PLUCKED STRING SOUND ANALYSIS AND PERCEPTION

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1 INTRODUCTION

Plucked string sounds from guitars and harps have been analysed with the aim to reveal those broad spectral and temporal features which characterise each family of instruments. Recorded sounds, manipulated sounds and sounds generated by simple models have been used in a variety of psychoacoustical tests to assess which features are of primary importance for distinguishing between instruments and, more importantly, which features are important for determining the "sound quality" in instruments of the same family. The overall aim is part of a long-term project at Cardiff to try to understand how makers might control the sound quality of an instrument (particularly the guitar) during construction.

In plucked string instruments energy is supplied to the system only during the period of interaction between the fingertip and string. Once the string is released, the instrument vibrates as a free, damped oscillator in a manner determined by the normal modes of the system as a whole, i.e. strings coupled to the body and body coupled to the surrounding air. Radiated sounds therefore comprise decaying spectral components dominated by relatively long-lived "string partials" but also including short-lived transient components; the latter are basically impulsively-excited "body modes". Rate of energy transfer to the listener depends on the coupling between the three sub-systems. Input admittance measurements at the bridge (along with the string's characteristic admittance) give information on string-body coupling but structural coupling to the surrounding air is more difficult to evaluate. It is the variation in form and frequency distribution of the various modes of vibration of the instrument's body and their associated radiation fields which account for the large variation seen in initial amplitudes and decay rates of partials from one note to the next and generic differences which account for our ability (as an educated listener) to distinguish between various classes of instruments.

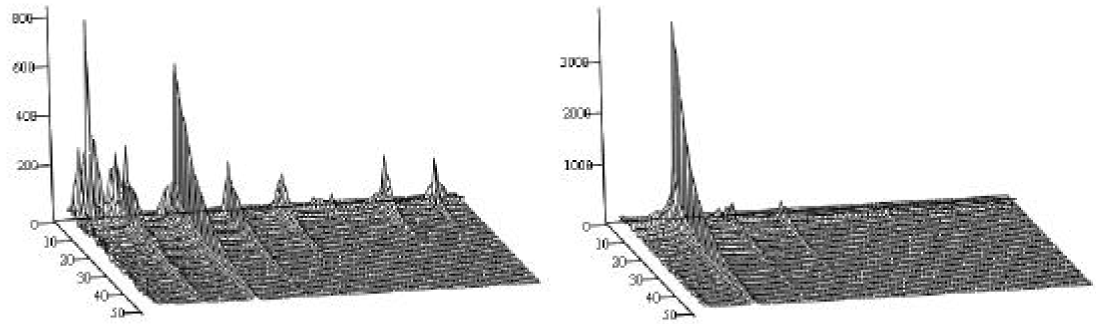
Long-term and short-term Fourier analysis provides a convenient method for determining the frequencies and time-histories of the various partials making up the decaying sounds. Such physical studies reveal a range of detail, such as various patterns of initial heights of spectral components from note to note, inharmonicity in the "string partials", the prominence of "body noise" components and various generalised forms of decays, including exponential decays (or "linear decays" when viewed on a logarithmic plot), dual-rate decays and beating decays. Signal processing allows various features of these "real sounds" to be modified in some way. Modifications can range from broad-band filtering of the whole signal to the extraction and replacement of individual partials (for example, replacing a beating decay with an exponentially decaying component). The processed and unprocessed tones were then presented to listeners, who were asked to rate perceived differences. In that way, we hope to highlight those features which are important to instrument or "quality" recognition, and also to assess the validity of a simple guitar model based on information of low-order body modes and radiation fields (described in Section 3).

The work described here is "on going" and is based on a number of fourth-year MPhys projects[1,2] and uses psychoacoustic-testing methodology developed by Wright[3,4] along with a revised transmission-line model for the guitar.

2 GUITAR AND HARP SOUNDS

2.1 Physical Analysis

Various plucked strings sounds across the whole playing range were recorded from a classical guitar and a concert harp. These were recorded in small, rather "dead" rooms with a single microphone about one metre from the front of the instrument. In each case, the instruments were played in the conventional manner (the guitar plucked with the nail and the harp with the finger-tip) except that all other unused strings were heavily damped. The sounds were analysed using long-term Fourier techniques to extract accurate frequencies of the partials (±0.7 Hz) and short-term Fourier techniques to extract the initial amplitudes and time histories of the decays of the partials. A "typical" pair of short-term Fourier transforms (waterfall plots) for the guitar and harp are shown in Figure 1. (Of course, there is nothing "typical" about any of these plots; the important point to stress is the variation in detail from one note to another.)



