Proceedings of the Institute of Acoustics

MATERIALS SELECTION FOR MUSICAL INSTRUMENTS

C Y Barlow

Cambridge University Engineering Department, Mill Lane, Cambridge CB2 1RX

ABSTRACT

This article explores some ways in which materials may be selected for use in applications including musical instruments. A powerful methodology is demonstrated, the idea of mechanical property maps of different classes of materials is introduced, and mention is made of the importance of "shape factors" and microstructure in the selection of a material. Examples are used to illustrate how a materials database presented in the form of maps can be used to optimise the choice of material for a given application.

1. INTRODUCTION

The musical instruments familiar to us nowadays are mostly little changed from their form some centuries ago. Many primitive instruments were made rather directly from materials which were to hand — such as drums made from tree trunks and stretched animal hide, or flutes made from bones. This tradition has been continued into recent times, for example with the invention of Caribbean steel pans in Trinidad during the second World War. The local population manufactured instruments from the oil drums which littered the island as a consequence of the oil refinery activities. However, not all instruments have been made from materials readily to hand, and even from earliest times there is evidence suggesting that some specialist supplies for musical instruments were imported from considerable distances (e.g. metal harp strings [1]).

It is increasingly becoming necessary to seek suitable materials to substitute for what has traditionally been used for musical instruments. The impetus comes from several directions. One factor is 'green' issues: some tropical hardwoods used in musical instruments are regarded as non-renewable resources. Another is the increasing scarcity of wood of acceptable quality due to such factors as acid rain and war (the war in Bosnia has affected a prime area for maple). It may even be that a substitute artificial material is better in some way than the natural material it replaces (metal-wound polymer strings, for example, have to a large extent taken over from gut).

The choice of what material to use for making a musical instrument is an issue which arouses strong emotions and is fraught with difficulty. People are used to seeing particular materials used, say for violins, and anything departing from the norm is not likely to be well received. It is a safe bet that a violin which looked as if it was made from fibreglass would not be perceived as sounding good by an audience who could see what was being played, no matter what it sounded like. This is just one example of a general phenomenon that evaluation of instruments is largely based on factors other than the sound of the instrument. Extracting quantitative selection criteria from players or makers about what makes a 'good' instrument is a minefield.

There are two approaches one can use when trying to find a substitute material. One is to seek a material (or combination of materials) which gives the same mechanical properties as what is being replaced. This approach automatically limits the choice of materials, so that if one is trying to replace a particular wood species, for example, the search is likely to suggest only other wood species (e.g. [2,3]). The other approach is to analyse the system to discover what the optimum properties of the material should be for it to function as efficiently as possible. This approach is potentially more general in that it makes no prior assumptions about the new material, but a systematic method has only recently been developed. As we will see, it is based on a graphical method which has great intuitive appeal.

2. EXAMPLES OF MATERIALS SELECTION.

Why are trees made of wood? Why wouldn't people buy a bicycle made from glass? Why are carbon fibre tennis racquets so desirable? Different materials have mechanical and physical properties which may differ by many orders of magnitude, making them suitable for different applications. Up to a point, our natural instincts about material properties lead us to make sensible judgements about what materials to use. Glass is clearly not the right material for a bicycle, but to understand exactly why this is so we need to consider which mechanical properties render it unsuitable. It turns out that glass does not have good enough tensile properties: its strength and toughness are too low. To decide which materials are suitable we need to analyse a number of factors, and choose materials with properties at least adequate for the task. Broadly, the stresses which the bicycle has to withstand will determine the strength required of the material; the amount of energy it can absorb on impact without disintegrating will determine the minimum necessary material toughness; the amount it is allowed to flex in use will determine the elastic modulus of the material; its weight will be influenced by the material density; and the whole design may be constrained by cost.

Optimisation of the choice of a material for a system will generally involve not a single material property but some combination of them. We can illustrate this by looking at a simple system: we will attempt to discover what sorts of materials would be suitable for making a tree. The first stage is to decide what the limiting factors are, and so to decide what material properties (or combinations of them) are important.

