

# DETECTION AND LOCATION USING AMBIENT NOISE

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## Abstract

Ambient noise is usually considered to be a nuisance, but even though it may sound like hiss, its properties such as spectrum, directionality, spatial and temporal coherence depend on the surroundings, i.e. the environment and discrete objects, as well as the sound sources. Recent work has shown that the seabed's reflection properties, sub-bottom layering, and location of target-like objects can be extracted from this hiss using forward operations such as FFT, beam-forming, cross-correlation, and spectral factorization. Quite generally the cross-correlation of the time series received on a pair of hydrophones in a noise field is closely related to the impulse response that would be received on replacing one of them by a sound source. This means that, in principle, an active sonar can be replaced by a pair of hydrophones, given some noise and some integration time. Also by using standard inversion techniques the reflection properties can be converted into geoacoustic properties either using the sub-bottom layering as a constraint or not. This paper briefly explains the theory and demonstrates some of these processes through experiment and simulation.

## 1 INTRODUCTION

Usually when underwater acoustics is used for any kind of detection, location, remote sensing, mapping, or exploration one requires a sound source as well as a receiver. In contrast the applications addressed here use ambient noise, mainly wind noise, as the source. Bearing in mind noise's uncertain power level and directionality and its spatial diffuseness<sup>1</sup> it is rather remarkable that it is any use at all. From another point of view ocean noise can be classified as formally chaotic with a positive Lyapunov exponent, nine degrees of freedom, and a prediction horizon of just a few samples<sup>2</sup>.

In fact there are various things one can do, either making use of the directional power, or making use of temporal coherence. Using power directionality alone it is possible to determine the reflection properties of the seabed as a function of angle and frequency from the incoherent directionality<sup>3</sup>. This can be converted into geoacoustic parameters by inversion and into layer separations by spectral factorization<sup>4</sup>. Alternatively the time series measured on upward and downward steered beams can be cross-correlated to reveal detailed sub-bottom layering that is almost the same quality as that produced by an active echo sounder or seismic boomer<sup>5,6</sup>. The same technique has been used to detect and locate nearby targets<sup>7</sup>.

This paper will concentrate on the cross-correlation technique and recent developments for geoacoustics and target location with particular emphasis on experiments at sea. Finally it considers the practicality of different experimental arrangements and other applications. First we give a brief overview of the simpler incoherent directionality techniques.

## 2 OVERVIEW OF INCOHERENT DIRECTIONALITY TECHNIQUES

### 2.1 Reflection properties

The obvious, but not so experimentally straightforward, method of measuring bottom reflection properties is the 'move-out' technique where one progressively changes the separation of a source and fixed receiver and deduces the angle from geometry<sup>8</sup>. Traditionally geoacoustic properties have been inferred by inverting propagation loss measurements<sup>9</sup>. Other methods include using ships of opportunity as sound sources<sup>10</sup> and light aircraft as a source of Doppler modulated lines<sup>11</sup>. Buckingham and Jones<sup>12</sup> extracted the seabed's critical angle from measurements of ambient noise spatial coherence on a standard sonobuoy vertical array (VLA). From an acoustic flux point of view, the upward and downward plane wave components of intensity are, by definition, related to the plane wave reflection coefficient at the seabed, so their ratio can be used to determine the seabed's reflection coefficient directly as a function of angle and frequency<sup>3</sup>. Figure 1(a) shows the result of inserting a VLA in the field, and Fig. 1(b) shows this ratio.

