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THEVENIN'S THEOREM AND ACTIVE CONTROL OF VIBRATION

C.J. Mazzola

NAMLAK, P.O.Box 804, Mamaroneck, New York 10543 U.S.A.

1. INTRODUCTION

This paper is an extension of previous work on active sound absorption by Mazzola [1]. Here we describe how Thevenin's theorem [2] is used to design a feedback strategy for extracting a maximum amount of power from a point on a vibrating structure. We then show that a modification of this strategy can be used to add or substract virtual mass or stiffness and change the resonant frequency and the Q of a mode. Also the damping of the mode may be set arbitrarily. In general we show that the feedback strategy implements an arbitrary linear operator for controlling the dynamics of a vibrating system.

For the most part the discussion will consentrate on the analysis and design of the feedback strategy to control a simple harmonic oscillator, and we will give some experimental results to support the work.

2. ACTIVE CONTROL OF A SIMPLE HARMONIC OSCILLATOR

2.1 Mechanical System

A simple harmonic oscillator with an active control force $F_C(t)$ is shown in Fig. 1. $F_E(t)$ is some

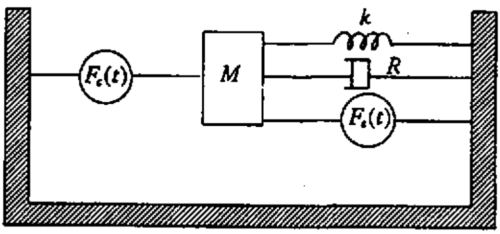

Figure 1. Harmonic oscillator with an active control force.

arbitrary external driving force, the parameters; M, k, and R represent the mass, stiffness, and mechanical resistance respectively. We develop a control strategy that generates a control force $F_C(t)$ equivalent to an arbitrary change of any system parameter. For instance we can add or subtract virtual mass or stiffness to the system and threreby change the resonant frequency and Q in a predictable way. Or we could specify that the active force be such that, for a sinusoidal excitation, the time average power into the control force transducer be a maximum.

In addition to the above control strategies, we could choose one to set the damping to some arbitrary value.

The first control strategy we consider is one that causes maximum power to flow into the force transducer in the sense defined above. We do this by removing the external force and estimating the impedance that the active control force "see". This is the Thevenin impedance of the system. It is well known that if the actual force is replaced by the complex conjugate of the Thevenin impedance and the external force is sinusoidal then the power into the load will be a maximum. We will show that there exists a feedback strategy that will adjust the force so that the control force transducer will look like the complex conjugate of the Thevenin impedance and consequently the power into the transducer will be a maximum.

2.1 Electrical Analog

The analysis is simplified if the electric analog of the mechanical system is used. See Figure2.

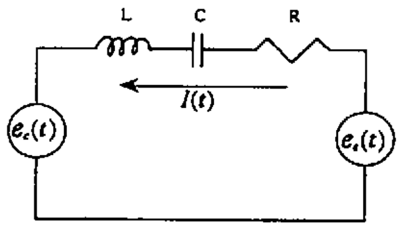

Figure 2. Electric analog of the mechanical system.

The voltages $e_c(t)$ and $e_e(t)$ are analogous to the forces $F_c(t)$ and $F_e(t)$ respectively. The inductance L, capacitance C, and the resistance R are analogous to the mass, compliance, and mechanical resistance respectively.

The Thevenin impedance is found as follows; short out $e_e(t)$, remove $e_c(t)$ and measure the impedance at the terminals to which $e_c(t)$ was previously connected. By this process the Thevenin impedance $Z_T(i\omega)$ is found to be

$$Z_T(i\omega) = i\omega + 1/i\omega C + R \tag{1}$$

where ω is the angular frequency. Having done this we replace the control voltage $e_c(t)$ by the complex conjugate $Z_T^*(i\omega)$ of the Thevenin impedance then the external voltage source $e_c(t)$ now "sees" a pure resistance

$$\bar{\bar{Z}} = 2R \tag{2}$$

It is well known that in this case and for a sinusoidal excitation the power into the load $Z_T^*(i\omega)$ is a maximum. Also we see that attaching $Z_T^*(i\omega)$ at the said location has the effect of cancelling both reactances in the system. The resultant system is completely "dead". If the voltage where shut off abruptly the current would stop immediately. Correspondingly in the mechanical system if the force is removed abruptly the velocity would go to zero immediately.

