect, however, that when several modes are simultaneously driven above resonance, the sound pressures radiated by the individual modes may add or subtract, and it has important consequences for sound radiation in the mid-frequency range, which is supported by some of the strongly radiating low-order modes. Finally, note that our use of the term "radiativity" is a little different from the context in which it is used by Weinreich [8]; his radiativities are functions of frequency and, for an individual mode, would be given by $\Gamma_{\ell m}(\omega) = y(\omega) G_{\ell m}$.

Sound fields were measured for a restricted set of low-order modes (with resonance frequencies up to about 600 Hz). Figure 1(b) shows that the reconstructed fields are reasonable fits up to this limit and, interestingly, give a prediction of the "average" response beyond this range (the rising line would turn over at some characteristic frequency if the finite size of the source were considered).

3. RESULTS AND DISCUSSION

3.1 IDENTIFICATION OF LOW-ORDER MODES

The three instruments shown in detail here, though of nominally similar dimensions, are built with quite different design philosophies. All are built to the highest standards and each have a unique sound which would suit varying requirements of professional players. The instrument by Ambridge is most easily described as of "standard Torres construction", with an internal soundboard strutting configuration similar to that shown in Figure 8(II); this is a very popular design used in the production of many hand-crafted and factory-made classical guitars. The instrument by Romanillos is also based on principles pioneered by Torres, but one in which the cross strut (or "harmonic bar") immediately below the sound hole is scooped out on its underside in two sections for about half its length. The relief allows the fan struts to run up the plate into the waist area as shown schematically in configuration VII of Figure 8. Because the bar is not glued along its entire length, this has the effect of reducing the stiffness of the soundboard in this region and tends to be accompanied by a reduction in mode frequencies [9]. The instrument by Fischer is of a radically different design. The soundboard is strutted using a lattice of wooden struts (along the lines of configuration III in Figure 8) and the sound hole is raised from its usual position and split either side of the fingerboard (as can be seen in Figure 4a). Further details of Fischer's strutting configuration can be found in reference [10]. Lattice strutting configurations, sometimes involving man-made materials such as carbon-fibre composites, are the subject of active experimentation by a number of renowned makers. Because of the increase in stiffness across the plate, lattice configurations tend to yield rather higher mode frequencies than Torres-style guitars.



(a) $T(1,1)_1$ 100 Hz (b) $T(1,1)_2$ 214 Hz (c) T(2,1) 238 Hz (d) $T(1,2)_2$ 429 Hz (e) T(3,1) 528 Hz

Figure 2. Holographic interferograms of selected modes on the guitar by Simon Ambridge (SA121).

Figures 2 to 4 show some of the modes of vibration of the three guitars visualised by means of holographic interferometry. Purely to aid identification, modes are named as (*m*,*n*) by counting "half waves" across and along the active regions of the plates. The prefix T and B are used to identify the top plate (soundboard) and back plate respectively. Where more than one mode yields the same name, a subscript is added. The modes are, of course, more complex than implied by this nomenclature, and some, e.g. the T(1,1)/B(1,1) combinations, involve significant motion of the ribs and couple with the air cavity (see [9]).

(a) T(1,1)$_1$ 88 Hz (b) T(1,1)$_2$ 172 Hz (c) T(2,1) 219 Hz (d) T(1,2)$_1$ 347 Hz

(e) T(2,2) 475 Hz (f) T(3,1) 568 Hz (g) T(3,2) 672 Hz

Figure 3. Holographic interferograms of selected modes on the guitar by José Romanillos "*La Casona*" (JLR677).

The mode shapes and frequencies identified in Figure 2 are very representative of many of the guitars studied in this and other projects. The five modes shown contribute significantly to the acoustical response of the guitar [2][3][5]; these modes are relatively strongly driven by the strings and are also good radiators. Figure 3 shows that the modified harmonic bar of the Romanillos guitar tends to lower the resonance frequencies of the low-order modes; at 88 Hz, the T(1,1)$_1$ is the lowest "air resonance" we have observed. The T(1,1)$_2$ also has a comparatively low resonance frequency (200 Hz would be a good average to quote for this mode). Stress relief near the cut-away sections of the harmonic bar cause perturbation of mode shapes (e.g. see Figure 3c), and the instrument shows a little more motion at the waist and upper bout than is normally observed. Bending stiffness across the soundboard is increased by the presence of a thin "bridge plate" which keeps the resonance frequency of the T(3,1) mode at a "conventional" position. In contrast, the resonance frequencies of the modes of the Fischer guitar are by far the highest of the ten instruments studied. It is interesting to note that the radical change in design has not modified the basic shapes and distributions of the modes, though modes occur in quite different orders. For example, the T(2,1) mode (split into two resonances here because of coupling with a comparable mode on the back plate) has a resonance frequency over an octave higher than in most guitars and unusually occurs well above the T(1,2) mode. Having made these observations, it is interesting to note that each instrument, whilst having its own unique playing quality, still accommodates professional performance requirements implying, as noted in other studies [2][3], that mode frequencies play a less important role in the determination of quality than is generally suggested by much of the musical-acoustics literature.