2.1. Example 1: Why wood grows on trees.

A tree is a very complex system, which has the fundamental purpose of surviving and propagating, often in competition with other trees. It needs to gather nutrients in order to grow, and one of the critical parameters for a successful tree may be to maximise the area of sunlight it can monopolise for photosynthesis. Long branches will therefore be important. It will also be important that the branch does not bend too much under its own weight: drooping branches will not stick out so far from the tree, so the efficiency of sunlight collection may be impaired, or the branches may touch the ground (whereupon they might be consumed by herbivores). The tree needs to manufacture its branches in the most energy-efficient way possible, so as to use the smallest mass of material. This will mean that the branches are as thin as possible, but the most general way of expressing this is to look for the material which will give the minimum mass of a branch.

Setting these criteria into analytical terms, we can try to choose a material to minimise the mass m of a branch with given length L, a certain maximum end deflection d, and variable diameter d.
The branch will bend under its own weight, which we can model as a uniformly distributed load (though the final result does not depend on how the load is distributed). The end deflection of a circular-section beam under distributed loading due to its own weight is:

$$\delta = \frac{2L^4 \rho g}{E d^2} \tag{1}$$

where E is the longitudinal elastic (Young's) modulus, ρ is the material density, and g is the acceleration due to gravity. The mass of the branch is given by

$$m = \frac{\pi d^2 L \rho}{4} \tag{2}$$

The branch diameter d is a variable which appears in both equations, and we have to eliminate it to obtain

$$m = \frac{\pi L^5 \rho^2 g}{2 \delta E} \tag{3}$$

The only variables in this expression for the mass of the branch are now the material constants E and ρ: we have fixed everything else. To minimise m we must maximise E/ρ^2. This parameter is called the *index of merit* of the system. If one has tables of data, it is a simple (but tedious) matter to calculate ratios for a range of materials and so find the optimum material. However, a more elegant method uses the map in Fig. 1. These maps have been invented and developed by Ashby [4].