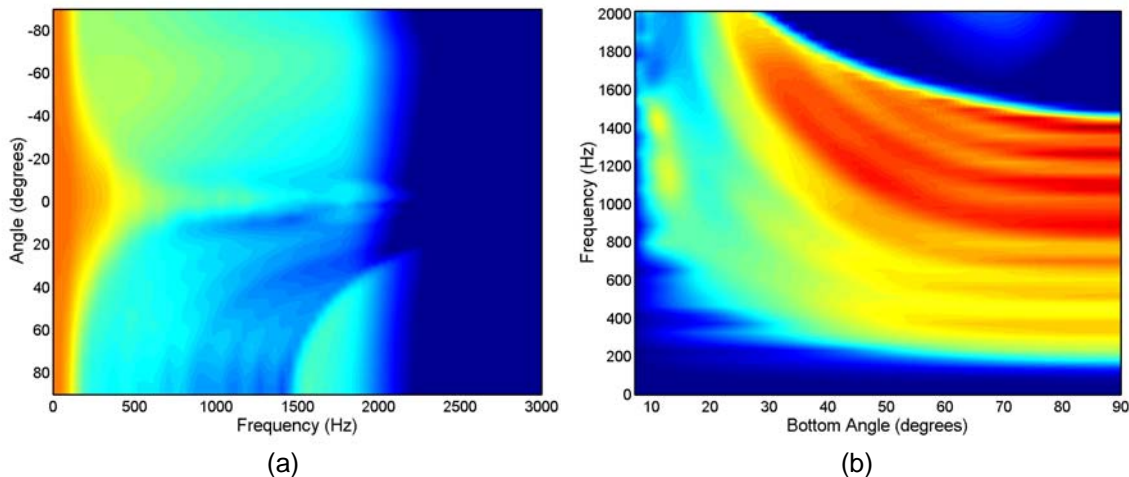


Figure 1 Example of (a) noise directionality measured on a VLA, and (b) reflection loss derived from the down-to-up-ratio.

### 2.2 Geoacoustic inversion of reflection loss $RL$

Because of the finite beamwidths the down-to-up ratio is not the exact reflection coefficient, but by using a model of reflection from multiple layers combined with a simple noise directionality model and known beams, one can find the geoacoustic layer parameters by inversion<sup>13,14</sup>. Having obtained the geoacoustic parameters the final best fit reflection loss also constitutes a cleaned up version of the original.

### 2.3 Sub-bottom profiling using $RL(f, \theta)$

In contrast with the cross-correlation technique discussed in Section 3 it is, under certain conditions, possible to obtain sub-bottom layer profiles (relative to the depth of the seabed interface) from the incoherent vertical beam data. Straight away one can see, through the Wiener-Khintchine theorem, that at each angle in Fig. 1(b) the function of frequency  $|R(f)|^2$  can be Fourier transformed into the autocorrelation function of the impulse response at that angle. It is less obvious that the frequency dependence of  $|R(f)|$  is enough to determine a (minimum) phase by spectral factorization<sup>15</sup>. This follows from the Cauchy-Riemann condition for any analytic function of a complex variable, whereby the imaginary part can be determined from the real part, or likewise the phase (except for multiples of  $2\pi$ ) can be determined from the modulus (i.e.  $|R(f)|$  in our case). Given a phase one can Fourier

transform the *complex*  $R$  into the *actual* impulse response. Thus reflection loss from a drifting VLA can be turned into a sub-bottom profile<sup>4</sup>. The conditions under which this process works are equivalent to the true phase being the same as the minimum phase, i.e. the reflection phase never completes more than half a revolution<sup>16</sup>. Alternatively, the leading peak in the impulse response must be stronger than later peaks so that one side of the autocorrelation of the impulse response is very similar to the impulse response itself.

### 3 CROSS CORRELATION OF NOISE IN GENERAL

The cross-correlation of the time series received on a pair of hydrophones in a noise field is closely related to the impulse response that would be received on replacing one of the pair by a sound source. More formally it can be stated that the Green's function between two points can be recovered from the cross-correlation of the noise time series measured at those points. Various conjectures, assertions and proofs exist in slightly different contexts. The ideas have their roots partly in ultrasonics<sup>17,18</sup>, and partly in seismology<sup>19,20</sup>. In underwater acoustics it has also been shown that one can obtain the Green's function and various derivative quantities from noise<sup>21,22,23,24</sup>. Here we follow Ref. 22 in asserting that the Green's function is proportional to the time derivative of the cross-correlation function. The credibility of this assertion can be reinforced by a simple example (taken from Ref. 22). The normalised cross-spectral density between a pair of points in a uniform noise field is  $\bar{C}(r, \omega) = \sin(\omega r / c) / (\omega r / c)$ . By taking the Fourier transform this becomes the cross-correlation function (– a function of time)

$$C(r, t) = \frac{1}{4\pi} \left\{ \int_{-\infty}^{\infty} \exp[i\omega(t + r/c)] / (i\omega r/c) d\omega - \int_{-\infty}^{\infty} \exp[i\omega(t - r/c)] / (i\omega r/c) d\omega \right\}.$$

The time derivative is then  $\frac{dC}{dt} = \frac{1}{4\pi r/c} (\delta(t + r/c) - \delta(t - r/c))$  which is the sum of the time domain Green's functions between the hydrophones, taken either way round.

