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BOUNDARY ELEMENT ANALYSIS OF PROBLEMS OF ACOUSTIC PROPAGATION IN VISCOTHERMAL FLUID

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1. INTRODUCTION

The study of acoustic propagation by taking in to account the viscosity and thermal conductivity of a fluid, has received a lot of interest during the last twenty years, with the development of miniaturised transducers. When the size of the cavity is in the order of the thickness of the acoustic or thermal boundary layer, the account of these effects becomes necessary [1-4]. This paper presents a variational formulation using integral equations for the resolution of problem of propagation in a viscothermal fluid. This approach gives after discretisation by finite boundary element method a linear symmetrical system of relatively small size. Numerical results predicted by this formulation are in good agreement with analytic ones.

2. EQUATIONS AND BOUNDARY CONDITIONS

The basic relations which govern the characteristic variables of a viscothermal fluid (Fig. 1.) such as the particle velocity $\vec{v}$, the variable part of the density ρ, the pressure variation p and the temperature fluctuation T, can be established by using fundamental mechanical equations and thermodynamical state equations. In the absence of external forces and rate of heat creation and at initial state, the fluid is at rest (i.e.: density, pressure, etc..., are uniform in space and time), the set of linear homogeneous equations governing small amplitude disturbances for harmonic motion ($\partial_t = -i\omega$), includes the following [3] :

$$p = p_a + p_h \tag{1}$$

$$\left(\Delta + k_a^2\right) p_a = 0 \tag{2}$$

$$\left(\Delta + k_h^2\right) p_h = 0 \tag{3}$$

$$T = T_a + T_h = \tau_a\, p_a + \tau_h\, p_h \qquad (4)$$

$$\vec{v} = \vec{v}_e + \vec{v}_t \qquad (5)$$

$$\left(\Delta + k_v^2\right) \vec{v}_t = 0 \quad ; \quad \operatorname{div} \vec{v}_t = 0 \qquad (6)$$

$$\vec{v}_e = \vec{v}_e^a + \vec{v}_e^h = \phi_a \,\overline{\text{grad}}\, p_a + \phi_h \,\overline{\text{grad}}\, p_h \qquad (7)$$

p_a is the acoustic pressure, p_h is the entropic pressure, T_a is the acoustic temperature, T_h is the entropic temperature, $\vec{v}_e$ is the irrotational velocity and $\vec{v}_t$ is the rotational velocity.

$$k_a^2 = k^2(1 - i\,k\,\ell_{vh} - k^2 \ell_h \ell'_{vh})^{-1},\; k_h^2 = \frac{ik}{\ell_h}(1 + i\,k\,\ell'_{vh})^{-1},\; k_v^2 = \frac{i\,k}{\ell'_v},\; k = \frac{\omega}{c}$$

$$\tau_{a,h} = \frac{\gamma - 1}{\beta\gamma}\left(1 + i\,\frac{c}{\omega}\,\ell_h\,k_{a,h}^2\right)^{-1},\; \phi_{a,h} = \frac{-i}{\rho_0\,\omega}\left(1 + i\,\frac{c}{\omega}\,\ell_v\,k_{a,h}^2\right)^{-1},\quad \ell_h = \frac{\chi}{c}$$

$$\ell_{vh} = \ell_v + (\gamma - 1)\,\ell_h, \qquad \ell'_{vh} = (\gamma - 1)(\ell_h - \ell_v), \quad \ell_v = \frac{\eta + \frac{4}{3}\mu}{\rho_0 c},\; \ell'_v = \frac{\mu}{\rho_0 c}$$

β is the increase in pressure per unit increase in temperature at constant density, μ is the dynamic viscosity, η is the bulk viscosity, ρ_0 is the density, χ is the thermal diffusivity, γ is the specific heat ratio, c is the adiabatic speed of sound.

We consider that the edge of domain Ω is composed of two surfaces, S_p when pressure $\overline{p}$ is imposed and S_v when the velocity $\overline{V}$ is imposed. Also it is assumed that the normal component of rotational velocity is negligible as compared to that of the irrotational velocity $\vec{v}_e$[5] and the temperature fluctuation is null on the boundaries . With these assumptions we have :

Over S_p :

$$p_a = \frac{-\tau_h \overline{p}}{\tau_a - \tau_h} \qquad (8)$$

$$p_h = \frac{\tau_a \overline{p}}{\tau_a - \tau_h} \qquad (9)$$

Over S_v :

$$\phi_a \frac{\partial p_a}{\partial n} + \phi_h \frac{\partial p_h}{\partial n} + v_t^n = \overline{v}_n \quad \text{(normal component)} \qquad (10)$$

$$T = \tau_a\, p_a + \tau_h\, p_h = 0 \qquad (11)$$

3. VARIATIONAL FORMULATION BY INTEGRAL EQUATION

An integral representation of acoustic and entropic pressure is used for solving the pressure calculation problem. It is based on determining the two potentials [6], one of simple layer and the other of double layer :

$$p_a(X) = \int_S \left(-p_{aY} \frac{\partial G_a}{\partial n_Y} + G_a(X,Y) \frac{\partial p_a}{\partial n_Y}\right) dS_Y \qquad X \in \Omega / S \qquad (12)$$

$$p_h(X) = \int_S \left(-p_{hY} \frac{\partial G_h}{\partial n_Y} + G_h(X,Y) \frac{\partial p_h}{\partial n_Y}\right) dS_Y \qquad X \in \Omega / S \qquad (13)$$

where $\vec{n}$ is the external normal vector of regions containing fluid; G_a and G_h

are the elementary solutions of equation (2) and (3) in the free space.