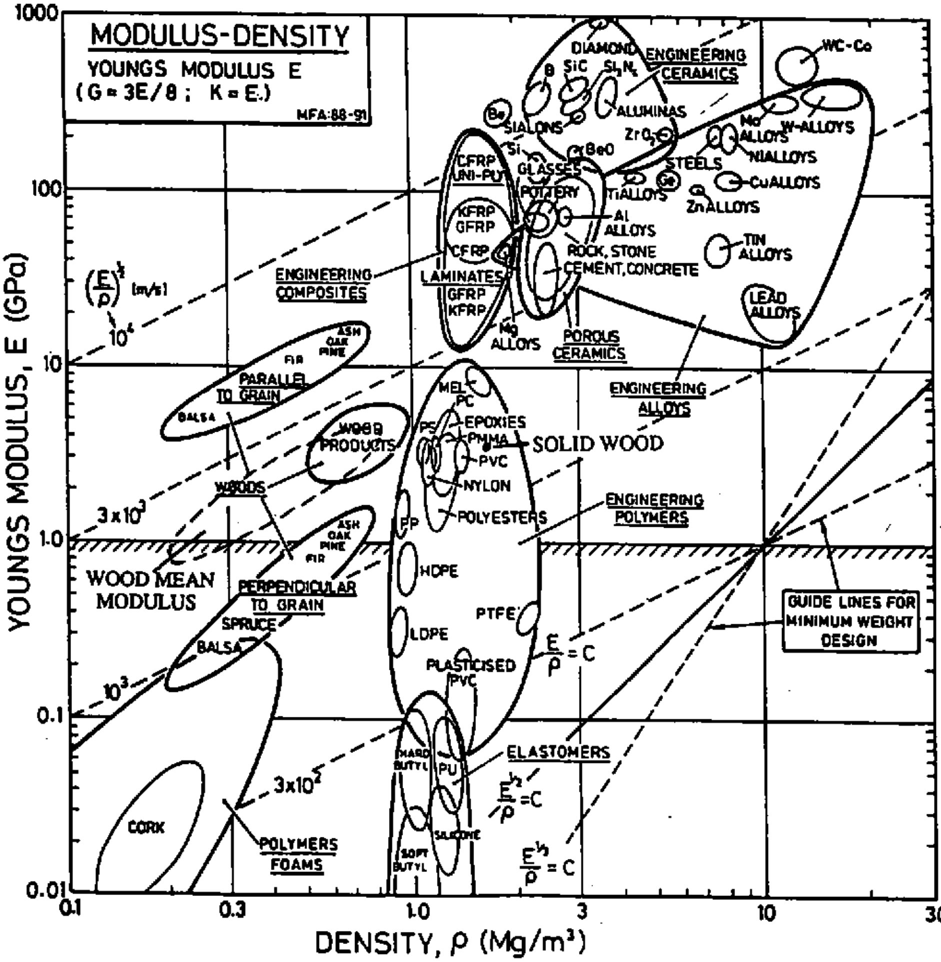

Figure 1. Map showing elastic modulus and density for the main classes of engineering materials. There is a huge amount of information on the map, and the examples used here look at only a small part of what is available. Two pieces of information are added to this map: the balloon labelled 'Wood mean modulus' is shown faintly between the two balloons for wood parallel and perpendicular to the grain, and the point for the cell wall properties in the longitudinal direction is labelled 'Solid wood'.

The map presents data from all the major classes of solids materials: any particular material will have certain values of E and ρ and be represented by a point on this diagram. Each material class is enclosed in a balloon, and inside the balloon are smaller bubbles corresponding to specific materials in the class. The axes are logarithmic, to cope with the enormous variation between different classes of materials. For example, up at the top of the chart we find an envelope labelled 'Engineering ceramics', within which there is a bubble labelled 'SiC', silicon carbide. This is familiar as a very hard, brittle material, used as an abrasive. In the middle of the chart we have 'Engineering Polymers', covering materials with a huge range of elastic modulus (differing by more than two orders of magnitude). PC (polycarbonate) in the upper part of the envelope is a rigid transparent material, about half the density of glass but much more bendy, used amongst many things for safety glasses. Plasticised PVC at the bottom is a very flexible material, used in thin films (clingfilm) or as a coating on fabric (e.g. for waterproof clothing).

To use the chart for the merit index derived above, we first note that for the parameter we wish to optimise, E/ρ^2, straight lines of slope 2 correspond to the locus of points which will give equally good performance. This simple fact is a consequence of the use of logarithmic axis scales. The optimum materials for our parameter lie towards the top left of the chart, and by drawing a set of lines with slope 2 we can choose the material which gives the highest value of E/ρ^2.

There are three classes of materials which perform about equally well on this criterion: woods parallel to the grain, engineering composites and engineering ceramics. Specifically, the materials which give the optimum properties are balsa wood, CFRP uniply, and diamond. Carbon Fibre Reinforced Plastic is related to fibreglass, but uses the lighter and stronger (and more expensive) carbon fibres in place of glass fibres. The 'uniply' requires all the fibres to lie parallel to each other: the mechanical properties perpendicular to the fibre direction are very poor. We now need some further input to decide which of these materials would be feasible. Common sense dictates that diamonds are small, expensive and difficult to join together, so this might not in fact be the ideal material for a large structure like a tree. It is encouraging to find that wood is actually as good as the much more expensive CFRP.

2.2. The importance of shape factors

If one were designing something like a tree branch one would not necessarily choose a solid cylindrical shape. An aircraft wing, for example, could well be designed on the criterion cited above, and to maximise the bending stiffness one would naturally choose the wing to have a solid shell around a substantially empty centre (the principle of the I-beam). Many natural material embody these principles to use their material more efficiently. Bamboo, for example, which needs to grow as tall as possible whilst minimising the mass of material used, is constructed of hollow tubes. A thin tube (like a drinking straw) tends to collapse flat; bamboo has circular internal bracing plates at intervals across the stem to prevent this happening.

Wood is an internally microstructured material. The microstructure of wood falls into two classes depending on whether it is a hardwood or a softwood. Softwoods (e.g. pine or spruce) have the simpler structure, being composed mainly of tracheid cells running vertically in the tree (fig. 2). These cells are hollow, and the effect is very much that of an array of parallel thin-walled tubes glued together. The structure is very stiff if the cells are compressed along their length, corresponding to the longitudinal direction in the tree, but if one compresses perpendicular to this direction (radial or tangential in the tree) then the cell walls can deform by bending, and the stiffness is much less. This leads to highly anisotropic properties, with a longitudinal elastic modulus which may be an order of magnitude or more greater than that in the radial direction.