To understand this generally, imagine the paths between a single noise source (on the sea surface, say) and two separated hydrophones. The travel time difference between these paths has a maximum or stationary point when the noise source is on the projection of the straight line through the hydrophones. Thus there is a sudden truncation in correlation offset time which corresponds exactly to the separation of the two hydrophones. The derivative with respect to time of this step function is a delta function, so the result is the time domain Green's function between the hydrophones.

### 4 SUB-BOTTOM PROFILING WITH NOISE

Siderius, et al.<sup>5</sup> applied this approach to the domain of sub-bottom profiling by cross-correlating all possible pairs of hydrophones from a drifting vertical array and then aligning their peaks in time according to position on the array. This clearly demonstrates the link between correlating hydrophone pairs and correlating beam-formed time series. Harrison and Siderius<sup>6</sup> derived a formula for the absolute value of the normalised cross-correlation of the upward- and downward-steered beam time series and also the decorrelated background.

The derivation assumed there to be uncorrelated time sequences emitted from random points on the sea surface, received by a  $M$ -element vertical array a distance  $z$  above the reflecting seabed of reflection coefficient  $R$ . The absolute peak value of the forward difference (indicated by  $\Delta$ ) of the normalised correlation of the time series (sampled at  $f_s$ ) is then

$$\Delta\{C(\tau)\} = \frac{c M \text{sign}(R)}{2 z f_s \beta} = \frac{L \text{sign}(R)}{z \gamma \beta} \quad (1)$$

where  $c$  is the speed of sound,  $L$  is the array length,  $\gamma$  is the ratio of sample frequency to design frequency for the array:  $\gamma = f_d/f_o$ . The term  $\beta$  is a factor, of order unity, that allows for the variation of noise spatial coherence across the (wide) processing band. In the intermediate term one can recognise the impulse response multiplied by  $c/f_s$ . In contrast the normalised background variance is the reciprocal of the number of independent samples in the time series. Given white noise this number corresponds to the total number of samples during the integration time. Otherwise with filtering or a finite band the number will be smaller and the background will be higher.

This formula for peak values and background levels was reconciled with a number of simulations using straight ray acoustics and a large number of random number sequences emanating from spatially randomised locations above a 32-element vertical array [Fig. 2(a,b)]. It was shown<sup>6</sup> that one can indeed recover a signed impulse response from multiple layers, and that a correlation peak is still obtained when the seabed is tilted although naturally one must use upward and downward beams (both) steered at the same angle as the tilt. Another processing option is to take the Hilbert transform of the cross-correlation (or its time derivative). The price for the added robustness is loss of the impulse response's sign [Fig. 2(c)].