(a) $T(1,1)_1$ 109 Hz (b) $B(1,1)_1$ 109 Hz (c)) $T(1,1)_2$ 248 Hz (d) $B(1,1)_2$ 248 Hz

(e) $T(1,2)_1$ 369 Hz (f) $T(2,1)_1$ 522 Hz (g) $T(2,1)_2$ 606 Hz (h) T(1,3) 672 Hz

Figure 4. Holographic interferograms of selected modes on the guitar by Paul Fischer (PF952).

3.2 MEASURED PARAMETERS

Measured acoustical parameters for the three instruments are shown in Tables 1 to 3. Again, the Ambridge guitar is the most typical of the ten instruments in this study. The $T(1,1)_2$ is usually by far the most dominant mode. Its low effective mass (typically about 0.1 kg) means that the strings couple strongly to this mode, so strongly in fact that the string modes are sufficiently perturbed to create a "wolf" note at this resonance [9]. The mode is usually characterised by strong monopole radiation, proportional to at resonance and above resonance. Note the opposite polarity of the $T(1,1)_1$ and $T(1,1)_2$, which is a consequence of the variable phase of the radiation from the sound hole and soundboard. As suggested by Christensen [5], the monopole contributions of the T(1,1), T(1,2) and T(3,1) modes are highly significant.

Table 1. Acoustical parameters for the Ambridge guitar (SA121).

Mode	f_0 Hz	*Q*	*m* kg	G_{00} m^2	G_{1x} m^3	G_{1y} m^3	G_{1z} m^3
$T(1,1)_1$	100	64	0.589	-0.0194	-0.0080	-0.0003	-0.0092
$T(1,1)_2$	214	31	0.100	0.0243	0.0042	0.0006	-0.0010
T(2,1)	238	85	0.180	0.0028	0.0002	0.0028	-0.0001
$T(1,2)_2$	429	51	0.509	-0.0343	-0.0055	0.0003	-0.0039
T(3,1)	528	37	0.776	-0.0478	-0.0075	-0.0004	0.0045

Table 2. Acoustical parameters for the Romanillos Guitar (JLR677).

Mode	f_0 *Hz*	*Q*	*m* *kg*	G_{00} m^2	G_{1x} m^3	G_{1y} m^3	G_{1z} m^3
$T(1,1)_1$	88	63	0·722	-0.0217	-0.0153	-0.0004	0.0179
$T(1,1)_2$	172	74	0·191	0.0036	0.0025	0.0002	0.0002
$B(1,2)_1$	212	35	0·750	-0.0612	-0.0132	-0.0011	-0.0089
T(2,1)	219	65	0·163	-0.0029	-0.0005	-0.0018	0.0003
$T(1,2)_1$	347	48	0·591	-0.0111	-0.0012	-0.0002	-0.0008
$T(1,2)_2$	419	54	0·847	0.0414	0.0094	-0.0005	-0.0034
T(2,2)	475	55	1·800	-0.0297	-0.0037	-0.0029	0.0013
T(3,1)	568	68	1·110	0.0025	0.0005	0.0001	-0.0003

Table 3. Acoustical parameters for the Fischer Guitar (PF952).

Mode	f_0 *Hz*	*Q*	*m* *kg*	G_{00} m^2	G_{1x} m^3	G_{1y} m^3	G_{1z} m^3
$T(1,1)_1$	109	52	1·375	-0.0739	-0.0200	-0.0020	0.0321
$T(1,1)_2$	248	37	0·182	0.0517	0.0063	0.0002	-0.0016
$B(1,2)_1$	259	21	0·166	-0.0370	0.0022	0.0004	-0.0014
$T(1,2)_1$	369	56	0·632	-0.0010	-0.0006	-0.0001	-0.0003
B(1,3)	445	72	1·658	-0.0252	-0.0085	0.0006	-0.0039
T(1,3)	460	55	0·528	-0.0049	-0.0007	0.0000	0.0006
$T(2,1)_1$	522	86	0·517	-0.0023	-0.0005	0.0026	0.0007
$T(2,1)_2$	606	59	0·504	0.0045	0.0008	0.0038	-0.0005

The Romanillos and Fischer guitars both required more mode information to satisfactorily fit the sound-pressure response curves. A notable characteristic of these instruments was a higher-than-average effective mass of the $T(1,1)_2$ mode. Though there are good arguments for keeping this mass low [11], the higher value exhibited by these two instruments ensures that the "wolf" note found on most guitars somewhere between E_3 and A_3 is somewhat tamed, giving a more neutral response in this playing range. The monopole radiation from the $T(1,1)_2$ mode was very strong in both the Ambridge and Fischer guitars, as can be seen in Figures 5 and 7, but an anomalous feature of the Romanillos guitar was the small size of for this mode (see Figure 6b). This seemed to be the result of a strong structural interaction between the soundboard and back plate.

