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Acoustical road parameters identification

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ABSTRACT

Many impedance models can be used for describing the acoustical behahviour of a road. These models rely on some intrinsic parameters (2,3 or 4), which are difficult to measure in the field. Therefore, the measurement of these parameters usually requires to destroy the structure. This is not always a good way to caracterize these parameters, because first those are only measured in a very local area, which may not have a global meaning, and secondly the way of building and the evolution in time of the road are not taken into account. The present paper deals with numerical simulations based on Simulated Annealing in order to set up a method for road acoustical parameters identification. The main idea is to consider any pavement, to make *in situ* acoustic impedance measurements and from these experimental data, to retrieve the intrinsic parameters of the road.

1. INTRODUCTION

The acoustic behaviour of road surfaces depends on intrinsic parameters like porosity or tortuosity. The access to these parameters is a key to understand the progressive degradation of the acoustic performance of road surfaces, and drainage pavements in particular.

Parameters identification is an aspect of the general inverse problem. It relies on a direct resolution, a model, which constitutes the starting line of the problem. That is why the better way the model describes the physical phenomenon, the more pertinent the identification is.

The predictive model here is fixed and is considered as a reference resolution of the direct problem. The main interest of the approach is to use acoustical measurements in order to access to the intrinsic parameters without destroying the structure. This non-destructive approach is even more interesting because laboratory tests on samples have no global meaning and are most of the time difficult to prepare. A method based on *in situ* measurements is of course closer to the behaviour of a "real" road. The approach is very similar to the work about surface waves on pavement[4] .

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This inverse problem approach on *in situ* absorption coefficient measurements of porous road pavements has already been investigated[1] . One of the conclusions of this previous work is that using the impedance Z instead of the absorption coefficient would ease the resolution of the inverse problem because Z contains much more information. The purpose of this paper is to study numerically a Z-based inversion.

The present inversion problem is non linear, the space to explore is of dimension higher than 3, and the cost function is non convex. This is why this inversion is not trivial and a numerical method has been used. This is method is the Simulated Annealing (SA), presenting the inversion like a optimization problem. The target value which is considered is not the absorption coefficient, but the impedance given by the model. This choice is explained hereafter.

The paper is organized as follows. First the impedance model of Hamet and Bérengier is recalled. Then the specifically designed SA algorithm is exposed. The impedance model is used with the numerical method. Lastly, results are presented and discussed.

2. IMPEDANCE MODEL

A. Description of the model

This identification work relies on the model of Hamet and Berengier[2] . This model is a fusion from two models, one to take into account viscous phenomena and a second to take into account thermal phenomena.
There are 4 parameters : the flow resistiviy σ, the tortuosity α_∞, the porosity Ω and the thickness d.
These parameters allow for characterizing the acoustical behaviour of porous asphalt.

Here are the formulas for this impedance model. f_μ and f_θ are homogeneous to frequencies : $f_\mu = \frac{1}{\pi}\frac{\sigma}{\rho_0}\frac{\Omega}{\alpha_\infty}$ $\quad$ $f_\theta = \frac{1}{\pi}\frac{\sigma}{\rho_0}\frac{1}{N_{Pr}}$
The wave number κ is given by :

$$\kappa = \frac{2\pi f}{c_o}\sqrt{K\gamma}\left(1+\frac{{f_\mu}^2}{f^2}\right)^{1/4}\left(\frac{f^2/\gamma^2+{f_\theta}^2}{f^2+{f_\theta}^2}\right)^{1/4}\mathrm{e}^{-\frac{\mathbf{i}}{2}\left(\arctan\frac{f_\mu}{f}+\arctan\frac{f f_\theta(\gamma-1)}{\gamma f^2+{f_\theta}^2}\right)} \quad (1)$$

The characteritistic Impedance Z_∞ :

$$\mathrm{Z}_\infty = \frac{\rho_0 c_0}{\Omega}\sqrt{\frac{K}{\gamma}}\left(\frac{1+\frac{{f_\mu}^2}{f^2}}{\frac{f^2/\gamma^2+{f_\theta}^2}{f^2+{f_\theta}^2}}\right)^{1/4}\mathrm{e}^{-\frac{\mathbf{i}}{2}\left(\arctan\frac{f_\mu}{f}-\arctan\frac{f f_\theta(\gamma-1)}{\gamma f^2+{f_\theta}^2}\right)} \quad (2)$$