(a) *(b)*

Figure 1. Short-term FFTs of (a) a guitar and (b) a harp each playing E_5 (fundamental frequency 660 Hz). In both cases the frequency scale (horizontal) runs from 0-5 kHz and the time steps are measured in units of 20 ms (total time duration 1 s). The amplitude scale (vertical) is arbitrary and plotted on a linear scale. No attempt was made to "normalise" the relative amplitudes of the plots so the absolute height scales should not be compared. Both notes were played at a strong musical dynamic.

A programme was written to extract decay rates of the "string partials" and "body noise" components. Although many components decay with a roughly exponential envelope (i.e. linear on a logarithmic scale), it is not uncommon to find examples of "beating decays" or "dual decays", as shown in Figure 2 (over the page). The origin of such features is well understood in terms of different string polarisations involving different structural coupling. However, for our purposes, we chose to fit to these data using an exponential model and to quote a single "average" Q-value for the decay, as shown in Figure 2.

Physical analysis showed considerable similarities between guitar and harp sounds. Whilst Q values of individual partials varied considerably from note to note on both instruments, in general the decay rates of comparable partials on guitars and harps were very similar; this is not surprising since the nylon, gut and overwound strings are of rather similar construction and have comparable internal damping, and it is the latter which tends to dominate the decay rates in these instruments. Beating and dual decays were no more prominent in one instrument than the other, nor were they a particular characteristic of either instrument (the same may well not be true had sympathetic strings been allowed to vibrate). The most obvious differences between the two instruments were that the guitar generally had strong fundamental string partials across its whole playing range and considerable energy in high-frequency components, whereas the harp had a clear cut-off at low frequencies (below about 200 Hz – two octaves above its lowest note) and far less prominent upper partials. The guitar also featured prominent "body noise" (cf. the lowest short-lived peaks below the string fundamental at 660 Hz in Figure 1a) whereas no comparable feature was found in the harp. These general features are evident in Figure 1.

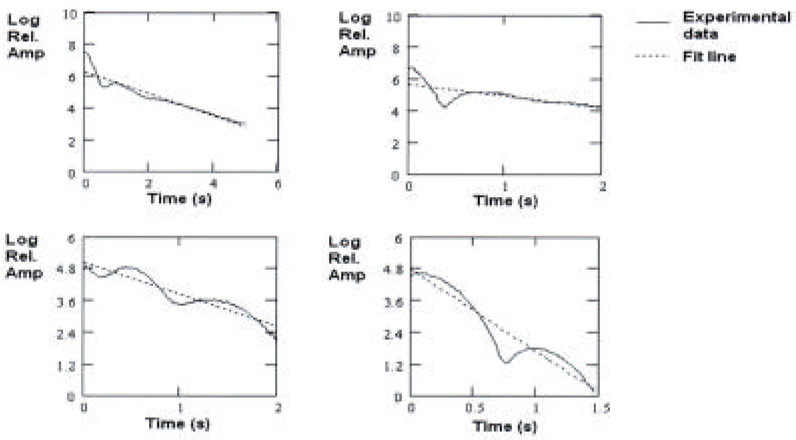


Figure 2. Decay profiles of various partials showing departure from simple exponential decay.

2.2 Psychoacoustical Tests

Some of the recorded sounds were processed or re-synthesised from the measured data and used as the basis of various psychoacoustical tests to determine which features were important for characterisation. Rolph and Weston[1,2] adopted the same methodology as Wright[3,4], presenting pairs of test tones and asking subjects to rate differences on a scale of 0-3. Most of the tests were undertaken by a group of 15 subjects of varying backgrounds (in the main local students) with a smaller, informal set of tests by a group of three subjects (BER, JR and MW). Only the main outcomes of these tests will be summarised here.

