A MODEL OF NOISE RESPONSE FOR BAFFLED HYDROPHONE ARRAYS

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ABSTRACT

Treating a hull mounted array such as a bow array as a set of omnidirectional hydrophones may lead to erroneous modelling of noise response versus steer angle; any tilt of the array from the vertical results in subtle environmental features (such as the 'noise notch') being swamped by noise from overhead. In contrast, the more realistic assumption of directional individual hydrophones (which would result from baffling) results in an array response that is indeed sensitive to environmental detail for moderate amounts of array tilt. A model is presented (CANARY) that can handle these effects by calculating the correlation function between a pair of arbitrarily orientated directional hydrophones and then finding array response by summing over all hydrophone pairs.

1. INTRODUCTION

Sonar designers put a lot of effort into reducing side lobe levels for arrays often implicitly assuming isotropic noise. In reality the noise field may be a long way from isotropic with a pronounced contribution in the vertical from ducts, pronounced holes (the 'noise-notch') in the vertical, and directional propagation and shipping lanes in the horizontal. If the directionality is known then obviously sonar performance can benefit. The corollary is that performance will be degraded if poor assumptions are made at the design stage.

There are important effects in realistic environments that cannot be modelled by the geometric phases and beam steering delays alone. An obvious example is that a baffled array cannot see behind the baffle, and this can be regarded as being caused purely by the individual hydrophone response.

A more serious case is one that we shall investigate later. Imagine a vertical line array of omnidirectional hydrophones in a noise field that is isotropic except for a noise-notch (a narrow band of angles near the horizontal plane where there is no noise).Given adequate angle resolution the array response plot versus steer angle will show a clear dip corresponding to this noise notch. If we now tilt the array a few degrees to the vertical but attempt to compensate by beam steering we will find that, no matter what we do, we cannot find the noise notch. The reason for this is that each steered beam is really a fixed angle cone about the array axis. When we thought some 'forward' part of the beam was looking in the noise-notch direction, some other part of the beam, in particular the 'backward' part, was looking upwards at the surface (where the noise sources are) and contaminating the beam response.

If we were to repeat the thought experiment using directional hydrophones (all aimed horizontally at one azimuth, eg 'forwards') then the array response would be uncontaminated and we would always see a steer angle where there was a noise-notch regardless of the array orientation. This demonstrates that there is a real effect that can give misleading results for signal-to-noise calculations if individual hydrophone directionality is not modelled adequately.

This paper describes a model of ambient noise, CANARY, that can calculate noise field directionality, noise coherence for a pair of hydrophones, and the correlation matrix, array response

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and noise gain for an array. The array can have arbitrary shape and be composed of directional hydrophones. This means that it can model baffled hydrophones typical of hull-mounted arrays.

After discussing the theory in Section 2 we go through some examples including a hypothetical bow array in Section 3.

2. THEORY

Our formulation for array response is a hybrid of the beam/directionality approach and the correlation sum approach. Therefore we start with some definitions and derive the formulae.

The Array Response AR for the entire array is

$$AR = \int N(\Omega)B(\Omega)d\Omega \tag{1}$$

where N is the directional noise field (assumed to be a function of θ only), and B is the beam pattern for the given steer direction.

The noise measured by a single hydrophone is

$$NL = \int N(\Omega)d\Omega \tag{2}$$

and the Noise Gain is

$$NG = \frac{\int N(\Omega)B(\Omega)d\Omega}{\int N(\Omega)d\Omega} \tag{3}$$

Now we assume that the jth hydrophone has a directionality $h_j(\Omega)$ and a weighting w_j and the power beam pattern for the entire array is

$$B(\Omega) = |b|^2 = \sum_i \sum_j e^{ikd_{ij}\cos\xi} h_i(\Omega)h_j(\Omega)w_i w_j \tag{4}$$

where d_{ij} is the distance between hydrophone i and j, and ξ is the angle between the incoming ray and the hydrophone separation axis.