The usual formula to compute the characteristic impedance Z_c is :

$$\mathrm{Z}_c = \mathrm{Z}_\infty \mathrm{cotanh}(\kappa d) \quad (3)$$

B. Analytical expression

The impedance can be expressed analytically as a complex quantity $\mathrm{Re}_{\mathrm{Z}_c} + \mathbf{i}\mathrm{Im}_{\mathrm{Z}_c}$:

$$\mathrm{Z}_c = \mathrm{Z}_\infty \left(\frac{\sinh\left(2\,d\,\mathrm{Im}_\kappa\right) - \mathbf{i}\sin\left(2\,d\,\mathrm{Re}_\kappa\right)}{\cosh\left(2\,d\,\mathrm{Im}_\kappa\right) - \cos\left(2\,d\,\mathrm{Re}_\kappa\right)} \right) \tag{4}$$

$$\mathrm{Re}_{\mathrm{Z}_c} = \frac{\mathrm{Re}_{\mathrm{Z}_\infty}\sinh\left(2\,d\,\mathrm{Im}_\kappa\right) + \mathrm{Im}_{\mathrm{Z}_\infty}\sin\left(2\,d\,\mathrm{Re}_\kappa\right)}{\cosh\left(2\,d\,\mathrm{Im}_\kappa\right) - \cos\left(2\,d\,\mathrm{Re}_\kappa\right)} \tag{5}$$

$$\mathrm{Im}_{\mathrm{Z}_c} = \frac{\mathrm{Im}_{\mathrm{Z}_\infty}\sinh\left(2\,d\,\mathrm{Im}_\kappa\right) - \mathrm{Re}_{\mathrm{Z}_\infty}\sin\left(2\,d\,\mathrm{Re}_\kappa\right)}{\cosh\left(2\,d\,\mathrm{Im}_\kappa\right) - \cos\left(2\,d\,\mathrm{Re}_\kappa\right)} \tag{6}$$

The new aspect here is only the presentation of the model under an analytical form for the formula of impedance Z_c.

3. SIMULATED ANNEALING

A. Principle of the method

The Simulated Annealing method is a combination between a random walk in a space and the Metropolis criterion. The name annealing comes from crystalline solids physics[5] . The idea is to start from a initial temperature T_0 with the same unit as the objective function and a cooling parameter a (about 0.9). Each iteration j involves a random choice of each parameter. Figure 1 gives the flow chart of the algorithm.

The Metropolis rule consists in the fact that the new set of parameters may be accepted if $r < \exp\left(-\dfrac{\mathrm{CS}(\mathrm{p}_j) - \mathrm{CS}(\mathrm{p}_i)}{\mathrm{T}}\right)$ where r is a random number from 0 to 1.

B. The key choices

As all numerical methods, SA is effective under some conditions.
The first thing is to define how to generate a "neighbour". In a multi-dimensional space, the term of "neighbour" depends strongly on the physical meaning of the parameters.
The second important condition is the temperature law. If the temperature T decreases too fast, the method is said to "freeze", that is the optimal solution is not reached, but the generated neighbours are too close to improve significantly the objective function. This happens when SA gets "stuck" in a local minimum. The choice of the temperature law is not easy because it is strongly relying on the way of choosing neighbours. In the present paper, the temperature law is the same for all parameters, but it is possible to make a specific law for each parameter.
A common cooling schedule is the exponential, this is the temperature law that is used in the present paper.