This simple account of the microstructural reasons for the elastic anisotropy of softwood can explain a feature of Fig. 1. In the cross-sectional plane, wood structure consists of an irregular honeycomb of thin plates. Suppose that a typical one of these plates has length L (in the cross-sectional plane) and thickness h. If the density of the cell-wall material is ρ_s, the mass of this typical plate is $Lh\rho_s$ per unit length along the grain. The cross-sectional area of typical cell is proportional to L^2, and it follows that the density of the wood, ρ, satisfies

Figure 2. Scanning electron micrograph of spruce (picea abies) cut to show the shape of the tracheids in the three main directions in the tree.

$$\rho \propto \left(Lh / L^2\right)\rho_s = (h / L)\rho_s \, . \qquad (4)$$

Now for compression of the wood in the axial direction, the stiffness is determined simply by the Young's modulus of the cell wall material in that direction, E_{s1} say, and the total proportion of wood rather than empty space in the cross-sectional plane. Thus the axial Young's modulus E_1 satisfies

$$E_1 \propto \left(Lh / L^2\right)E_{s1} \propto (\rho / \rho_s)E_{s1} \, . \qquad (5)$$

In the transverse direction the stiffness of the wood is governed by the bending behaviour of the small plates. It is well known that bending rigidity scales with the cube of thickness, so that the Young's modulus E_2 in the transverse direction satisfies

$$E_2 \propto (h / L)^3 E_{s2} \propto (\rho / \rho_s)^3 E_{s2} \qquad (6)$$

where E_{s2} is the cell-wall modulus in the transverse direction. If we now assume that the cell-wall material of all wood species has rather similar mechanical properties, then we expect the balloon in the modulus-density diagram for woods parallel to the grain to lie along a straight line of slope 1 (from eq. (5)), while the balloon for woods perpendicular to the grain should lie along a straight line of slope 3 (from eq. (6)). This is true to a good approximation, as Fig. 1 shows. Gibson and Ashby [5] give more details, and analyse the corresponding scaling laws for other material properties of wood.

The cell walls of wood are made up of natural polymeric materials, which are structured at several different levels to give a material with properties which lie far outside the 'engineering polymers' balloon. The matter which makes up the cell walls has an elastic modulus of 35 GPa in the longitudinal direction, about half that value in the radial direction, and a density of 1.5 Mg m^{-3}, and these points are marked on the map in Fig. 1. The cellular structure discussed above reduces the density of the material, whilst allowing a high elastic modulus to be retained in one direction. The cell walls are made of something which is really an engineering composite material, with fibres embedded in a polymer matrix. The cellulose fibres are constructed of long polymer molecules aligned along the fibre axis giving a very high elastic modulus in this direction. These fibres are then embedded as helical windings in the cell walls. The anisotropic properties of the cell wall depend on the helical winding angle.

2.3. A different criterion for trees.

We started by looking at a criterion for designing trees involving the lightest possible branch for a given amount of elastic bending. In practice one often needs more than one merit index to define the optimum material. In this case, there are many other criteria we could use. One additional requirement for the material of a branch is that it must not break under the load. The mechanical property of interest here is the failure strength, σ_f. Strong materials are at the top of the map shown in Fig. 3, with diamond emerging as the strongest material of all. A bit lower down, we have the 'engineering alloys' including steels, for example, but on a level with these we also have glasses. The caption tells us that the strength of the glasses has been measured in compression: as mentioned before, their strength in tension is low, and this is related to their low toughness. Strong materials take a large stress before cracks run through them and they break; tough materials absorb a lot of energy, and so cracks do not run so readily in them. Glass cracks very easily in tension because of its low toughness, but in compression cracks can be stable and the glass shows high strength. Toughened glass has its surface in compression to stop cracks from forming. A useful engineering material tends to need both strength and toughness.

