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FREQUENCY- AND SPACE-AVERAGED INTENSITY MEASUREMENTS

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1. INTRODUCTION

The concept of measuring vibrational energy flow, or structural intensity [1,2], is an appealing one. If the vibrational energy flow in a structure can be measured, sources and transmission paths can be readily determined and ranked. This can greatly simplify the selection of appropriate vibration treatments. Unfortunately, in practice there are many problems. Some of these concern the fundamental principles of structural intensity measurement; for example the need to estimate internal forces from external deformations [3]. Others are a direct consequence of the limitations of transducers or signal processing, such as phase mismatch between measurement channels [4].

In addition to these difficulties, there are also practical problems in implementing the measurements and displaying the results in a useable form. Structural intensity is a continuous function of frequency and space, and can change rapidly with these variables. It is therefore necessary to measure the intensity at all frequencies and, in two-dimensional structures, at all locations, to obtain a complete description of the intensity field. Even when restricted by practical considerations to discrete frequencies and locations, numerous such measurements may be required and these can be extremely time-consuming to obtain.

In reality, however, knowledge of the precise variations of intensity with location and frequency is often of little practical use to the engineer. Of more relevance is the spatial- and frequency-averaged intensity that characterises the energy flow on a larger scale. From this viewpoint, 'comprehensive' intensity measurements often contain a wealth of detail that cannot be used or is not required. Naturally, the unwanted detail can be eliminated through suitable post-processing, however this ignores much of the information that has been so painstakingly obtained. The question therefore arises as to whether similar results can be obtained from a smaller data set, allowing the number of measurements to be reduced. This paper considers the circumstances under which the more general intensity information typically desired by the engineer can be inferred or estimated from a relatively small number of measurement points using spatial- and frequency-averaging.

2. WAVES AND ENERGY FLOW IN PLATES

Consider a plate under flexural vibration. It can be shown that, in a region where far-field conditions exist and with dimensions that are small relative to the distance from any discontinuities or sources of excitation, the displacement can be approximated as the sum of plane propagating waves [5]. All propagation directions may be present, however for clarity an arbitrary number of discrete propagation directions will be assumed. The displacement can thus be expressed as

$$w(x,y,t)=\sum_n A_n \exp\left(-ik_p\left(x\cos\theta_n + y\sin\theta_n\right)\right)\exp(i\omega t)\ , \qquad (1)$$

where $A_n = |A_n|\exp(i\phi_n)$ is the complex amplitude of the wave propagating in direction θ_n, ω is the angular frequency and k_p is the flexural plate wavenumber. In the absence of damping the time-averaged intensity in the x-direction due to this wavefield is given by [1]

$$\langle I_x\rangle = D\omega k_p^3\left(\sum_n |A_n|^2\cos\theta_n + 0.5\sum\sum_{m\neq n}|A_m||A_n|f_{m,n}g_{m,n}\right), \qquad (2)$$

where D is the flexural stiffness of the plate,

$$f_{m,n}(\theta_m,\theta_n)=\cos(\theta_n)(1+\cos(\theta_n-\theta_m))+\nu\sin\theta_n\sin(\theta_n-\theta_m), \qquad (3)$$

$$g_{m,n}(\phi_m,\phi_n,\theta_m,\theta_n,x,y)= \cos\left(\phi_n-\phi_m+k_px(\cos\theta_m-\cos\theta_n)+k_py(\sin\theta_m-\sin\theta_n)\right), \qquad (4)$$

ν is Poisson's ratio, and $\langle\ \rangle$ denotes a time-average. A similar expression can be written for time-averaged intensity in the y-direction.

The expression for intensity (equation (2)) can be considered as composed of two sets of terms - a set of spatially invariant terms, which constitute the ideal 'spatial average' of the intensity, plus a set of terms that are position-dependent. The spatially invariant terms give the intensity that would be present if each propagation direction was considered in isolation, while the spatially varying terms are a consequence of the interference between waves propagating in different directions. For a given pair of waves, (m,n), $f_{m,n}$ represents the influence of the propagation directions on the magnitude of the spatially varying component of intensity, and $g_{m,n}$ provides the spatial modulation with changing relative phase.

