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THREE CONSERVATIVELY OR TWO NON-CONSERVATIVELY COUPLED SUBSYSTEMS: IS A DUAL INTERPRETATION POSSIBLE?

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1. INTRODUCTION

Statistical Energy Analysis (SEA) [1] is a useful tool for investigating transmission paths for sound and vibrations. A well-known disadvantage of SEA is that it is difficult to treat systems in which high losses occurs between subsystems. The topic of this paper is how to deal with non-conservative coupling in energy flow models.

2. THEORY

The case of three coupled subsystems has been dealt with in e.g. references [2,3], where a complete energy-flow model was proposed. It was shown that two additional, so called indirect, couplings must be introduced into an energy-flow model for the three-subsystem case to yield a complete solution. The onset of this paper is to examine the case of two non-conservatively coupled subsystems as a special case of the three-subsystem case. The U-shaped plate case of ref. [2,3] is used to help yield suitable examples. Theory is, unfortunately, too lengthy to be repeated here in complete form and the approach is instead to give a brief outline of the features of this case. A more in-depth examination is submitted in ref. [4].

Uncoupled subsystem energy

Following the procedure that was outlined in the derivation of [2,3], the flow of energy between the three subsystems in Fig. 1(a) are

$$S_{P_{1->2}}=\omega\eta_{12}n_1\theta_1+\omega\eta_{13}^{'}\, n_1(\theta_1-\theta_3), \text{ and } S_{P_{2<-3}}=\omega\eta_{32}n_3\theta_3+\omega\eta_{31}^{'}\, n_3(\theta_3-\theta_1) \quad (1a,b)$$

respectively, when no excitation occurs in subsystem 2, i.e. when, by definition, $\theta_2 = 0$. Unfortunately, eq.(1a,b) does not support 'energy flow reciprocity' which makes it less attractive for introduction into a non-conservative two-subsystem model.

However, it can be observed from eq.(1a,b) that the energy flow that is communicated between subsystems 1 and 3 yields a transmission path that fulfils the requirement of 'energy flow reciprocity' and is

$$S_{P'_{1->3}} = \omega\eta'_{13}n_1(\theta_1 - \theta_3) \ . \quad (2)$$

The case that is considered to produce positive values for $Sp'_{1\text{->}3}$ is when energy flows in the direction from subsystem 1 to subsystem 3 Eq.(2) applies for energy-flow across the *fictitious* subsystem interface of Fig. 1(b). The power balance can, after some steps of algebra, be stated as

$$S_{P_{in,1}}=\omega\left(\eta_1+\eta_{12}+\eta'_{13}\right)n_1\theta_1 \text{ , and } S_{P_{in,3}}=\omega\left(\eta_3+\eta_{32}+\eta'_{31}\right)n_3\theta_3 \text{ ,} \tag{3a,b}$$

for subsystems 1 and 3, respectively (Fig. 1(c)).

Introducing the weak coupling approximation $E_j = n_j\theta_j$ for the coupled subsystem energy yields, after some steps, the approximate power balance

$$\begin{Bmatrix} S_{P_{in,1}} \\ S_{P_{in,3}} \end{Bmatrix} = \omega \begin{bmatrix} \left(\eta_1+\eta_{12}+\eta'_{13}\right) & -\eta'_{31} \\ -\eta'_{13} & \left(\eta_3+\eta_{32}+\eta'_{31}\right) \end{bmatrix} \begin{Bmatrix} E_1 \\ E_3 \end{Bmatrix} . \tag{4a,b}$$

This shows that the non-conservative case can be introduced into SEA, if indirect Coupling Loss Factors and suitable radiation factors are derived.

Coupled subsystem energy

Most measurements or calculations refer to systems in which a response occurs in the receiving subsystem. It is obvious that such cases, by definition, must refer to coupled rather than uncoupled subsystem energy. The weak coupling approximation of the coupled subsystem energy may not be sufficient for all of these cases. The experience from previous examinations is that one should be cautious not to take the benefits of the uncoupled subsystem energy for granted also for coupled subsystem energy. A separate evaluation is therefore motivated for this kind of subsystem response coordinate.