The index of merit giving the minimum weight for maximum strength of a bending branch turns out to be $\sigma_f^{2/3}/\rho$ (see Ashby [4]), and a guideline for this criterion is shown in Fig. 3. The optimum material again will be one towards the top left of the chart. We are concerned with the properties of wood parallel to the grain, and we see that wood is not actually spectacularly good on the merit index we have chosen. Interestingly, though, all woods are roughly equally good, and the merit index line passes through the long axis of the wood balloon. Presumably, trees have evolved to produce an acceptable level of strength for normal conditions.

While we are working with material strengths, we might consider why balsa wood, the lightest of all woods, is not more widely used for structural applications. As an example, let us imagine a violin front made out of balsa, and check whether the loading provided by the feet of the bridge would be sufficient to crush the wood. The downward force of the strings carried by the bridge is approximately 100 N. With the area of each foot of the bridge typically about 50 mm^2, the stress under the bridge feet will be about 1 MPa. The crushing stress of balsa (now across the grain if the wood is used in its traditional orientation for a violin front) is very variable, but is typically somewhat below 1 MPa [5]. Balsa is ruled out of the list of suitable materials for many applications, by such a strength requirement. When working with a property map, it is very straightforward to put in a criterion such as the minimum strength one we have derived here. On Fig. 3 it would be a horizontal line, and we would search only the region above the line when selecting a material using a further criterion or merit index.

2.4. Example 2: What might violins be made of?

Violins are ordinarily made of wood: spruce for the front and maple for the back. If a violin-shaped instrument were to be invented today, would wood be the obvious choice of material, given the huge range of artificial materials now available? A very simple criterion can be used to derive a crude index of merit for the top plate of a violin: we will seek the material which allows the loudest sound to be made from an instrument of more-or-less conventional design . The major purpose of the soundboard of any stringed instrument is to take a proportion of the energy from the vibrating string and convert it into radiated sound. The radiated sound pressure will be governed by the