The array response is then

$$AR = \int N(\Omega)\sum_i \sum_j e^{ikd_{ij}\cos\xi} h_i(\Omega)h_j(\Omega)w_i w_j d\Omega \tag{5}$$

Rearranging, this becomes

$$AR = \sum_i \sum_j w_i w_j \rho_{ij} \tag{6}$$

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where ρ_{ij} is the unnormalised correlation function

$$\rho_{ij}(d_{ij}) = \int N(\Omega)h_i(\Omega)h_j(\Omega)e^{ikd_{ij}\cos\xi}d\Omega \tag{7}$$

We now have all the fundamental quantities that need to be calculated, but there are some more terms in the integral that result from breaking down the solid angle integral into θ (elevation angle at the receiver) and ϕ (azimuth), $d\Omega \rightarrow \cos\theta\, d\theta\, d\phi$.

Assuming the hydrophone pair to be separated along a line at azimuth β and elevation angle γ we have

$$\begin{aligned} d_{ij}\cos\xi &= d_h\cos(\phi-\beta)\cos\theta + d_v\sin\theta \\ d_v &= d_{ij}\sin\gamma \\ d_h &= d_{ij}\cos\gamma \end{aligned} \tag{8}$$

Equation 7 can be written as

$$\rho_{ij}(d_{ij}) = \int N(\theta)F(\theta)e^{ikd_v\sin\theta}\cos\theta\, d\theta \tag{9}$$

with all the ϕ dependence contained in F through

$$F(\theta) = \int h_i(\theta,\phi)h_j(\theta,\phi)e^{ikd_h\cos(\phi-\beta)\cos\theta}\, d\phi \tag{10}$$

The environmental dependence taking account of multipath reflections, and so on, is contained in N (see Harrison 1995, 1996)

$$N(\theta) = R_o\sin\theta_s / \left[1 - R_s(\theta_s)R_b(\theta_b)e^{-as_c}\right] \tag{11}$$

where

$$\begin{aligned} R_o &= e^{-as_p} \quad &;\quad \theta > 0 \\ &= R_b(\theta_b)e^{-a(s_c-s_p)} \quad &;\quad \theta < 0 \end{aligned} \tag{12}$$

The ray angles θ_s and θ_b at the surface and bottom ray turning point respectviely, are related to θ by Snell's law. R_s and R_b are the corresponding losses at the surface and bottom turning point (not necessarily the sea bed); a is the volume absorption, and s_c, s_p are the full cycle and part-cycle ray path length from surface to receiver.

If the $h_i(\theta,\phi)$ are independent of ϕ, then eq 10 reduces to the usual $2\pi\, J_0(kd_h\cos\theta)$ (as in Cron and Sherman, 1962) multiplied by the residual θ dependence of h_ih_j, and the whole formula (eq 9) is identical to that in Harrison (1995a, 1996). Otherwise we can solve eq (10) in several ways. One is a straightforward numerical integration as it stands. Another is to convert the exponential into a Bessel function series (Abramowitz and Stegun, 1972).

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$$\left.\begin{aligned}
e^{iA\cos\phi} &= \cos(A\cos\phi) + i\sin(A\cos\phi) \\
\cos(A\cos\phi) &= J_0(A) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(A)\cos(2k\phi) \\
\sin(A\cos\phi) &= 2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(A)\cos((2k+1)\phi)
\end{aligned}\right\} \quad (13)$$

The advantage of this approach is that rapid oscillations associated with A are taken out of the ϕ integral and we are left with a soluble set of trigonometric integrals, even for cases with arbitrary integral limits. This is effectively the same approach as Cox's (1973) describing $h_i h_j$ in cylindrical harmonics.

CANARY has four options. The first assumes omnidirectional hydrophones ($h_i = 1$ for all i). The second assumes each h_i to be a given axisymmetric function defined in terms of the polar angle α_i to this given axis . The third option is

$$h_i = A\cos\alpha_i + B \quad ; \quad 0 \le \alpha \le \pi \qquad (14)$$

and with this formula, one could construct a cardioid by putting B = 1 and A = 1. The fourth option differs from the third in that there is always a flat response at the back of the hydrophone, ie

$$\begin{aligned}
h_i &= A\cos\alpha_i + B \quad ; \quad 0 \le \alpha \le \pi/2 \\
&= B \quad ; \quad \pi/2 < \alpha \le \pi
\end{aligned} \qquad (15)$$

With B = 0 and A = 1 we can construct the front half of a dipole (cosine), effectively a well baffled hydrophone.