The power that is injected into the subsystems and the kinetic energy for the plates can be written as

$$\begin{Bmatrix} S_{P_{in,1}} \\ S_{P_{in,3}} \end{Bmatrix} = \begin{bmatrix} \mathrm{V}_1 & 0 \\ 0 & \mathrm{V}_3 \end{bmatrix} \begin{Bmatrix} \mathrm{S}_1 \\ \mathrm{S}_3 \end{Bmatrix} = \left[\mathrm{V}\right] \begin{Bmatrix} \mathrm{S}_1 \\ \mathrm{S}_3 \end{Bmatrix} , \tag{5}$$

$$\begin{Bmatrix} S_{E_1} \\ S_{E_3} \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{13} \\ R_{31} & R_{33} \end{bmatrix} \begin{Bmatrix} \mathrm{S}_1 \\ \mathrm{S}_3 \end{Bmatrix} = \left[R\right] \begin{Bmatrix} \mathrm{S}_1 \\ \mathrm{S}_3 \end{Bmatrix} , \tag{6}$$

respectively. The rain-on-the-roof excitations are denoted as S_j. Eq.(5) can be rewritten as a function of subsystem energy (eq.(6)), which yields

$$\begin{Bmatrix} S_{P_{in,1}} \\ S_{P_{in,3}} \end{Bmatrix} = \left[V\right]\left[R\right]^{-1} \begin{Bmatrix} S_{E_1} \\ S_{E_3} \end{Bmatrix} = \omega \begin{bmatrix} \left(\eta_1+\eta_{12(1)}+\eta'_{13}\right) & -\left(\eta'_{31}+\eta_{32(1)}\right) \\ -\left(\eta_{12(3)}+\eta'_{13}\right) & \left(\eta_3+\eta'_{31}+\eta_{32(3)}\right) \end{bmatrix} \begin{Bmatrix} S_{E_1} \\ S_{E_3} \end{Bmatrix} . \tag{7}$$

Two additional, (off-diagonal) radiation factors can be noted in eq.(7). No terms in the matrix $[V][R]^{-1}$can be separated purely from a knowledge of the power inputs and subsystem responses. To access the complete information content in the right hand side of eq.(7) requires that the energy flows and the dissipative powers are known. Therefore, the input power balance must be given the simplified interpretation

$$\begin{aligned} \begin{Bmatrix} S_{P_{in,1}} \\ S_{P_{in,3}} \end{Bmatrix} &= \omega \begin{bmatrix} \left(\alpha_1+\alpha_{12}+\alpha'_{13}\right) & -\alpha'_{31} \\ -\alpha'_{13} & \left(\alpha_3+\alpha_{32}+\alpha'_{31}\right) \end{bmatrix} \begin{Bmatrix} S_{E_1} \\ S_{E_3} \end{Bmatrix} \\ &= \omega \begin{bmatrix} \left(\alpha_{1tot}+\alpha'_{13}\right) & -\alpha'_{31} \\ -\alpha'_{13} & \left(\alpha_{3tot}+\alpha'_{31}\right) \end{bmatrix} \begin{Bmatrix} S_{E_1} \\ S_{E_3} \end{Bmatrix} \end{aligned} \tag{8}$$

Note that the power input is explicitly balanced by eq.(8). It is only the function of the *pseudo* coupling and loss factor data that is inaccurately interpreted with respect to the derivation of the energy-flow. The form of eq.(8) is identical with the results of other authors, e.g. [5,6], even though types of subsystem response coordinates other than coupled subsystem energy have been used in these examinations. Furthermore, comparison of the simplified interpretation (eq.(7)) and the weak coupling approximation (eq.(4a,b)) shows close similarity. Examination of a number of cases, however, demonstrates that the success of eq.(8) does not necessarily depend on the coupling strength. What matters is how large the on-diagonal radiation factor is in comparison with a subsystem's Dissipation Loss Factor (DLF). Cases in which one subsystem's DLF dominates over its radiation factors can be shown to yield satisfactory estimates of the energy flow when this subsystem is the receiving subsystem, while poor results are obtained when it is the excited subsystem.

In short, such cases can be expected when a subsystem is unable to destroy the power transfer from an indirectly connected subsystem (which is where the source is located). This type of cases is invoked when the receiving subsystem is either lightly damped or is unable to accumulate energy. Examples of the latter are when the subsystem is rigid, limp or at anti-resonant frequencies.

The experience from numerical examination of several cases suggests that eq.(8) does not depend on whether the indirect energy flow is weak or strong. As mentioned above, it is the re-radiation strength of the receiving subsystem that matters. The energy flow that is communicated between subsystems 1 and 3 can be quite weak and eq.(8) still yield valid results as long as the re-radiation is small in comparison with the dissipation. Thus, it does not matter whether large distances separate subsystem 1 and 3 or the intermediate element is heavily damped.