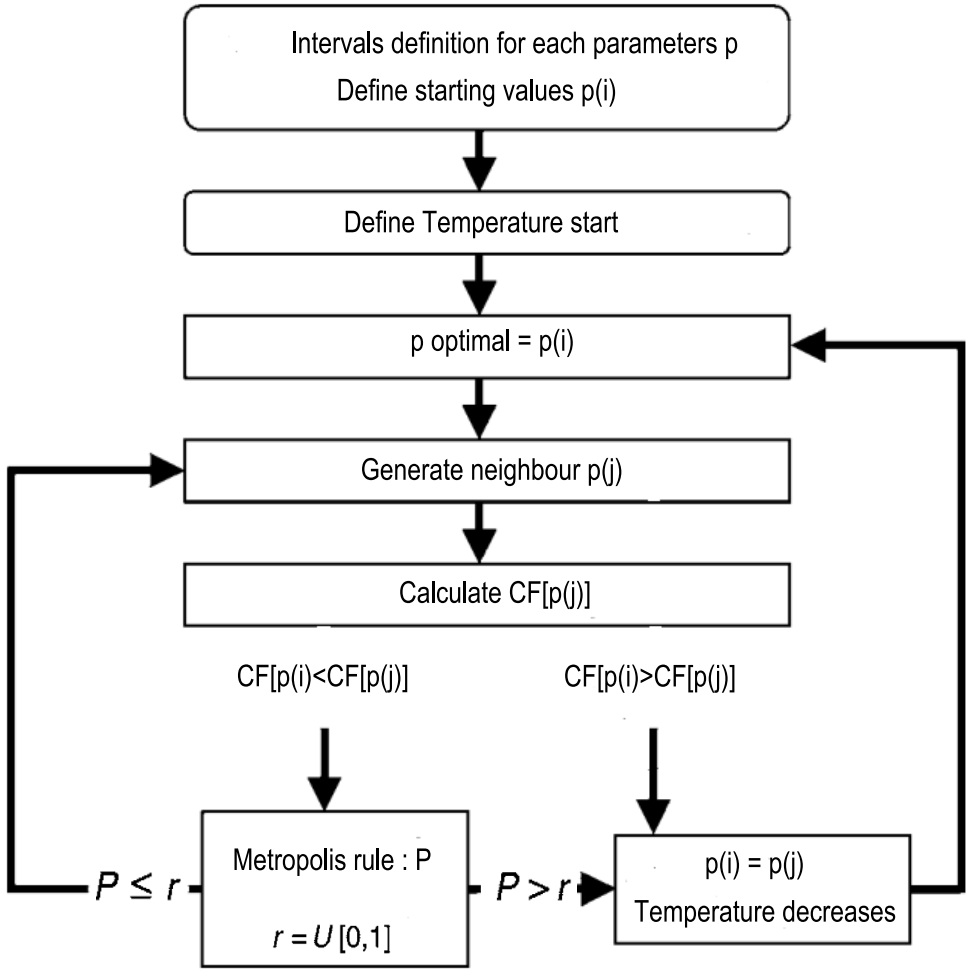

Figure 1: Description step by step of the principle of Simulated Annealing method. CF stands for Cost Function

4. THE INVERSION PROBLEM

A. Generating neighbours

Generating neighbours in a space of dimension 4 for continous parameters needs some abstraction. The fact is that this step is crucial for the efficiency of the method on the one hand, and on the other hand the physics of parameters should be taken into account. This last point is difficult and was not treated in the present paper.

The research of neighbours is done here through a perturbation way, that is for a current point x_i, a neighbour x_{i+1} is generated by :

$$x_{i+1} \;=\; x_i \;+\; \left(1 - \mathrm{e}^{-T}\right)\; x_i\, \eta_1\, \frac{\arctan(\eta_2)}{\arctan(\eta_1)} \tag{7}$$

where η_1 and η_2 are random numbers in $[-1, 1]$.

This approach differs from the Fast Simulated Annealing[3], but for our application FSA generation of neighbours gives very often values corresponding to the minimum or the maximum values allowed for the parameters. This generating function is smoother at edges. Therefore, the case of generation of extreme values seldom occurs.

B. Temperature law

The temperature law is the exponential classic one. It is important to note that the generation of neighbours has been also modified in its temperature dependance, in replacing T by $1 - \mathrm{e}^{-T}$.
It is possible at this stage to change the temperature law, which can contribute to a better exploration of the parameters space, but it can also leeds to a excessive number of computations.

C. Cost function

The cost function is computed from the analytical expressions f_{re} and f_{im} given by (5) and (6). The frequency discretization has been fixed from 100 Hz to 4000 Hz with a step of 5 Hz. $\mathrm{Real}Z_c$ and $\mathrm{Im}Z_c$ are computed on this frequency scale.

In this numerical study, a set of realistic target values has been chosen : $\sigma^{\mathrm{target}} = 50901$, $\alpha_\infty^{\mathrm{target}} = 2.4$, $\Omega^{\mathrm{target}} = 0.252$ and $d^{\mathrm{target}} = 0.12$. Then $\mathrm{Real}Z_c^{\mathrm{target}}$ and $\mathrm{Im}Z_c^{\mathrm{target}}$ has been computed.
The aim is to start from $\mathrm{Real}Z_c^{\mathrm{target}}$ and $\mathrm{Im}Z_c^{\mathrm{target}}$ on the frequency band and to retrieve the target values σ^{target}, $\alpha_\infty^{\mathrm{target}}$, Ω^{target} and d^{target}.

The chosen cost function is a quadratic error function at each frequency. The better the solution, the lower the error. For numerical considerations and because of the large scale of

variation, this error has been put in a $\log$ function to avoid very big numbers.

$$\begin{aligned} \mathrm{Real}Z_c &= f_{re}(\sigma, \alpha_\infty, \Omega, d) && (8)\\ \mathrm{Im}Z_c &= f_{im}(\sigma, \alpha_\infty, \Omega, d) && (9)\\ \text{Cost Function} &= 10 \log \left[\sum_f \left(\mathrm{Real}Z_c - \mathrm{Real}Z_c^{\mathrm{target}}\right)^2 + \left(\mathrm{Im}(Z_c - \mathrm{Im}Z_c^{\mathrm{target}}\right)^2 \right] && (10) \end{aligned}$$

D. Starting conditions

Each value lies in an interval. This interval comes from the physical sense of the parameter :

$$\begin{aligned} 1 \leq \alpha_\infty \leq 10 && (11)\\ 0 \leq \Omega \leq 0.5 && (12)\\ 10000 \leq \sigma \leq 70000 && (13)\\ 0.02 \leq d \leq 0.5 && (14) \end{aligned}$$

Then a starting value is necesseray for each parameter as described in table 1 :

	α_∞	Ω	σ	d
target values	2.4	0.252	50901.	0.12
starting values	1.2	0.4	40000	0.035

Table 1 : Starting values.