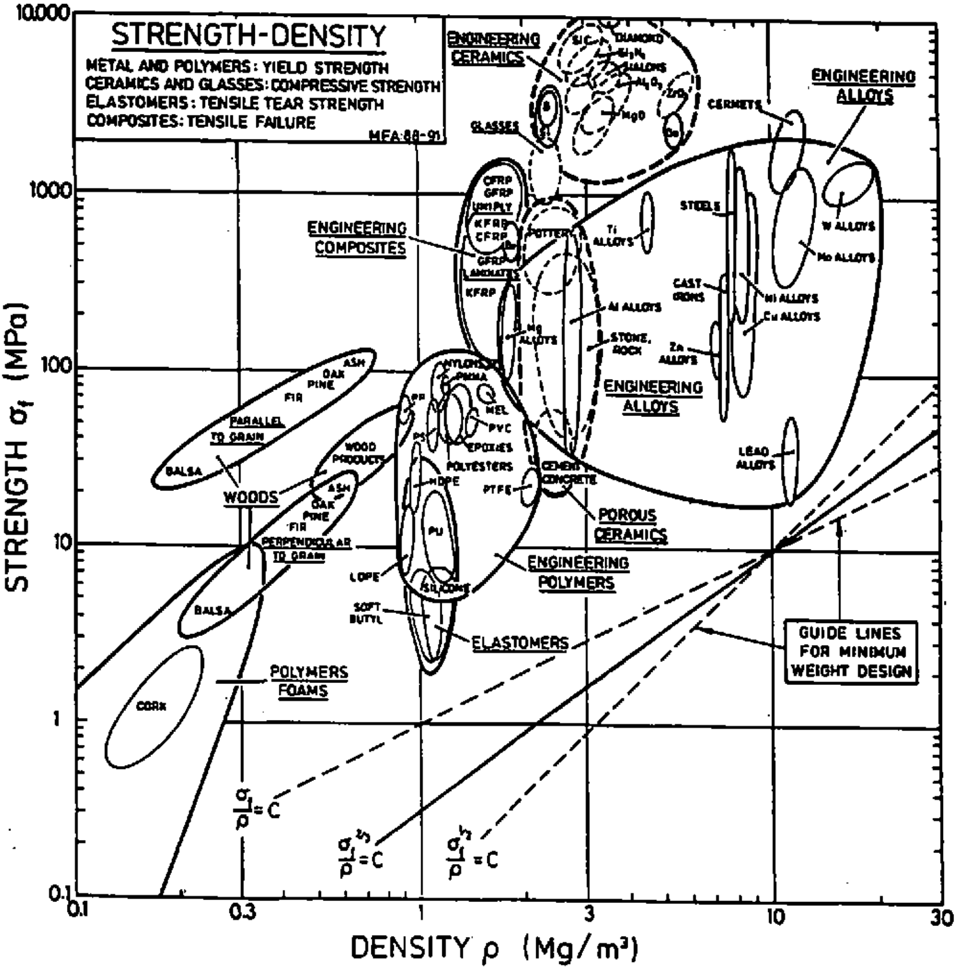

Figure 2. Map showing strength and density for the main classes of engineering materials.

amplitude of vibration of the plate, for a given frequency and mode shape. A string of given tension, vibrating at given amplitude and passing over a bridge of given geometry will exert certain forces on the top plate (at the bridge feet). So we seek to maximise the vibrational response of the plate to a given applied force.

We will suppose that the length and width of the instrument body are fixed (being governed by the ergonomics of playing), but that the plate thickness might need to be varied depending on the material. This thickness will be governed by a requirement that the vibration resonances occur at roughly the usual frequencies — any big deviation from the norm is this respect is likely to produce an instrument which doesn't 'sound like a violin' (e.g. the phonofiddle, or Caldersmith's 'Almas' [6]). It is not realistic to hope to fix all the individual resonant frequencies, but it is easier to ensure that the *modal density* has the correct value [7]. For an isotropic bending plate of any geometry and boundary conditions, the density of modes (in frequency space) is given by

$$n(\omega)=\frac{A}{2\pi h}\sqrt{\frac{3\rho\left(1-\nu^2\right)}{E}} \tag{7}$$

where E is the Young's modulus, ρ is the density, ν is Poisson's ratio, A is the area of the plate, and h is its thickness. Now, a violin top is neither a bending plate nor isotropic. The arched shape means that the plate deformation involves stretching as well as bending, and as we have seen the mechanical properties of wood are far from isotropic. The former objection is probably not very severe, at the rather sweeping level of approximation in use here. The second problem requires a modification to the formula to allow the possibility of anisotropic materials such as wood or carbon-fibre reinforced plastics: if the Young's moduli in the two principal directions, along the grain and across the grain if we are thinking of wood, are E_1, E_2 respectively, then

$$n(\omega)\approx\frac{A}{2\pi h}\sqrt{\frac{3\rho\left(1-\nu^2\right)}{\left(E_1E_2\right)^{1/2}}} \tag{8}$$

The forced vibration response level of the plate, which we wish to maximise, is governed by the admittance, or frequency-response function. A typical example is shown in Fig. 4. It shows a driving-point response on a violin front, suspended by rubber bands in order to give it "free" boundary conditions. Amplitude is plotted on a logarithmic (decibel) scale. Using Skudrzyk's "mean value theorem" [8], the mean value of this logarithmic plot (shown as the solid horizontal line on the figure) is known to be the same as the driving-point impedance of an *infinite* plate: it is

$$Y=\frac{1}{4h^2}\sqrt{\frac{3\left(1-\nu^2\right)}{E\rho}} \tag{9}$$

for an isotropic plate, or

$$Y\approx\frac{1}{4h^2}\sqrt{\frac{3\left(1-\nu^2\right)}{\rho\left(E_1E_2\right)^{1/2}}} \tag{10}$$

for an orthotropic plate [9]. Skudrzyk's theory also tells us that the peaks of the response rise above this mean level by a factor of the order of Q, the Q-factor of the mode in question. (Also the dips fall below the average by approximately the same factor.) These "envelope" levels are indicated by the dashed lines in the figure.

Thus to maximise the average response level, and hence average loudness, of the violin we would choose the material which maximises Y once h has been substituted in terms of n from (8): ignoring quantities assumed given, this means that we must maximise the index of merit

$$M_1=\frac{\left(E_1E_2\right)^{1/4}}{\rho^{3/2}} \tag{11}$$

This quantity, equal to $E^{1/2} / \rho^{3/2}$ for an isotropic material, is often called the "radiation ratio".
If instead we wish to maximise the *peak* response, then we should maximise the alternative index

$$M_2 = \frac{Q(E_1 E_2)^{1/4}}{\rho^{3/2}} \tag{12}$$

where Q is a representative Q-factor. Since the damping behaviour is also anisotropic, we might guess

$$Q \approx \sqrt{Q_1 Q_2} \tag{13}$$

by analogy with (8) and (10), where Q_1, Q_2 refer to the two principal directions. Indices of merit related to M_1 and M_2 have been discussed before in this context [2,10,11], derived by somewhat different arguments.