The derivation of the above mentioned equations does only rely on the form of the power balances. It may therefore be speculated that they hold also for the case in which the intermediate element (subsystem 2) encircles subsystems 1 and 3. A possible use of eq. (8) may therefore be as an approximate tool in the survey of dominant transmission paths in structures that are not (too) lightly damped.

CONCLUSIONS

Two non-conservatively coupled subsystems was demonstrated as a special case of the three-subsystem configuration. The physical subsystem interfaces cannot be directly applied in the non-conservatively coupled two-subsystem model for the simple reason that only a single interface can be introduced. A *fictitious* interface was therefore introduced to account for the transfer of energy between the non-conservatively coupled sending and receiving subsystems. The SEA temperature analogy was shown valid for the energy flow across the *fictitious* interface.

Two kinds of non-conservatively coupled energy flow models were derived for cases in which uncoupled and coupled subsystem energies are used. The first type, that refers to uncoupled subsystem energy, can be used in a 'forward' fashion when setting up a SEA model, while the second type, that uses coupled subsystem energy, is appropriate for computational or experimental analysis procedures where the flow of energy is searched for. It was shown that the non-conservatively coupled two-subsystem energy flow model needs change when the response coordinate is altered from uncoupled to coupled subsystem energy. One change is that two additional radiation factors must be added to the model for coupled case.

Two versions of the power balance were deduced for the coupled case. The first version contains a complete information set that cannot be revealed from the power inputs alone. A second, simplified, version therefore had to be applied for analysis of the energy flows. This simplification can yield perfectly valid results when exciting in one of the subsystems and truly fail when the other subsystem is excited. The degree of approximation depends on how large the receiving subsystem's dissipation-loss and radiation factors are in comparison with each other.

A few speculations about extending the use of the non-conservative power balance were made to suggest directions for further research. The non-conservative two-subsystem case may be applied in the search of dominant transmission paths in structures that are not too lightly damped. However, an un-cautious use of the simplified power balance is not recommended as it yields the energy flow by approximation only.

REFERENCES

[1] R.H. Lyon, *Statistical Energy Analysis of Dynamical Systems, Theory and Applications* , (MIT Press, MA USA, (1975).

[2] "Energy flow within three subsystem configurations", C.R. Fredö, *Inter-Noise 94*, (1994).

[3] "Indirect Coupling: An Ordeal or Benefit for Energy Flow Models?, C.R. Fredö, *Inter-Noise 95*.

[4] "Energy flow in three-subsystem configurations, Part 3: Non-conservative coupling", C.R. Fredö, *submitted to J.SoundVib*, (1996).

[5] J.C. Sun, L.C. Chow, N. Lalor and E.J. Richards, Power flow and energy balance of non-conservatively coupled structures, II: Experimental verification of theory, *J.SoundVib.* **112**(2). (1987)

[6] Kishimoto and Bernstein, Thermodynamic modelling of interconnected systems, Part II: Dissipative coupling, *J.SoundVib.* **182**(1), (1995)

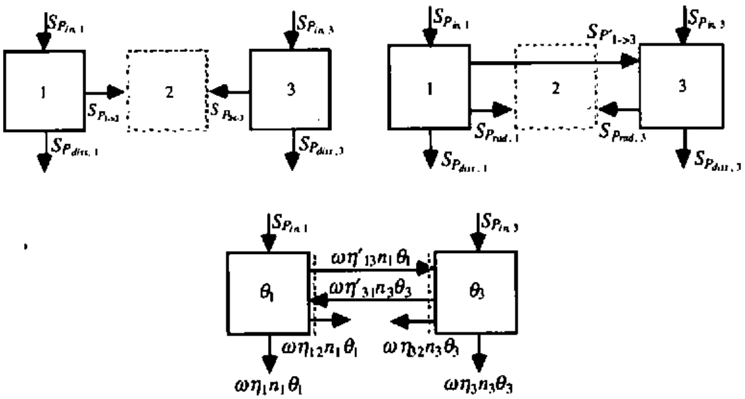

Figure 1 a) The energy flow model in which the physical subsystem interfaces are used. b) The conceptual energy flow balance fin which the energy flows of Fig. 1(a) are re-expressed into one indirect and two radiating path s. c) The non-conservatively coupled energy flow model the way it can be expressed in SEA. The dashed lines symbolise the fictitious subsystem interface that must be introduced into the model.