Low-pass filtering of guitar and harp sounds showed that guitar tones have important spectral information up to about 6-7 kHz, whereas the harp sounds cut off at about 3 kHz. Whether this is related to the method of excitation (with the rounded fingertip close to the centre of the string rather than nail close to the bridge) or the structural response of the harp was not investigated (though it is more likely the latter). Filling out the upper spectral range of harp sounds with guitar components made the harp much less easy to distinguish from the guitar. (Similarly, the guitar sounds could be made harp-like by reducing the upper frequencies.) At the other end of the spectrum, the harp had no prominent "body noise" components. Adding noise components extracted from guitar tones made the harp sound more guitar-like, but it was clear that it is not the presence of these body noise components which distinguishes one instrument from the other. Adding both "body noise" and short-lived decays above 3 kHz to harp tones made them far less distinguishable from guitar tones.

Beating and dual-decaying partials were replaced in a number of tones with exponential decays. These modifications were audible only when the replacements were made in the lowest partials (primarily the first or second "string partials"). Although "non-linear" decays are, on occasions, noticeable, they do not appear to be an important contributor to either guitar or harp tone generally (as they are in some instruments, e.g. the kantele[5]). Simple models (as described later in Section 3), which model only exponential decays, are therefore not fundamentally flawed. In most sounds, it is the decay rates of only the first two or three partials which have a prominent effect on the perceived tone; altering the Q-values of higher (much shorter lived) partials had a much less effect.

Frequency shifts due to the typical string dispersion found in plain and wound guitar strings were also not important perceptually on either instrument (though this finding is somewhat at odds with the experience of most guitarists who find the inharmonicity of the G_3 string barely acceptable).

We concluded that the primary difference between harp and guitar sounds is one of initial spectral amplitudes, the harp having a much more limited frequency response, cutting off above about 3 kHz and below about 200 Hz. These differences were much less prominent after the short time it takes for the upper partials to decay (0.2 s); guitar and harp tones in which the first 0.5 s were removed were virtually indistinguishable. "Body noise" components are clearly important to guitar sounds, though not a primary distinguishing feature. What they do appear to add (from informal observations) is a "percussive" element to the initial transient, which gives the instrument extra definition and "attack". The guitar generally has a far more "immediate" sound than the harp.

3 MODELLING GUITAR SOUNDS AND FURTHER TESTS

3.1 Transmission-Line Guitar Model

The current guitar-synthesis model used at Cardiff is based on a transmission-line representation of a lossy string stretched between points 0 and *L* (the bridge) connected to a load impedance $Z_L(\omega)$ representing the mechanical modes of the body. The model incorporates mechanical and acoustical parameters which can be extracted from experimental data for the body modes and their radiativities[6,7].

The load impedance is given by $Z_L = \sum 1/Y_B$ where

$$Y_B(\omega) = \frac{\mathrm{i}\omega}{M_B(\omega^2 - \omega_B{}^2 + \mathrm{i}\omega_B\omega/Q_B)}, \tag{1}$$

and M_B, ω_B and Q_B are the effective masses, frequencies and Q-values of the individual body modes.

Transmission line theory for the transfer admittance between a plucking point at position *x* and the bridge is given by

$$Y_T(\omega) = \frac{\sin(k'x)}{Z_L(\omega)\sin(k'L) + \mathrm{i}Z_0'\cos(k'L)}, \tag{2}$$

where k' and Z_0' are "lossy" versions of the wavenumber for transverse waves on the string and the string's characteristic impedance respectively, given approximately by $k' = k\sqrt{1+\mathrm{i}/Q_s}$ and $Z_0' = Z_0\sqrt{1+\mathrm{i}/Q_s}$, where Q_s is the Q-value of the string. Woodhouse[8,9] stresses that it is important that the Q-values of strings are made frequency dependent; in this case suitable functions were derived from Valette[10]. Equation (2) predicts, in the frequency domain, all the frequency shifts of the "string modes", their decay-rate modifications and the "body noise" elements expected of real guitar tones, but because the model has only one degree of freedom it does not predict beating or dual decays. However, the evidence presented in Section 2 suggests that this is only of secondary importance.