2.2 Control Law

For the system shown in Figure 2 the equation of motion, in the frequency domain, is

$$(i\omega + 1/i\omega C + R)I(i\omega) = e_e(i\omega) - e_c(i\omega) \tag{3}$$

If in this system the control source voltage $e_c(i\omega)$ is removed and $Z_T^*(i\omega)$ is put in its place, see Figure 3,

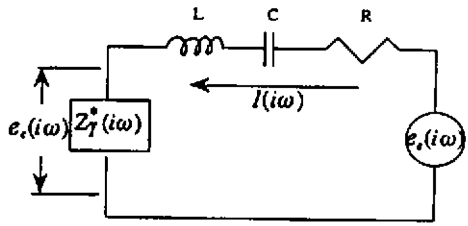

Figure 3. $e_c(i\omega)$ source replaced by $Z_T^*(i\omega)$

we get for the voltage drop across $Z_T^*(i\omega)$

$$e_C(i\omega) = Z_T^*(i\omega)I(i\omega)$$

or

$$e_C(i\omega) = (-i\omega l - 1/i\omega C + R)I(i\omega) \tag{4}$$

We have used the same symbol for the voltage drop across $Z_T^*(i\omega)$ and the voltage of the controlled voltage source. We can do this because we have not specified the control voltage source in the first place. Equation (4) now specifies the control voltage source as being equivalent to the voltage drop across $Z_T^*(i\omega)$. Substituting (4) into (3) gives the equation of motion for the modified system,

$$2RI(i\omega) = e_e(i\omega) \tag{5}$$

The motion (current) of the modified system is controlled by the damping only, there are no reactances and the power into the load is a maximum

In (4) we have already noted that $e_c(i\omega)$ is the voltage drop across $Z_T^*(i\omega)$ when the current through it is $I(i\omega)$. If instead of installing the $Z_T^*(i\omega)$ as described above we simply left the control voltage source in its place but adjusted its voltage according to (4) we would get the same results, that is we would have created a situation in which maximum power would flow into the control voltage transducer. Looking at the problem in this way we recognize that (4) is a <u>control law</u>. We can think of $Z_T^*(i\omega)$ as the linear operator that maps the current - which is measurable - into a control voltage in such a way that the power into the control voltage transducer is a maximum.

It is an easy matter to design a feedback strategy to implement the control law.

3. CONTROL SYSTEM

3.1 Background

The essential idea underlying control system theory is the differential operator shown in Figure 4. $X(i\omega)$ is the system input and $Y(i\omega)$ is the output. The objective of the control system

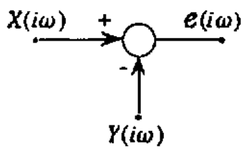

Figure 4. Differential operator.

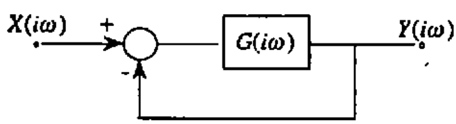

Figure 5. Control system in canonical form.

is to force $Y(i\omega)$ to track $X(i\omega)$. The better the tracking the smaller the error $e(i\omega)$. A control system in canonical form is shown in Figure 5. The corresponding input output ratio is given as follows

$$\frac{Y(i\omega)}{X(i\omega)} = \frac{G(i\omega)}{1 + G(i\omega)} \tag{6}$$

where $G(i\omega)$ is the loop gain. It is the nature of the control system that as the loop gain is increased one of two things will happen; either the error $e(i\omega)$ will approach zero and the output will follow the input exactly, or at least one of the zero's of $1 + G(i\omega)$ will move onto the right hand side of the complex plane and the system will become unstable. In this discussion we assume that the system is always stable.

3.2 Control System for the Harmonic Oscillator

We now set up the control system for the harmonic oscillator. Figure 6 shows the electric analog

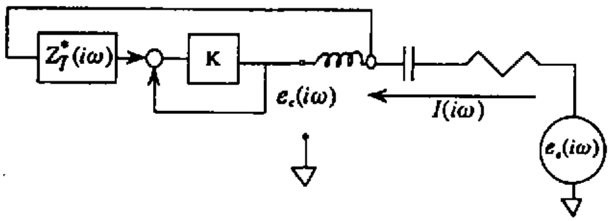

Figure 6. Control system attached to the electric analog system.

system. The control voltage source $e_C(i\omega)$ has been replaced by control system as shown. Once the control system is implemented and the gain increased indefinitely the output $e_C(i\omega)$ track $Z_T^*(i\omega)I(i\omega)$ and maximum power will flow into the control system. This is demonstrated by showing that when $K \to \infty$ the control law is satisfied. The transfer function connecting $e_C(i\omega)$ and $I(i\omega)$ - these are the output and input of the control system - is

$$e_C(i\omega) = \frac{KZ_T^*(i\omega)I(i\omega)}{1+K} \tag{7}$$

$$\lim_{K \to \infty} e_C(i\omega) = Z_T^*(i\omega)I(i\omega) \tag{8}$$

From (8) it is apparent that the control system shown in Figure 6 does indeed implement the control law.