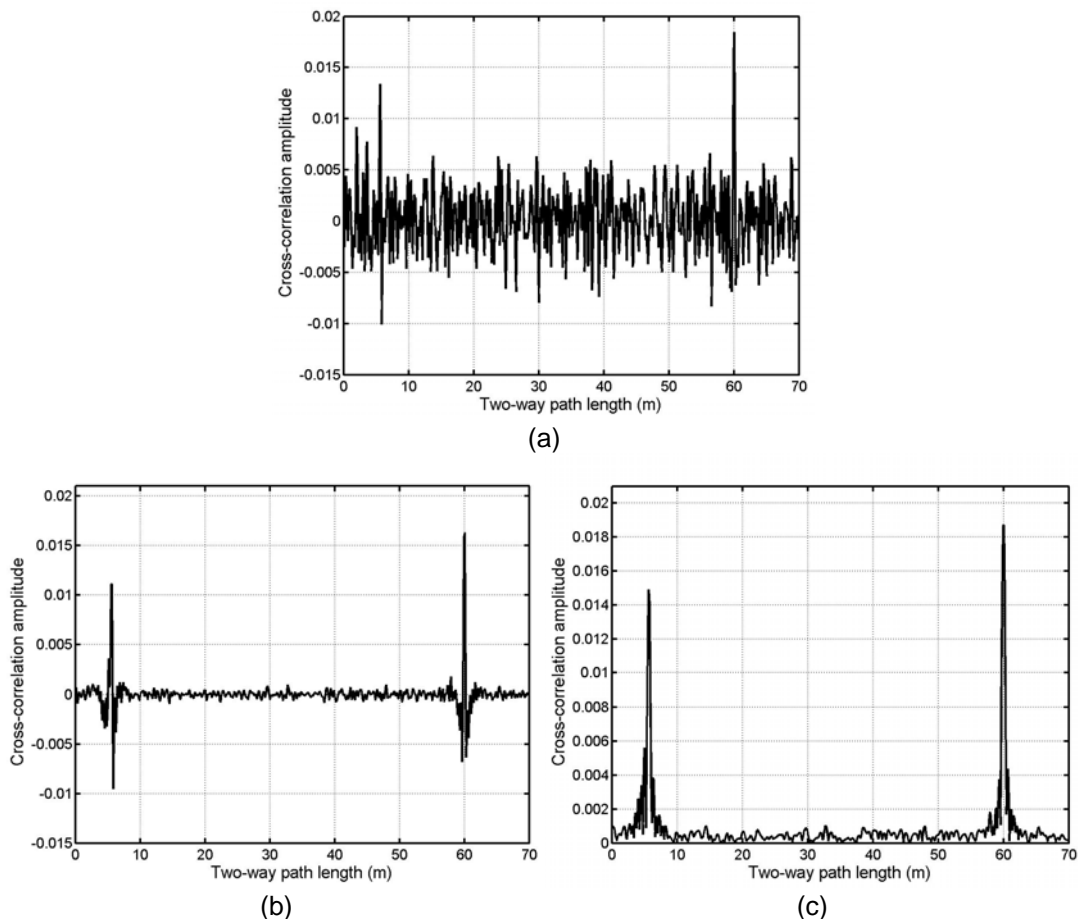


Figure 2 Simulation of the cross-correlation resulting from (a) a single 11s file showing a peak at two-way travel time corresponding to 60m, (b) 81 coherently added 11s files, (c) the Hilbert transform corresponding to 81 files.

The technique has produced sub-bottom profiles with a drifting VLA in four series of experiments to date BOUNDARY2002, BOUNDARY2003, BOUNDARY2004 and CLUTTER2007 in the vicinity of the Malta Plateau and Ragusa Ridge south of Sicily. Snapshots of layering can also be derived from various moored VLA experiments. These profiles have been verified against seismic boomer records along the same drift track<sup>5</sup> and layering has been seen as deep as 55m inside the

sediment. In addition the correlation amplitudes were shown by Harrison and Siderius<sup>6</sup> to agree well with the predictions of Eq. (1). An example from 2003 using a 32 element array with a band between 2 and 4kHz is shown in Fig. 3. The correlation during each 10 second file was incoherently averaged over about 10 files, i.e. a couple of minutes in total, which provides adequate spatial resolution at this drift speed. The Hilbert transformed cross-correlation, i.e. the impulse response, for time 23:00:00 is shown in Fig. 4(a). Figure 4(b) shows a blow-up of the main signed peak near 115m path length with its Hilbert envelope. One can see that there are phase differences between the various layer arrivals.

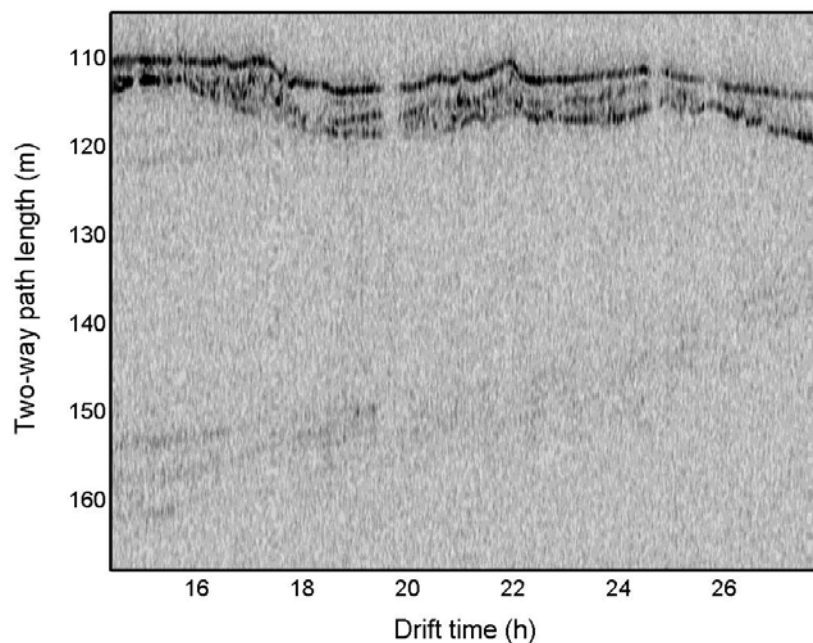


Figure 3. Sub-bottom profile from drifting MFA on the Malta Plateau (2003) showing deep echoes.