3. EXAMPLES

3.1 A tilted line array

In the introduction the tilted line array was presented as a case where a directional noise field could still be resolved by a beam steered array of directional hydrophones but could not be resolved by a steered array of omnidirectional hydrophones. Here we take this as a test case for a numerical demonstration of the effect.

Part of the reason for the interest in this case is that, as we shall see, the bow array contains downward tilted panels of hydrophones or staves arranged like window panes in an airport control tower. The bow array behaves in the same way, but it is difficult to see wood for trees. We have a

14 element unshaded array at half wave length spacing operated at 4500Hz in a Baltic environment (Fig 1). Details of the environment are given in Hamson (1988) and Harrison (1995a).

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3.1.1 Omnidirectional hydrophones

Figure 2 presents the fundamental problem that with steered omnidirectional hydrophones the low noise region cannot be seen if the array is tilted. The solid line in this plot of Noise Gain (eq 3) shows the vertical line array case (tilt angle $\varepsilon = 0°$) where we can see a clear noise-notch between steer angles of about $\pm 8^{o}$, peaks at about 13^{o} due to the surface duct, direct path on the right, and once-bottom-reflected on the left (see Harrison, 1996).

The dashed line ($\varepsilon = 10°$) and the dotted line ($\varepsilon = 20°$) show less and less resemblance to the vertical case. Here we are steering at zero azimuth ('forwards') with elevation angles as shown, and the array is tilted so that its broadside points *down* from the horizontal in the forwards direction. Thus a noise feature from the forward direction at elevation θ appears at the same steer elevation angle as in the $\varepsilon = 0$ case, ie θ, whereas from the backwards direction the same feature at θ appears at steer angle θ-2ε, ie offset by twice the angle of tilt. This accounts for the general widening of the plot with tilt ε and the drift of the low noise region to the left.

3.1.2 Cosine hydrophones

Figure 3 shows the same tilted array but now with 'cosine' hydrophones always looking horizontally (A=1 and B = 0 in eq 15). The tilt has only a very minor effect, but there is still a tendency for the middle null to shift to the left. If the array is tilted backwards (ε= -10°- 20°) we find the null shifts to the right instead, and this is consistent with the interpretation in Section 3.1.1 (Harrison 1995b).

Similarly, we can keep the hydrophone beams locked perpendicular to the array axis (Fig 4) and the differences from Fig 3 are small for the central region. In effect the relatively wide hydrophone beam is narrow enough to cut out surface noise contamination from the back of the array but not narrow enough to modify the middle part of the steered beam pattern.

3.1.3 'Searchlight' beam

In contrast, if we use a narrower hydrophone beam

$$h_i = (1 + \cos 3\alpha)/2 \quad ; \quad \alpha < \pi/3$$

$$= 0 \quad ; \quad \alpha \geq \pi/3$$

perpendicular to the array axis as in Fig 5 we can still resolve the main features of the noise field, but now the middle part of the steered beam pattern is beginning to distort, as can be seen by the relative height of the two middle peaks.

3.1.4 Azimuth variation

For completeness we show in Fig 6 the azimuth variation corresponding to Fig 3 (cosine hydrophone beams pointing horizontally). As we change azimuth the sharp peaks and troughs seen in Fig 3 shift gradually in elevation angle. If we look at a fixed elevation angle ($\theta = 0°$) these changes result in the rather unlikely looking shapes seen in Fig 6.

3.2 A typical bow array

To investigate these effects on a more realistic but hypothetical bow array we have assumed that there are 616 hydrophones divided into groups of 4 (horizontally) by 14 (vertically) on plane panels. Eleven panels are then arranged in a horseshoe curving in azimuth to ± 65°. Each panel is tilted down from the vertical by about 20°, and it is this tilt that makes the hydrophone directionality important.

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3.2.1 Omnidirectional hydrophones

Figure 7 shows the bow array noise gain using unshaded omnidirectional hydrophones in the same Baltic environment as the tilted line array. Again we see contamination of the noise-notch by nearby surface noise sources.