3.3 Beyond the Maximum Absorption Control Strategy

The control law which is implemented by the control system in Figure 6 maps the current into the control voltage in the following way

$$e_C(i\omega) = Z_T^*(i\omega)I(i\omega) \tag{4}$$

The result is that the control system behaves as a virtual power sink and absorbs a maximum of power from the primary system. This is a direct result of using $Z_T^*(i\omega)$ as the linear operator in the control system. We are free however to change the linear operator at will and get any one of a number of control strategies. In general the control law can be written as follows

$$e_C(i\omega) = L(i\omega)I(i\omega) \tag{9}$$

where $L(i\omega)$ is some arbitrary linear operator, and would replace $Z_T^*(i\omega)$ in Figure 6. For instance if $L(i\omega) = R$ then the control system is a virtual resistor and extracts power from the primary system in the same way that a resistor would.. On the other hand if $L(i\omega) = i\omega L$ then the control system is a virtual inductor and will reduce the resonance frequency of the primary system. These are just two examples of a large variety of linear operators that may be used in the control system to place some arbitrary virtual load on the primary system.

In the next section we give some results achieved with a mechanical oscillator using the control strategies discussed above.

4. EXPERIMENT

5.1 Background and Experimental Set Up

A schematic of the experiment is shown in Figure 7. It consists of a mass-spring-dashpot

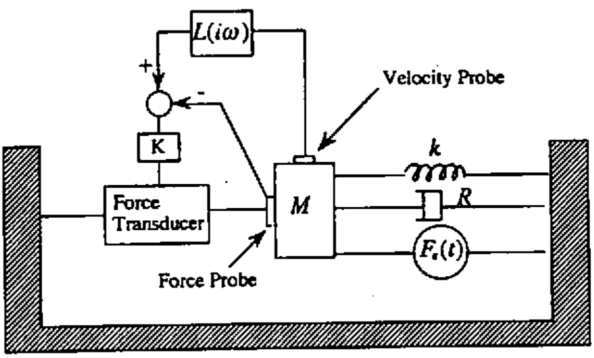

Figure 7. Harmonic oscillator with active control.

oscillator and an active control system of the type described above. All of the above analysis applies to this system if we replace the electrical system parameters by their mechanical analogs, and the voltage and current by force and velocity. The parameters defining the system are as follows

M = 4.57 kgm　　　　k = 88.4 newt/m　　　　R = 11.9 kgm/sec

The resonant frequency is 0.7 Hz and the Q is 1.75. The mass rides on a track, and the control and external forces are provided by two motors attached to the mass and driving a stationary rack through a pinion.

The linear operator - see Figure 7 - is

$$L(i\omega) = -i\omega A - B / i\omega + C \tag{10}$$

In this experiment the parameters A, B, and C take on only positive values. With this operator we can add to the primary system negative virtual mass and or stiffness to some desired level by adjusting the magnitude of A and B respectively. In addition we can add virtual damping by adjusting C.

5.2 Experimental Results

The hardware was constructed to implement the control system shown in Figure 7 and an experiment was performed on the harmonic oscillator described above. The results are shown in Figure 8. The curve B is the response of the oscillator to the external force with no active

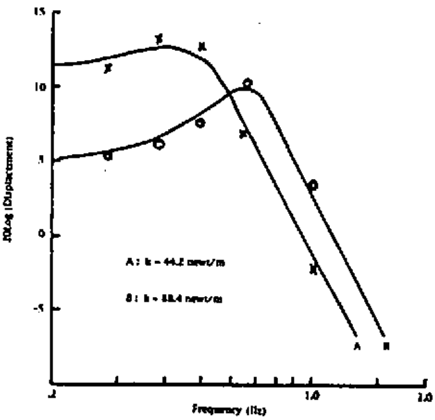

Figure 8. Response of the oscillator with and without active control.

control ($L(i\omega) = 0$). The solid line is the estimated response and the 0's are the measured responses. Curve A is the response with the linear operator set to add negative virtual stiffness equal to approximately to one-half of the real stiffness. In this case $L(i\omega) = -44.2 / i\omega$. The solid is the estimated response and the x's are the measured responses.

5. REFERENCES

[1] C. J. MAZZOLA, "Active Sound Absorption", New York, NAMLAK, 1993.
[2] E. A. GUILLEMIN, "Introductory Circuit Theory", New York, John Wiley and Son, 1953.