3. ESTIMATION OF SPATIALLY AVERAGED INTENSITY

In many circumstances, and particularly at higher frequencies, it is the spatially invariant component, or spatial average, of intensity that is of most interest to the engineer. It is this component that characterises the net energy flow, while the spatially varying component simply results in local perturbations of that flow. Furthermore, under broad-band excitation the intensity at a given frequency is often of less relevance than its average over a frequency band. The spatial average of frequency-averaged intensity is thus, in many cases, of particular interest, and is generally estimated from a number of point measurements of frequency-averaged intensity. Clearly, the number of measurements required is dependent on the manner in which the frequency-averaged intensity varies with location. We are therefore interested in this variation, and the consequent relationship between the local frequency-averaged intensity and its spatial average.

It is apparent from equation (2) that the intensity varies with both frequency and location through the terms $g_{m,n}$. Unfortunately, the relationship is complicated by the fact that other terms in equation (2) are also frequency-dependent. In particular, the wave amplitudes can vary rapidly with frequency, especially in lightly damped structures with low modal overlap. However, while many practical situations will come into this category, it is informative to consider first a case where the wave amplitudes vary slowly with frequency.

Consider a plane wave, of amplitude A, incident on the (conservative) boundary of a semi-infinite plate. Furthermore, assume that the reflection coefficient phase, ϕ, and the energy carried by the wave, $D\omega k_p^3|A|^2$, are independent of frequency. If the coordinate system is as shown in Figure 1, the intensity is purely in the x-direction, and is given by

$$\langle I_x \rangle = 2D\omega k_p^3|A|^2 \cos(\theta)\left(1+\left(\cos^2\theta + \nu\sin^2\theta\right)\cos\left(\phi + 2k_p y\sin\theta\right)\right). \qquad (5)$$

Under these circumstances the spatial average of intensity (over a sufficiently large region) is independent of frequency. We are thus interested in how the frequency-averaged intensity at a point is related to the spatially averaged intensity (over a large region) at a discrete frequency. It is apparent that there is equivalence between varying location and varying the wavenumber. In fact, varying the wavenumber is equivalent to moving the measurement point along a line running from the origin (or, more correctly, from any point at which the relative phase of the waves does not change with frequency) through the observation point. Since the wavenumber is proportional to the square root of the frequency, averaging over a number of evenly spaced spectral lines is equivalent to spatial averaging with a non-uniform separation, however if the bandwidth is relatively small compared to the centre frequency the effects of this non-uniformity will be small.

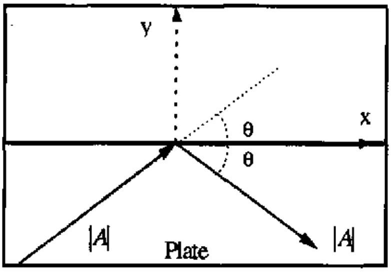

Figure 1: Coordinate system for semi-infinite plate

The relationship between change in wavenumber and change in location is given by

$$dy = y_0\, dk_p / k_{p0} \,, \tag{6}$$

where y_0 and k_{p0} are the initial coordinate and initial wavenumber respectively. Note that, for a given relative change in wavenumber, the equivalent spatial shift is dependent on the initial coordinate, y_0. This is because the relative phase of the waves is fixed at the boundary by the reflection coefficient, and their phase at any other point can be found in terms of the distance that the waves have propagated from that boundary. In general terms, the sensitivity of relative phase to changes in the wavenumber is dependent on the difference in absolute distance that the waves have travelled from the point or points where their phase relationship was independent of frequency. If these distances or 'effective path lengths' are equal then changing the wavenumber will have the same effect on both paths, and the relative phase will remain the same. In contrast, if the effective path lengths differ greatly the relative phase will be very sensitive to changes in the wavenumber.

In a uniform wavefield, spatial averaging is, in effect, averaging over a range of phases. This means that the distance of the measurement point from discontinuities or lines of symmetry has a considerable influence on the relationship between the frequency-averaged intensity at a point and its spatial average. Near these features the relative phase of certain wave components will be very insensitive to changes in the wavenumber. Conversely, remote from discontinuities and lines of symmetry of the structure the effective path lengths will generally be dissimilar, and the relative phase of wave components will be sensitive to changes in the wavenumber. As a consequence, the bandwidth required for the frequency-averaged intensity at a point to give an acceptable estimate of its spatially averaged value increases in the vicinity of these features.