As described in Hill *et al.*[6], radiation from individual modes to a position r, θ, ϕ is given by

$$p(\omega) = -\frac{\mathrm{i}\rho_0\omega F}{4\pi r}Y(\omega)\,\mathrm{e}^{\mathrm{i}kr}\left[G_{00}\frac{\mathrm{e}^{-\mathrm{i}kr_0}}{1-\mathrm{i}kr_0} + \ldots \right.$$
$$\left.\left(\frac{1}{r} - \mathrm{i}k\right)\left(\frac{2\,\mathrm{e}^{-\mathrm{i}kr_0}}{2 - \mathrm{i}2kr_0 - k^2r_0{}^2}\right)(G_{1x}\cos\theta\cos\phi + G_{1y}\sin\theta\cos\phi + G_{1z}\cos\theta\right], \tag{3}$$

where the various *G* are measured monopole and dipole radiativities and where $k = w/c$. The above calculation incorporates finite-sized sources of radius r_0.

There is no simple way in this model to calculate the radiation when the system is driven from the string; the transfer admittance requires the load to incorporate all body modes whilst the radiation calculations are generated, in effect, one mode at a time. However, dividing Equation (3) by Equation (1) (both summed over all body modes of interest) gives the sound field per unit velocity at the bridge. A good approximation to the transfer function $H(w)$ representing the sound pressure response at a given field point for a force applied to the string at the plucking point is then obtained as follows:

$$H(w) = \frac{p(w)}{Y(w)} Y_T . \qquad (4)$$

The Fourier transform of $H(w)$ gives the time-domain radiated sound signal generated by a d-function force applied at string position *x*. For a string released from stable equilibrium (a step force function) $H(w)$ can be pre-processed by multiplying by $1/w$ (which is what was used for the following evaluations). In practice, neither a d-function nor a step function adequately represents the plucking action[11].

3.2 Psychoacoustical Evaluations using the Guitar Model

The above model was used to generate "open string" sounds. The "base set" used experimental data for the first seven body modes (in the range 90-450 Hz) from one instrument (see Hill *et al.*[6]). Pairs of tones were then generated by introducing specific variations in the model parameters and these pairs of tones were then used in psychoacoustical listening test with 21 subjects. Subjects were asked to rate similarities/differences on a scale from 0-2 to determine which were the most significant parameters. The following changes were investigated: (i) body modes frequencies were raised or lowered by a semitone and a tone, (ii) effective masses of the first, second and fourth modes (each considered significant for radiation[6,7,12]) were individually halved and doubled, (iii) Q-values of the body modes were halved and doubled, (iv) the sizes of the source radii were modified from 5 cm (tentatively determined experimentally[6]) to zero, and (v) the frequency-dependent string damping functions were replaced with a constant Q-value.

Two pairs of tones were generated for each parameter variation, one involving one of the lower, "bass" strings (E_2, A_2, D_3) and one involving one of the three upper, "treble" strings (G_3, B_3, E_4). The distinction between these two groups of strings is important for two reasons: the internal damping is significantly greater in the plain strings (roughly a factor of three) and one of the principal modes of the body usually has a resonance frequency close to this transition. This test determined whether changes in mode parameters had a greater or lesser effect in the "treble" or "bass" playing ranges of the instrument. To check the listeners' accuracy and consistency, a number of pairs of identical sounds were also included in the test in addition to the pairs with changes outlined above. The first three pairs in the test were example sounds and were included to aid the listener in giving more consistent results. Rating categories were:

0 – they sound identical,
1 – there is a small difference,
2 – there is a large difference.

There then followed 31 pairs of sounds consisting of three identical pairs and one pair for each of the changes outlined above.

Figure 3 (below) shows a subset of the responses: the proportion rated at 2 (large difference). In this chart effective mass changes are denoted by *M* and the number beside it is the factor by which it has been changed (e.g. 1st Mode M*0.5 represents a change in the effective mass of the first body mode by a factor of 0.5). Other nomenclature is self-explanatory.