It can be noticed here that values have been selected quite "far" from the target values.

5. RESULTS

A. Number of shots

The number of shots here is not fixed. What is fixed is the value of the cost function, that is a minimum to reach. Table 2 is the result with a minimum for the cost function fixed to 20, which has taken about 7000 shots. The temperature is increased regulary and the algorithm is restarted from the last best solution.

	α_∞	Ω	σ	d
target values	2.4	0.252	50901.	0.12
SA values	2.3931158	0.2516366	50825.757	0.1201745

Table 2 : Results of the SA for about 7000 shots.

As it can be noticed, results are quite in good agreement with target values, except for the flow resistivity. This is why a second simulation has been carried on with a lower condition on the cost function. Results are presented in table 3. The number of shots has been multiplied by 2 and the flow resitivity is still not the exact one. On the convergence point of view, the method is quite slow. The difficulty is that it is not possible to know exactly the required computation time.

	α_∞	Ω	σ	d
target values	2.4	0.252	50901.	0.12
SA values	2.4041784	0.2522196	50946.121	0.1198947

Table 3 : Results of the SA for about 14000 shots.

B. Graphical results

Figure 2(a) describes the behaviour of the cost function during the simulation with 14000 shots. It decreases much faster at the begining. On figure 2(b), there is a zoom in order to illustrate the fact that the temperature is increased at each 100 shots in order to keep finding better solutions.

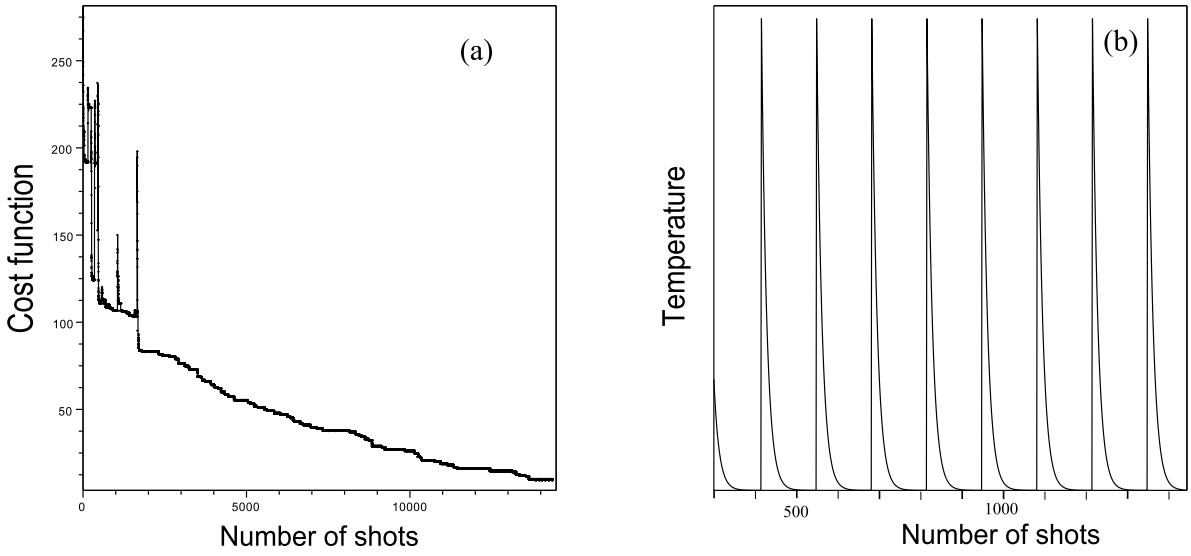

Figure 2 : (a) Evolution of the Cost function related to the number of shots. (b) Illustration of re-annealing by increasing temperature.

6. CONCLUSION

The method has been shown to work on a simulated measurement data set. The consideration of the impedance as a target value with the Simulated Annealing as an optimization algorithm allows for finding the intrinsic road's parameters.
Of course this is only a numerical method tested on simulated data. It has too be first tested with noise on impedance values and then on real experimental data.

The numerical inversion used here is a general procedure than can be carried on with other impedance models.

When using global optimization tools, it is difficult to know *a priori* what is the most efficient algorithm for a given inverse proble. From this application of SA, it can be seen that a genetic algorithm could be a good replacement, because the generation of new solutions can be designed to automatically respect to constraints on the definition intervals of each parameter to be optimized.

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