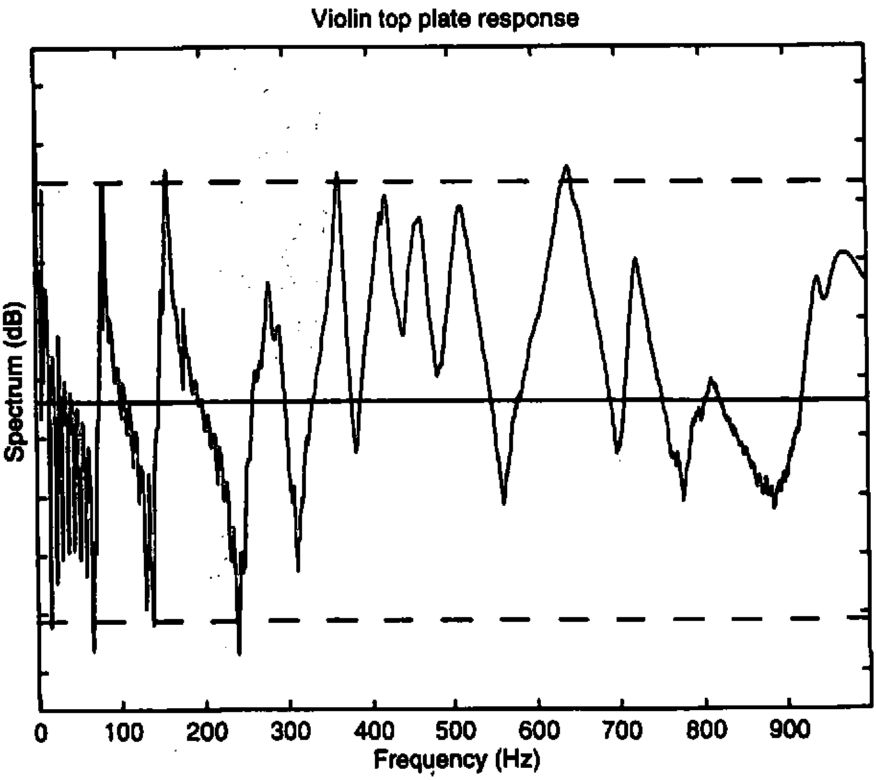

Figure 4. Driving point response of a violin front.

Using eq. (12) is not straightforward with the simple maps we have here, as it contains three parameters: to plot this index we need the electronic version of the database. What we will do here is to use Fig. 1 to plot the merit index from eq. (11). First we have to add the balloon corresponding to $(E_1E_2)^{1/2}$: this lies exactly midway between the two 'wood' balloons, on the logarithmic scale. The optimum material is now found using a line of slope 3, and the maximum value of this parameter is found towards the top left of the map. The furthermost material balloon to be intersected is the 'mean modulus' one for wood. The best wood according to this criterion alone is balsa, but we can rule it out as we have already discovered in section 2.3 that balsa is not strong enough to be useful for soundboards. Remarkably, modern materials cannot yet out-perform wood according to this criterion.

CONCLUSIONS

We have given a brief introduction to the use of 'merit indices' in conjunction with maps of mechanical properties for identifying suitable materials for specific applications. We have used only two different maps out of a huge range of maps which can be plotted, and the electronic version of the database allows one to produce customised maps for any combinations of properties. It is not only physical and mechanical properties which may be important: cost, for example, is an important factor for most applications, and this can be built in to the maps. When faced with a materials selection problem, it is always worth using the approach of constructing a 'merit index' and seeing if the maps give a quick indication of an answer.

ACKNOWLEDGEMENTS

I am indebted to Jim Woodhouse for the analysis in section 2.4.

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