In the more general case, the variation of wave amplitudes with frequency must also be considered. It is apparent from equation (2) that averaging the intensity over a suitable region will reduce the influence of the spatially varying terms, and the frequency-average of spatially averaged intensity in the x-direction is

$$\overline{\overline{\langle I_x \rangle}^{x,y}}^{\omega} = \sum_n \overline{D\omega k_p^3 |A_n|^2 \cos\theta_n}^{\omega} . \qquad (7)$$

In contrast, the frequency-average of intensity at a point is given by

$$\overline{\langle I_x \rangle}^{\omega} = \sum_n \overline{D\omega k_p^3 |A_n|^2 \cos\theta_n}^{\omega} + 0.5 \sum \sum_{m \neq n} \overline{D\omega k_p^3 |A_m||A_n| f_{m,n}(\theta) g_{m,n}(\phi)}^{\omega} . \qquad (8)$$

We can approximate equation (8) as

$$\overline{\langle I_x \rangle}^{\omega} = \sum_n \overline{D\omega k_p^3 |A_n|^2 \cos\theta_n}^{\omega} + 0.5 \sum \sum_{m \neq n} \overline{D\omega k_p^3 |A_m||A_n|}^{\omega} f_{m,n}(\theta) \overline{g_{m,n}(\phi)}^{\omega} . \qquad (9)$$

This approximation is valid if the amplitude and phase of the waves are statistically independent and the bandwidth is sufficiently large. In modal terms its validity depends on the mode shapes with natural frequencies that lie within the bandwidth being spatially uncorrelated, since spatial correlation between mode shapes implies a similar phase relationship between the waves at resonance. Equation (9) gives a reasonable approximation provided that the frequency band contains a sufficient number of resonant modes, this number being largely determined by the distance of the measurement point from discontinuities, and that the term $\sum \sum_{m \neq n} \overline{D\omega k_p^3 |A_m||A_n|}^{\omega} f_{m,n}(\theta)$ changes only slowly with centre frequency. Under these circumstances the difference between the frequency-averaged intensity at a point on a plate and its spatial average is given by

$$\overline{\langle I_x \rangle}^{\omega} - \overline{\overline{\langle I_x \rangle}^{x,y}}^{\omega} = 0.5 \sum \sum_{m \neq n} \overline{D\omega k_p^3 |A_m||A_n|}^{\omega} f_{m,n}(\theta) \overline{g_{m,n}(\phi)}^{\omega} . \qquad (10)$$

Averaging over a sufficiently broad frequency band, and hence over a range of wavenumbers, will make the term $\overline{g_{m,n}}^{\omega}$ in equation (10) tend to zero, and thus give equality between the local frequency-average of intensity and its spatial average under the specified assumptions.

Many realistic situations will lie between the extremes given by these two examples. The wave amplitudes will vary strongly with frequency due to resonant behaviour, and the assumption of statistical independence between wave amplitude and phase will not strictly be valid. Commonly this is a consequence of the proximity of boundaries or lines of symmetry and/or limitations on the bandwidth. The resulting spatial correlation between mode shapes will mean that the energy flows at resonance of the various modes exhibit similar spatial distributions, and thus the fluctuating components will not tend to cancel. The frequency-averaged intensity therefore retains a spatial variation under these circumstances [6], and the separation of the measurement points must be reduced.

4. CONCLUDING REMARKS

In practical intensity measurements on structures subjected to broad-band excitation, particularly at high frequencies, the spatial- and frequency-averaged intensity is often of more interest than the precise variation of intensity with frequency and location. If there is a sufficient number of spatially uncorrelated modes within the averaging bandwidth, frequency-averaged measurements of local intensity may be used to estimate the spatial average of frequency-averaged intensity, thus reducing the number of measurements required. The proximity of discontinuities or lines of symmetry increases the spatial correlation between mode shapes, and thus increases the bandwidth necessary for equivalence between the frequency-averaged intensity at a point and its spatial average. Conversely, for a given bandwidth, the separation of the measurement points should be decreased in the vicinity of discontinuities or lines of symmetry owing to the reduced effectiveness of frequency-averaging in eliminating the spatial variation of intensity in these regions.

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