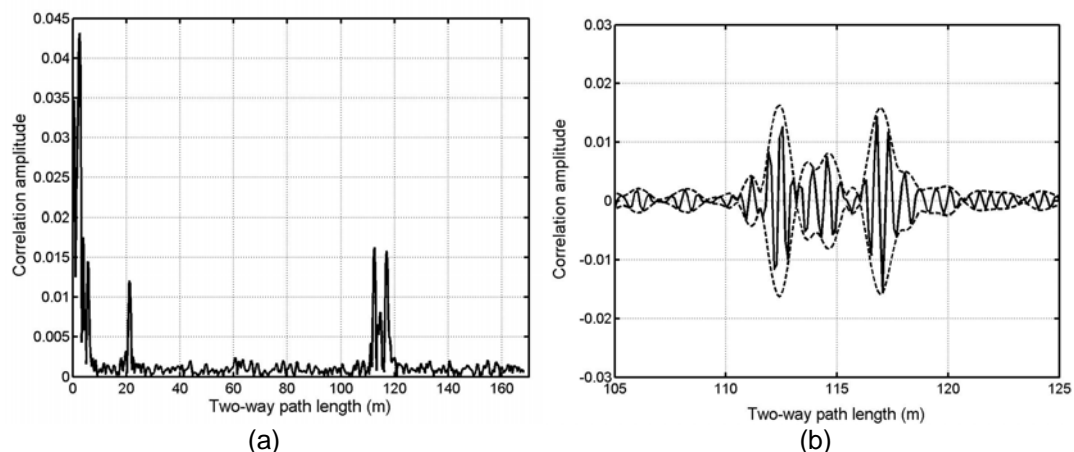


Figure 4(a) Hilbert transformed impulse response (average of 100 files) during the 2003 drift (time 23:00:00), (b) blow-up of the main peak showing correlation amplitude with Hilbert envelope.

## 5 TARGET LOCATION WITH NOISE

It was noted in the 2003 and 2004 experiments<sup>6</sup> that there was a persistent echo at respectively 21m and 24m two-way path length. This was suspiciously close to the path length expected for the

array's ballast weight. During the CLUTTER2007 experiment an effort was made to measure the positions of all potential scatterers, floats, weights, etc. hanging from the array assembly, in order to make a more systematic check of travel times. In addition, experiments were done<sup>7</sup> with and without three glass spheres attached below the array. Figure 5(a) shows that reflections were indeed seen from the ballast weight at 19.5m below centre (39m round trip), the stainless steel end cap of the array at 6.5m (13m round trip), and two of the three glass spheres when they were attached [Fig. 5(b)]. Whether one thinks of the source as a small patch of sea surface noise sources above the array or an effective point impulse source in the middle of the array one might expect the lower spheres and the ballast weight to be slightly obscured by the top sphere. This is believed to be the reason for the unchanged end cap response but weakening lower sphere and ballast responses in Fig. 5(b) when compared with Fig. 5(a).