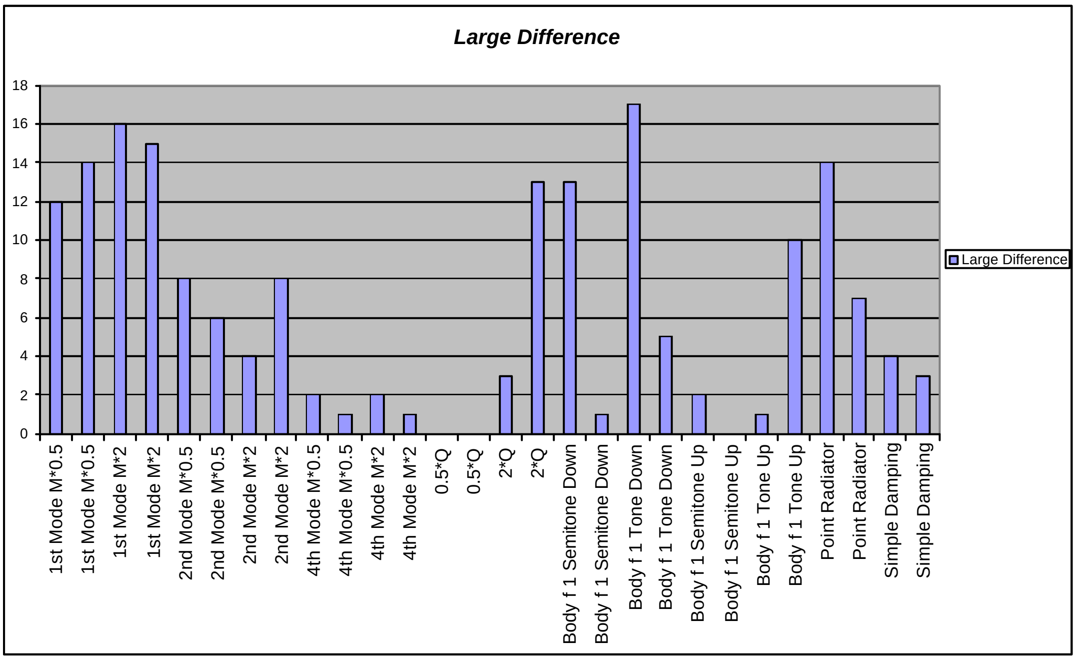


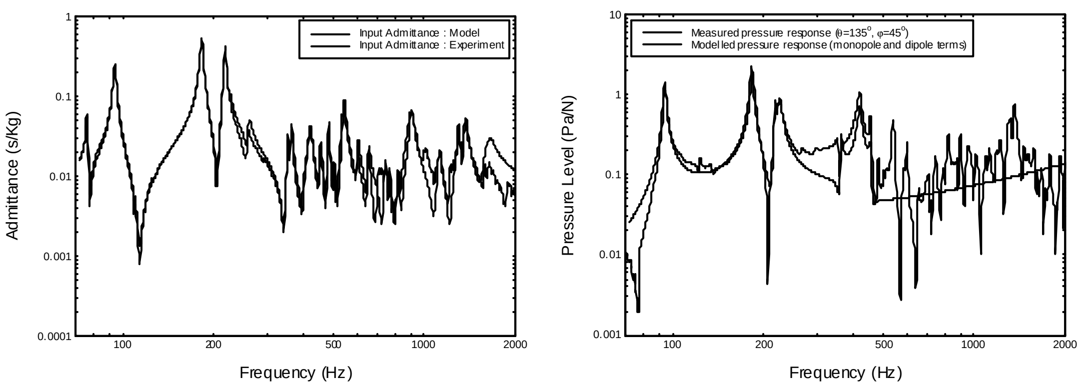
Figure 3. Total number of responses (out of 21) rated at 2 ("Large difference"). For each parameter change, there is a pair of histogram bars. The one on the left shows the perceived response from tones generated on the "bass" or wound strings and the one on the right shows the perceived response to tones generated on the "treble" or plain strings.

The data shows results consistent with those found by Wright[3,4]. He observed that changes in mode frequencies and damping tended to produce what are best described as "localised" effects (e.g. shifting the strength or pitch of "wolf notes"), whereas variations in effective masses and mode radiativities had a more "global" effect. As expected[3,4,6,12], changing the effective masses for the first and second body modes produced perceptible changes in both "treble" and "bass" playing ranges of the instrument. Changes to the fourth mode were less noticeable. In general, changes in the Q-values of the body modes had little effect. The large difference noted in the 2*Q of body modes in the "treble" playing range is perhaps a "localised effect" brought about by coincidence of "string" and "body" modes. Similar effects are considered at work when mode frequencies are moved. Shifting mode frequencies can have large perceptual effects, but what this normally does is simply to push up or down the initial amplitude of one partial, which, whilst audible, is of little overall consequence to the playing qualities of the instrument. These tests imply that the exact functions used for string damping are not of huge significance (see "simple damping"). In contrast, modifying the radii of the acoustic sources is perceptually significant (see "point radiator").