3.2.2 Cosine hydrophones

If we introduce cosine hydrophone beams perpendicular to the panels (Fig 8) we can resolve features of the noise field again. Behaviour in azimuth is shown in Fig 9, and the rise in level away from the centre is the same phenomenon as seen for the tilted line array in Fig 6.

3.2.3 Cosine hydrophones: Chebyshev shaded array

The resolution of the noise field is improved further by using Chebyshev weighting (Fig 10). Here the weights are purely a function of hydrophone height, ie there are 14 different Chebyshev weights. Now the noise-notch of 16 to 18dB is clearly visible.

4. CONCLUSIONS

We have demonstrated that the model, CANARY, can calculate array response and noise gain for arbitrary steered arrays of individually directional hydrophones. The method involves first calculating the correlation matrix for the given environment and then adding terms with appropriate weighting.

There are very large differences in performance between arrays of omnidirectional hydrophones and arrays of baffled hydrophones when the axis of the array is tilted from the vertical. These can be seen by comparing Figs 2 and 3 for a tilted line array and Figs 7 and 8 (or 10) for a bow array.

There are two corollaries. One is that when modelling the signal-to-noise ratio for a real system we need to take care with the details of the hydrophone directionality as well as the steering. The other is that a small amount of baffling (eg Fig 3) in a real or modelled system has the beneficial effect of improving the resolution of the noise field and therefore providing usable regions of low noise.

ACKNOWLEDGEMENT

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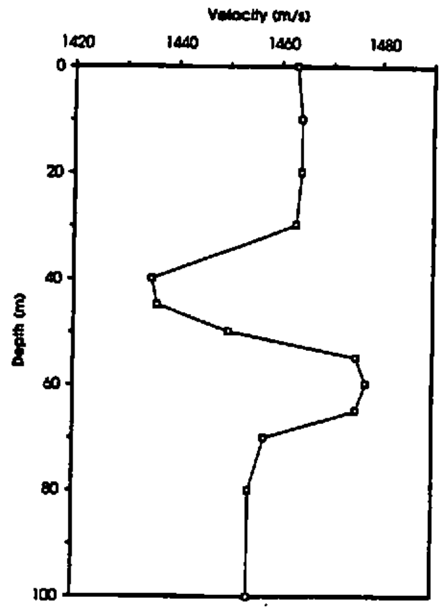

Figure 1 - Baltic soundspeed profile

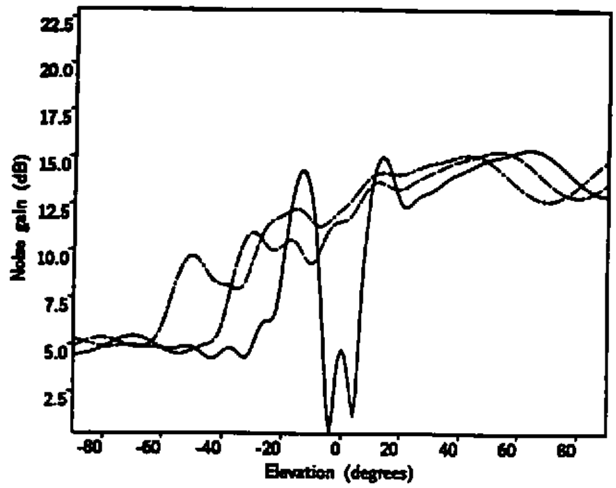

Figure 2 - Tilted Line Array Noise Gain with omnidirectional hydrophones; tilt = 0° (solid), 10° (dashed), 20° (dash-dot).

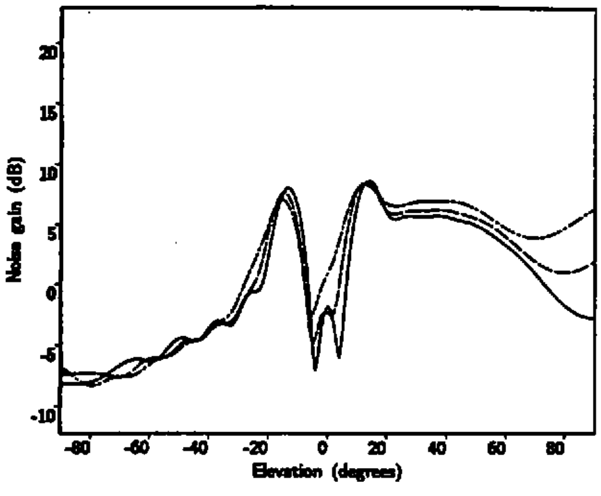

Figure 3 - Tilted Line Array Noise Gain with horizontal cosine hydrophones; tilt = 0° (solid), 10° (dashed), 20° (dash-dot).