The application of boundary conditions (8) and (9) over S_p, (10) and (11) over S_v, at integral representations of p_a and p_h and their normal derivatives, gives the system of equations which enables us to determine the unknowns of the problem : $\frac{\partial p_a}{\partial n}$ and $\frac{\partial p_h}{\partial n}$ over S_p, p_a and $\frac{\partial p_a}{\partial n}$ over S_v.

Instead of resolving this system by the usual collocation technique which presents the difficulties at the level of numerical computation associated with integral singularities of Green functions (G_a and G_h), we suggest to associate to it a variational formulation which avoid these difficulties. The discretisation of this formulation for the problems with revolution geometry by finite boundary element, gives a linear symmetrical algebraic system of relatively small size [7] :

$$[D]\{\tilde{P}\} = \{F_a\} \qquad (14)$$

4 NUMERICAL RESULTS

In this section we consider a cylindrical cavity filled with air (Fig. 2.) whose physical properties are: ρ_0 = 1.1614 Kg/m^3, c = 340 m/s, μ = 184.6 10^{-7} Ns/m^2, η = 110 10^{-7} Ns/m^2, β = 458 N/(m^2 °K), χ = 22.5 10^{-6} m^2/s, γ = 1.403. The fluctuation temperature is assumed to be null at all boundaries. The pressure is zero at the boundary r = R. For z = L/2, the normal speed is imposed to $\overline{V} J_0 (\chi_{01} \frac{r}{R})$ where $\overline{V} = 1 m/s$. Finally, at z = - L/2, the speed is null.

All results presented in this paper are nondimensionalized by referring them to the following parameters : ρ_0 (the density), c (the adiabatic speed of sound), L (the height of the cavity) and $T_0 = 293°K$ (the ambient temperature).

Fig. 3 shows the non-dimensional modulus of the acoustic pressure at point A1 (Fig. 2.), versus to the non-dimensional frequency. The obtained peak corresponds to the first radial mode (f = 0.003827). Numerical and analytical results of viscothermal model are in good agreement and are clearly different from those of perfect fluid model. One clearly sees the damping of the first radial mode and the decrease of the first resonance frequency which is about 15 %.

For the frequency 0.0037 (close to the first resonance frequency of the cavity with perfect fluid model), we have carried the non-dimensional modulus of acoustic pressure according to radial position (z=0) in Fig. 4. This later shows the validity of our numerical results and illustrates, clearly the difference between the results of perfect fluid model and those of viscothermal model.

Acknowledgments

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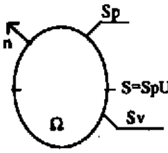

Fig. 1.

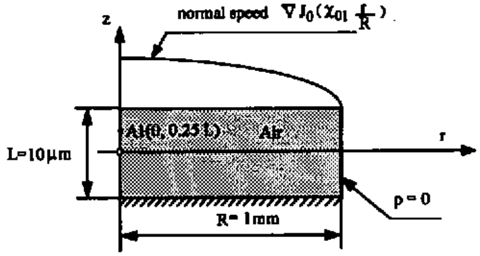

Fig. 2.

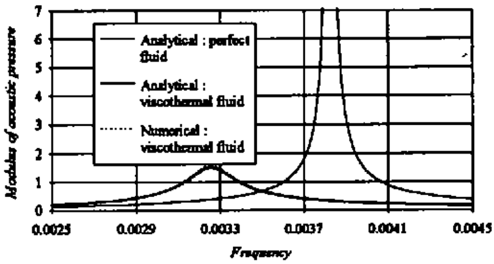

Fig. 3.

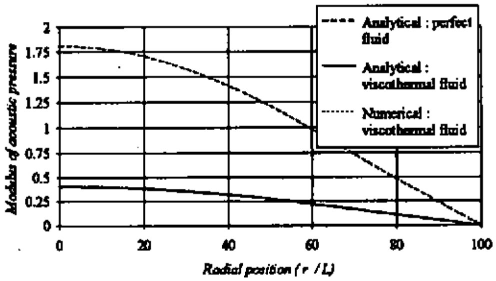

Fig. 4.

References

[1] P. M. Morse and K. Uno Ingard, "Theoretical acoustics," Princeton University Press (1986).

[2] M. Bruneau, "Introduction aux théories de l'acoustique," Université du Maine Editeur, Le Mans, France (1983).

[3] M. Bruneau, Ph. Herzog, J. Kergomard and J. D. Polack, "General formulation of dispersion equation in bounded visco-thermal fluid, and application to simple geometries," Wave Motion 11, 441-451 (1989).

[4] M. Bruneau and G. Plantier, "Heat conduction effects on the acoustic reponse of a membrane separated by a very thin air film from a backing electrode," J. Acoustique 3, 243-250 (1990).

[5] E. Dokumaci, "An integral equation formulation for boundary element analysis of acoustic radiation problems in viscous fluids," J. Sound Vib. 147 (2), 335-348 (1991).

[6] M. A. Hamdi, " Formulation variationnelle par équations intégrales pour la résolution de l'équation de Helmholtz avec des conditions aux limites mixtes," C.R. Acad. Sci. Paris, Série II, t. 292, 17-20 (1981).

[7] C. Karra, M. Ben Tahar, "Formulation variationnelle par équations intégrales pour la résolution des problèmes de propagation acoustique dans un fluide thermovisqueux". Proceeding of Strucome 95, Paris, (1995), 432-443.