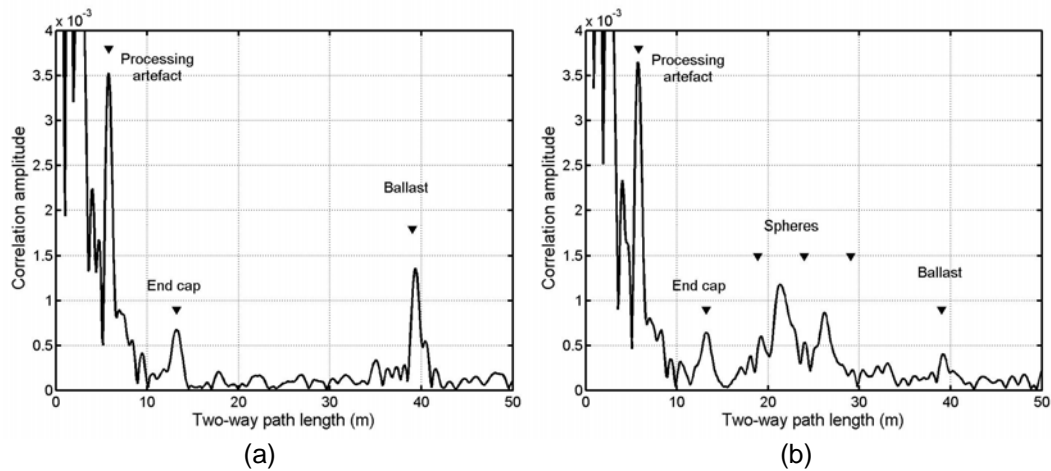


Figure 5. Correlation amplitude, to be interpreted as an impulse response, showing labelled returns (a) excluding the glass spheres, (b) including the glass spheres. Black triangles indicate measured two-way path lengths to the ballast weight, the array end cap, and the three spheres.

The strength of the echoes from the end cap, the unobscured ballast weight, and the upper glass sphere could be predicted by a slightly modified version of Eq. (1). Using the normalisation implicit in Eq. (1) the bottom echo is independent of reflection coefficient magnitude since, with up and down beam time series denoted respectively by  $U$ ,  $D$ , the cross-correlation numerator is  $U * D$  and the denominator is the product of the standard deviations  $\{\langle U^2 \rangle \langle D^2 \rangle\}^{1/2}$ . Thus the magnitude of the reflection coefficient, which is contained only in  $D$ , cancels. One could just as easily have used a normalization where the denominator was  $\langle U^2 \rangle$  instead. Thus the magnitudes of the noise sources still cancel but the result does depend on the magnitude of the reflection coefficient. In the context of demonstrating a target detection it is desirable to avoid the first normalization because, although the peak height in the numerator would depend only on the target (through  $D$ ), the standard deviation of  $D$  in the denominator would also depend on reflections from the seabed. With the second normalization the resulting peak height depends only on the target.

Thus for a point target of target strength  $TS = 20 \log_{10}(s)$  at a depth  $z$  beneath the array the peak strength is

$$P_T = \frac{2L s}{z^2 \gamma \beta} \quad (2)$$

Dependence of this amplitude on distance goes as  $z^2$  exactly as one would expect for a point scatterer a distance  $z$  from a monostatic active sonar.

In these experiments the targets were immediately below the array, but it can be seen (by analogy with the tilted seabed simulation<sup>6,7</sup>) that targets can also be detected and ranged when away from the vertical line. The fact that in laboratory ultrasonic experiments (on centimetre scales) targets have been detected by autocorrelation of a single transducer adds credibility to this assertion<sup>25</sup>.

Indeed this is not the first underwater demonstration of target detection with noise, but the cross-correlation approach is novel in resolving the target range – it operates like a passive radar. In this respect it is quite distinct from acoustic daylight<sup>26,27,28</sup> which, being an analogue of daylight vision, resolves a two-dimensional angle. The passive radar range resolution is dependent on the bandwidth and is not particularly dependent on frequency. Its angle resolution just depends on the array size, and in principle, the cross correlation process still functions with a single hydrophone, though with complete absence of angle resolution.

## 6 PRACTICALITIES

In considering the design of an ideal system there are a number of issues depending on the chosen processing. Generally:

- *Source strength, beam-forming and shipping interference:* In principle any sheet source is adequate. Nearby sources such as ships may spoil the reflection loss technique by being vertically asymmetrical. Loud sources at long range are more problematic for the beam-forming than for the technique since they can leak into side lobes and thus spoil the up / down ratio. This can be improved to a certain extent by adaptive beam forming provided there is not too much tilt. A rule of thumb for reflection loss is that there should be white caps for adequate wind noise, however this is not necessarily a restriction for the correlation technique.
- *Ideal frequency range:* All techniques require a significant bandwidth, but the centre frequency is a compromise between avoiding shipping frequencies, array size and bottom penetration.

The incoherent techniques are relatively immune to motion and array tilt problems, but the coherent techniques are more sensitive.