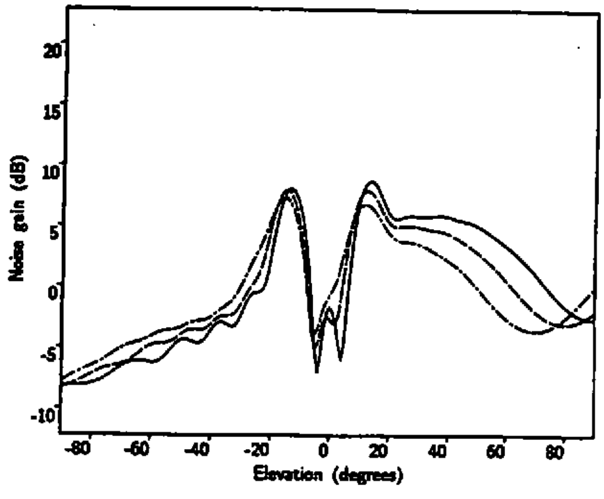

Figure 4 - Tilted Line Array Noise Gain with perpendicular cosine hydrophones; tilt = 0° (solid), 10° (dashed), 20° (dash-dot).

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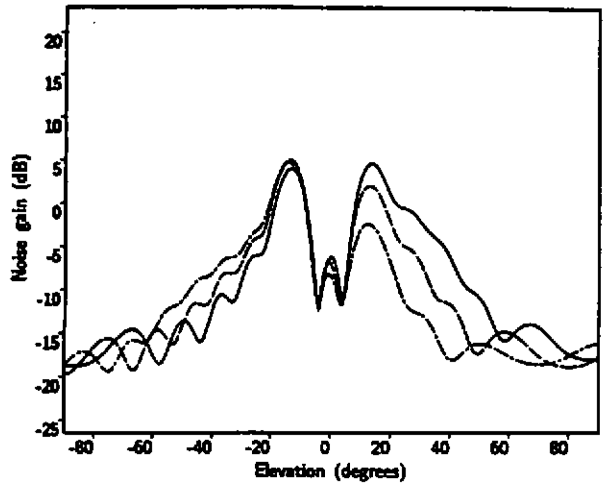

Figure 5 - Tilted Line Array Noise Gain with perpendicular 'searchlight' hydrophones; tilt = 0° (solid), 10° (dashed), 20° (dash-dot).

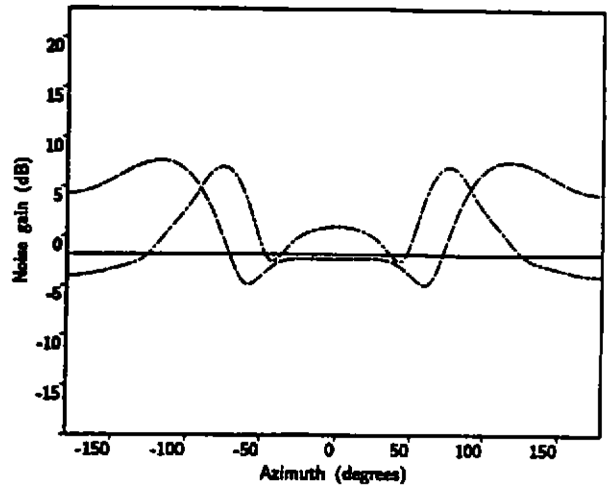

Figure 6 - Tilted Line Array Noise Gain azimuth behaviour with horizontal hydrophones; tilt = 0° (solid), 10° (dashed), 20° (dash-dot).