Like the variations in the time-history of the excitation force function, the source sizes influence the high frequency content. Their presence in Equation (4) act a little like low-pass filters whose cut-off frequencies decrease for increasing r_0. As noted in Section 2, subjects were very sensitive to the presence or absence of "high-frequency" content of guitar and harp tones, so it is not surprising that these tests shows similar results. To some extent this is perhaps a simple limitation of the current model which uses data only for the lowest body modes of the system; it would be expected that higher modes would assist radiation in the mid- to high-frequency ranges. Variations in effective masses (and mode radiativities, which are currently under investigation) create "global" changes and can be seen to raise or lower the peak amplitudes of partials across wide spectral ranges effectively altering the overall perceived loudness, but it is clear from these results that these are not as noticeable as the reduction or increases in mid- to high-frequency content of the tones brought about by variations in the source sizes.

4 CONCLUSIONS

Like many acousticians interested in the construction of stringed musical instruments, we have for some time concentrated our efforts on the "low-frequency" characterisation of the body. It is tempting to argue that because these low-order modes are the only ones which makers can control with any certainty, that their acoustical parameters somehow control the overall response of the instrument. Considerable effort has been placed into measuring modes (mode shapes, frequencies and damping factors as well as radiativities). We have also argued (Richardson[12]) that the ratio of the G/M (what we have labelled the "acoustical merit" of body modes) have an important influence not just close but resonance but also well above the resonance of some of the lowest modes. These generate the tall, low-frequency peaks in admittance and sound pressure response evident in Figure 4. Whilst these modes undoubtedly play an important role in the mechanical and acoustical action of an instrument like the guitar, it seems clear from these studies that the low-order modes alone are not able to predict the mid- and high-frequency response of the instrument adequately and that the initial amplitudes in the 1-5 kHz range play an important role in determining sound quality. Hence, these regions now require further attention. The model overlap in these regions (shown partly in Figure 4) suggests that some broad statistical measures may be adequate (it is not viable to made the sound field measurements described by Hill et al.[6] in these more densely-packed modal regions).



(a) *(b)*

Figure 4. Measured and reconstructed (a) input admittance at a guitar bridge, and (b) sound-pressure response of a guitar.

4 REFERENCES

1. J. Rolph, Unpublished MPhys Project, Cardiff University. (2007).
2. M. Weston, Unpublished MPhys Project, Cardiff University. (2007).
3. H.A.K Wright, The acoustics and psychoacoustics of the guitar. PhD Thesis, University of Wales. (1996).
4. H.A.K. Wright & B.E. Richardson, On the relationships between the frequency response of the guitar body and the instrument's tone, Proc. Inst. Acoust., Vol.19(5), 149-154. (1997).
5. C. Erkut, M. Karjalainen, P. Huang & V. Välimäki, Acoustical analysis and model-based sound synthesis of the kantele, J. Acoust, Soc, Am., Vol.112(4), 1681-1691. (2002).
6. T.J.W. Hill, B.E. Richardson & S.J. Richardson, Acoustical parameters for the characterisation of the classical guitar, Acta Acustica united with Acustica, Vol. 90, 335-348. (2004).
7. T.J.W. Hill, B.E. Richardson & S.J. Richardson, Input admittance and sound field measurements for ten classical guitars. Proc. Inst. Acoust., Vol.24. (2002).

8. J. Woodhouse, On the synthesis of guitar plucks, Acta Acustica united with Acustica, Vol. 90, 928-944. (2004).
9. J. Woodhouse, Plucked guitar transients: comparison of measurements and, Acta Acustica united with Acustica, Vol. 90, 945-965. (2004).
10. C. Valette, The mechanics of vibrating string, Chapter 4 of Mechanics of Musical Instruments (Ed. Hirchberg, Kergomard & Weinreich), Springer-Verlag. (1995).
11. M. Pavlidou, A physical model of the string-finger interaction in the classical guitar. PhD Thesis, University of Wales. (1997).
12. B.E. Richardson, Simple models as a basis for guitar design. J. Catgut Acoust. Soc., Vol.4(5), 30-36. (2002).