Figure 5. Ambridge SA121. Sound fields for the $T(1,1)_1$ mode (left) and $T(1,1)_2$ mode (right) driven with a standard force at resonance. For the production of Figures 5 to 7, the instrument

stands upright with the soundboard facing an observer on the left.

Figure 6. Romanillos JLR677. Sound fields for the $T(1,1)_1$ mode (left) and $T(1,1)_2$ mode (right).

Figure 7. Fischer PF952. Sound fields for the $T(1,1)_1$ mode (left) and $T(1,1)_2$ mode (right).

Q-values of modes are generally in the range 30 to 80. Tests on strips of wood used in guitar construction (e.g. spruce, western red cedar and rosewood) show Q-values in excess of 120 for transverse vibrations involving bending along the grain but more in the region of 60 when the bending is across the grain. Q-values of the order of 80 imply that the damping of these modes is largely structural. In other cases, particularly the $T(1,1)_2$ mode, radiation damping significantly reduces the Q-value, the primary exception being the $T(1,1)_2$ mode in the Romanillos guitar, which is much higher than usual, consistent with its small monopole radiativity.

Our analysis to date has not yet extended to looking in detail at the dipole radiation. What is clear from the above tables is that, as expected, modes such as the T(2,1)s have a strong dipole component orientated along the line of the bridge and the T(1,2)s radiate strongly along the line of the strings. The $T(1,1)_1$s also have comparatively strong dipole radiation, presumably a consequence of the strong radiation from the sound hole. It is very evident that dipole radiation is important, a point we had not fully appreciated at the outset of this project. Without these components, a simple "monopole" model, as used by Christensen [5] for example, overestimates the size of the coefficients. Higher order radiation ( ) has been collected in this study, but the reconstructed sound fields are not significantly different in the absence of these terms, and we consider them to be unimportant in the frequency range studied. It is clear from Figure 1(b), however, that dipoles play an increasingly important role at higher frequencies, and the relative sizes of the monopole and dipole components of the low-order modes may well be a factor in helping to balance the "treble" and "bass" response of an instrument.

3.3 IMPLICATIONS FOR THE MAKER

Guitar making appears to be a precarious balancing act. Once the string is released, the system acts as a free oscillator, gradually dissipating energy in various forms, including radiated sound. A strong response – a requirement of any instrument to be played in a modern concert hall – requires a body which is readily set into vibration (high admittance) and which strongly radiates (large). In fact, most guitars react rather too strongly to the string, leading to inharmonicity of radiated "string" partials and rather rapid decays [9]. Choosing the right compromise between competing requirements requires great skill and is subject to personal taste.

At a strong body resonance, the input admittance is dominated by the response of that mode with a peak value given by . The corresponding monopole radiation is proportional to . In order to tame "wolf" notes, it is advantageous to limit the value of the input admittance, which can be achieved either by maintaining reasonably high resonance frequencies or by increasing the effective masses of modes (lowering the Q-values is another option, but the maker usually has little direct control over these). Increasing the effective mass, however, reduces the radiated sound, so it is one of the acoustical parameters which requires very careful choice. Above resonance – and the majority of sound radiated by the higher string partials will always be "above resonance" for the low-order modes – the monopole sound radiation is governed by (and a comparable sum exists for the dipole radiation of the form). Again, the effective mass is seen to feature as an important control parameter. What is interesting about the mid-range response of the guitar is that its "mean value" appears to be set by radiation from the low-order modes, and the relative sizes and *polarities* of the terms are clearly important. Although the largest contribution to the monopole radiation is generally made by the $T(1,1)_2$ mode, the mid-range value is tempered by modes with negative s. Although it is generally assumed that it is desirable for the strings to couple to as many modes as possible, it is by no means clear that this is necessarily advantageous.

A strong case has been presented for the importance of control of effective masses. For a given mode, the effective mass varies with the position of excitation, being largest near a node. A great many modes have nodal lines which fall in the vicinity of the bridge, and it is easy to see how small changes in the construction of the instrument might move the positions of nodal lines and therefore allow the makers some control of the instrument's dynamics. This is especially true of modes such as the T(2,1)s and T(1,2)s. Asymmetric bracing, such as that shown schematically in configuration V (Figure 8), is known to shift the upright node in the T(2,1) mode to an off-centre position; this clearly modifies the coupling at the various string positions at the bridge. Whether or not this particular change is significant is debatable, but it gives an indication that mode shapes can be manipulated systematically if required. (The main effect of the additional, angled bar in configuration V seems to be to add stiffness and raise mode frequencies.)