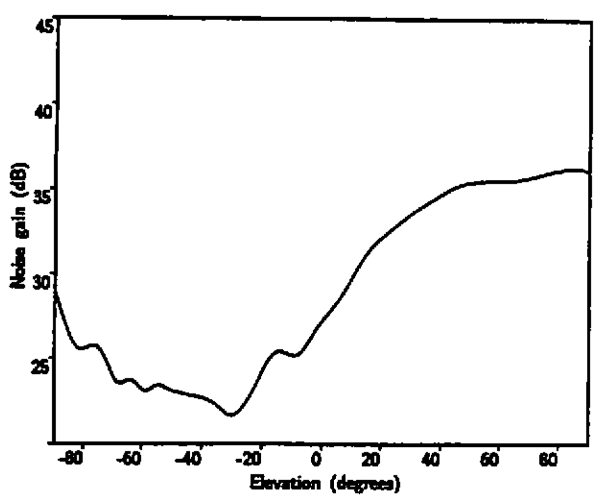

Figure 7 - Bow Array Noise Gain with omnidirectional hydrophones.

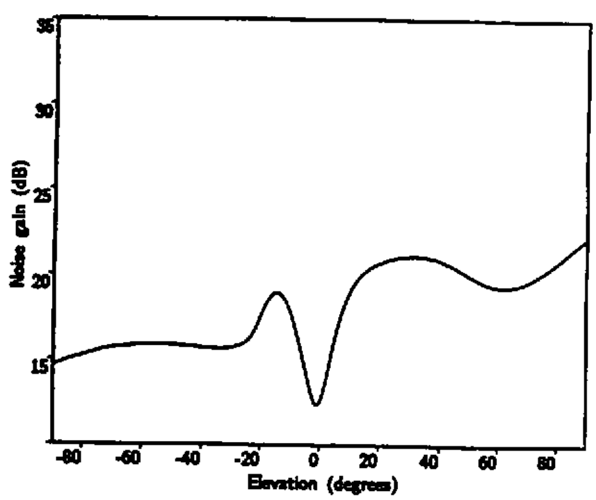

Figure 8 - Bow Array Noise Gain with perpendicular cosine hydrophones.

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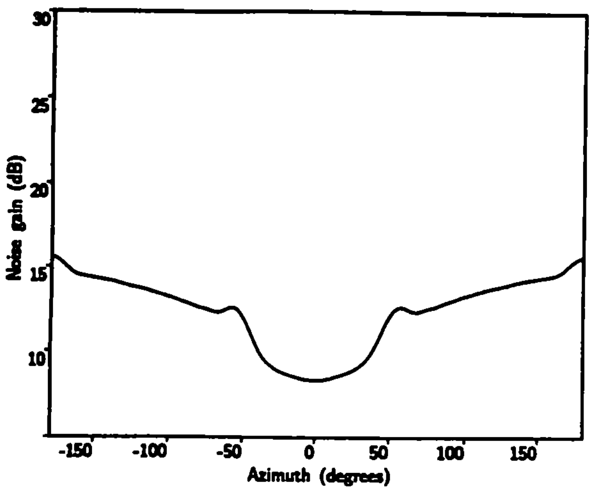

Figure 9 - Bow Array Noise Gain azimuth behaviour with perpendicular cosine hydrophones.

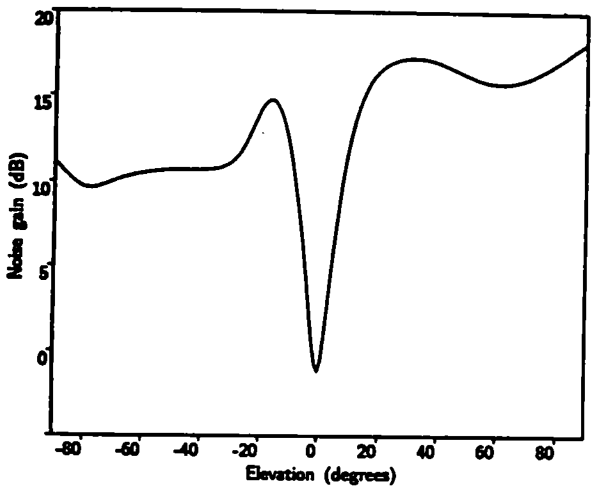

Figure 10 - Bow Array Noise Gain with Chebyshev weighted perpendicular cosine hydrophones.