- *Array tilt:* There are two conceptually different effects with cross correlation. One is the coherent addition of hydrophone arrivals in the cross-correlation, the other is the rejection of decorrelated noise by forming up and down beams. In fact both degradations are avoided if the tilt is less than the beam width.
- *Horizontal motion – over a smooth surface:* Since the impulse response does not change as the array moves horizontally over a smooth surface there is no effect on the technique.
- *Horizontal motion – over rough surface:* Some degradation is to be expected depending on speed. Imagine  $dC/dt|_1$  averaged for a very short time;  $dC/dt|_2$  for the next short time, etc. Each  $dC/dt$  corresponds to a Green's function  $G_1$ ,  $G_2$ , etc. for each array position. One cannot distinguish them because they are buried in the decorrelated background, but one could still coherently add them, getting the usual  $dC/dt$  for the usual integration time with the usual background level. Clearly this is the coherent sum (average) of all the different Green's functions. It is equivalent to coherent ping-to-ping addition of an echo sounder's return as it moved over the rough surface. As an alternative the summation could be incoherent, resulting in incoherent summation of the Green's functions. In both cases the result is effectively spatially smoothed with a corresponding degradation – probably worse with coherent addition.
- *Array vertical motion – spread:* The array and its continuous flexible float (spar buoy) bobs up and down like a fishing float with a period of about 10 seconds and a potentially large amplitude, unless damped somehow. Long term averages of  $dC/dt$  correspond to similarly averaged impulse responses.
- *Array vertical motion – Doppler:* If the array is, for instance, moving instantaneously upwards the up beam will have an extremely small upward Doppler shift but the down beam will have a corresponding downward Doppler shift. In the time domain these shifts correspond respectively to a shrinking and stretching of the time series, so that they will not match as well as expected in the cross-correlation. By simulation this effect can be shown to be much weaker than other spreading mechanisms in the data gathered so far.
- *Local bangs, clicks, etc.:* Normally the resolution of the cross-correlation is the true impulse response convolved with the autocorrelation function of the source, which one would hope would be very short since it is derived from the noise's hiss. If during the experiment there were other undesired impulse-like sounds, particularly emanating from the point immediately above the array or

nearby they may raise the correlation amplitude but also convolve the response by the autocorrelation of the new sources. As a rule, this will degrade the spatial resolution.

## 7 OTHER HYDROPHONE ARRANGEMENTS: POTENTIAL

The cross-correlation of steered beams can be treated as the coherent (i.e. temporally aligned) sum of the correlations of all possible hydrophone pairs from the array<sup>5</sup>. Although this is more computationally intensive than just correlating the two beam time series it demonstrates that one can, in principle, achieve at least some of the  $M$ -fold gain of Eq. (1) with any physical hydrophone arrangement (i.e. array shape). Thus horizontal arrays, planar arrays and 3D arrays are possible contenders, mounted on fixed or moving platforms. Note also that the peak height to background ratio goes as  $M \times N_s^{1/2}$  so that there is more benefit in increasing the number of hydrophones than increasing the integration time by the same factor. These possibilities suggest such applications as silent echo sounders on moving platforms provided flow noise and self noise are low enough and provided the motion is not too fast.

## 8 CONCLUSIONS

Even though ocean ambient noise may be dominated by a random hiss from breaking waves, splashes, spray, popping bubbles, and so on, it still contains usable information about the environment. This information resides on the one hand in its spatial coherence, which is equivalent to its directionality, and on the other, in its temporal coherence or correlation. It is not so surprising that correlations with long time offsets exist in ambient noise, after all, there are many boundary reflections and distributed scatterers in the ocean. However it is surprising that these correlations can be extracted in the form of impulse responses or Green's functions.

This paper briefly overviewed techniques that use only incoherent intensity measurements from steered beams. Reflection loss is obtained simply by comparing the noise intensity in the beams. Spectral factorization can be used to reconstruct a reflection phase which enables the complex reflection coefficient to be Fourier transformed into a relative depth impulse response.

A more powerful technique is to cross-correlate the upward and downward steered beams from a vertical array. The result is equivalent to the impulse response that would have been obtained with a pulsed source at the centre of the array. So a drifting array can produce a sub-bottom profile similar to that produced by an echo sounder. A formula was given for the correlation amplitude, and this was verified with some examples of simulations and experimental results from the Malta Plateau south of Sicily. It has also been demonstrated that the same technique can be used to detect and locate targets. The positions of various reflectors – three spheres, an array end cap, and the ballast weight were carefully measured and shown to coincide in delay time with the correlation peaks. Also the estimated target strengths agreed with the correlation amplitude. A large number of experiments have been completed at various sites with various drifting and moored arrays. Some of the practical issues for coherent and incoherent processing were discussed. The ratio of correlation peak height to background depends on the number of samples in the average, and this suggests that other physical hydrophone arrangements may be practical; their potential was discussed.

## 9 ACKNOWLEDGEMENTS

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