It is clear that the T(1,1) modes play an important role in determining the mechanical and acoustical action of the guitar. Obviously, the properties of these modes are largely governed by the maker's initial choices of materials (density and elastic moduli), the general size of the plates, their thickness and bracing height. However, these are not simple "plate" modes but involve quite complex interactions between the different components of the system – the soundboard, back plate, ribs and air cavity. Although lumped-parameter models give a good overview of the corporate motion of the sound box, these do not give sufficient detail to explore the variations in effective mass and radiativity which might be expected from different construction. This is an area where new research might be extremely helpful.

There are subtle variations which can be introduced into the properties of the T(1,1) modes. The effective mass of the T(1,1) mode can be approximated using the following integral, , where is the mode shape, is the volume density of the plate and its thickness. A related expression for the volume displacement of the mode is given by . The ratio of is comparable to the ratio of introduced earlier. What is notable in these expressions is that is linear in one equation but squared in the other. This shows that the mode shape itself plays a part in determining .

II **III** **V** **VI** **VII**

Figure 8. The fundamental soundboard mode T(1,1) for each of five strutting configurations in common use. The same soundboard, glued to a "rigid" edge constraint, was used throughout these experiments; only the bracing was modified. It is shown here with no backing air cavity. The configurations are: II "standard Torres", III Lattice, V "Ramirez", VI Radial and VII Extended Torres (the numbering system is for consistency with other experimental work in progress). Note that the amplitudes are chosen arbitrarily. Data from Lewney [12].

Table 4. Modal parameters for the experimental guitar shown in Figure 8.

	Configuration				
	II	III	V	VI	VII
Frequency (Hz)	176	238	181	150	175
Effective mass (kg)	0.069	0.054	0.067	0.088	0.071
Q-value	65	63	72	94	60

We have not systematically investigated how construction affects the ratios of , though we would like to undertake such a programme. Lewney [12] conducted a simpler set of experiments to investigate how different strutting configurations affected the mechanical modal parameters. Some of these are presented in Figure 8 and Table 4, which summarise the results for the T(1,1) mode without a backing air cavity (Lewney presents data with and without the backing cavity). The variations in mode frequencies might well have been anticipated from consideration of the stiffness distribution of the braces, but it is very interesting to note the large variation in effective mass; the rather "pinched" mode shape of configuration III produces the lowest value, and there is some evidence to suggest this configuration might be expected to yield high values of .

4. ACKNOWLEDGEMENTS

We gratefully acknowledge financial support from the Leverhulme Trust (ref: F/407/R) and from the EPSRC (SJR). We would also like to thank Simon Ambridge, John Taylor and John Mills and the other makers and players who have loaned their instruments for these studies.

5. REFERENCES

[1] T J W Hill, B E Richardson and S J Richardson, "Measurements of acoustical parameters for the classical guitar", *Proceedings of the International Symposium on Musical Acoustics, Perugia, Italy*, **Vol.2**, pp. 417-420, 2001.

[2] H A K Wright and B E Richardson, "On the relationships between the frequency response of the guitar body and the instrument's tone", *Proceedings of the Institute of Acoustics* **19**(5), pp. 149-154, 1997.

[3] H A K Wright, *The acoustics and psychoacoustics of the guitar.* PhD Thesis, University of Wales, 1997.

[4] S J Richardson, *Acoustical parameters for the classical guitar.* PhD Thesis, University of Wales, 2001.

[5] O Christensen, "An oscillator model for analysis of guitar sound pressure response", *Acustica* **54**, pp. 289-295, 1984.

[6] E B Arnold and G Weinreich, "Method for measuring acoustic radiation fields", *Journal of the Acoustical Society of America* **68**, pp. 404-437, 1980.

[7] T J W Hill, B E Richardson and S J Richardson, "Acoustical parameters for the characterisation of the classical guitar", paper to be submitted to *Acustica*, 2002.

[8] G Weinreich, "Sound hole sum rule and the dipole moment of the violin", *Journal of the Acoustical Society of America* **77**(2), pp. 710-718, 1985.

[9] B E Richardson, "The acoustical development of the guitar", *Journal of the Catgut Acoustical Society* **2**(5) Series II, pp. 1-10, 1994.

[10] J Morrish, *The Classical Guitar: a Complete History*, Outline Press Ltd, p. 8, 1997.

[11] B E Richardson, "The classical guitar, tone by design", *Proceedings of the International Symposium on Musical Acoustics, Leavenworth, USA*, pp. 115-120, 1998.

[12] M Lewney, *The acoustics of the guitar.* PhD Thesis, University of Wales, 2000.