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# VIBRATION CONTROL USING COLOCATED ACTUATOR/SENSOR

C S Chou & C C Wu

Institute of Applied Mechanics, National Taiwan University, Taipei, Republic of China

## 1.INTRODUCTION

Vibration control based on the concept of active attenuation with colocated sensor and actuator is studied in this paper. The response of a structural element described by hyperbolic equations is represented by a traveling wave which can be cancelled by a secondary wave of opposite signature. This control scheme is similar to the approach adopted in noise control application[1]. Mace[2] worked on active control of flexural vibration on a beam by using wave absorber to absorb as much of the wave traveling toward the actuator as possible. Von Flotow[3] applied the wave absorbing control to shunt the vibrational energy away from the sensitive locations to areas where it can be dissipated. In either cases, they require a measurement technique to divide the vibration into different direction and local reflection and transmission behavior around the sensor/actuator pair need to be modeled.

A control system comprised a pair of colocated sensor/actuator and an inverting amplifier is used to control a vibrating string. The traveling wave of the vibrating body detected by the sensor is inverted and amplified then fed to the actuator to generate a control wave. Experimental and numerical results demonstrate that the amplitudes of vibration of string are substantially suppressed.

## 2. THE CONTROL SYSTEM

The transverse vibration of a undamped string is governed by the differential equation

$$T\frac{\partial^2 u}{\partial z^2} + f(z,t) = \rho\frac{\partial^2 u}{\partial t^2} \quad (1)$$

where $u(z,t)$ is the transverse displacement, $f(z,t)$ is the external force, $T$ is the tension and $\rho$ is the mass per unit length. For free vibration, $f(z,t)=0$, the transverse displacement of a infinite long string can be written in the form

$$u(z,t)=F_1(z-vt) + F_2(z+vt) \quad (2)$$

where $F_1$ and $F_2$ are wave profiles traveling to the right and to the left, respectively, with the wave propagation velocity $v=\sqrt{T/\rho}$.

For a finite string of length L and fixed at both ends, the u(z,t) must satisfy the boundary conditions u(0,t)=u(L,t)=0. The natural frequencies are $\omega_n=\sqrt{T/\rho L^2}$, n=1,2,....

Consider a string under a harmonic excitation force acting at z=L, the transverse displacement is $u(z,t)=u(z)e^{i\omega t}$, and u(0,t)=0, $u(L,t)=u_L e^{i\omega t}$.

A control force, $f_\varsigma^c=-Gu(\varsigma,t)$, is applied at z=ζ, where G is a proportional constant. Substituting those into Eq.(1), we obtain

$$T\frac{\partial^2 u}{\partial z^2}=-\omega^2\rho u(z)+Gu(\varsigma)\delta(z-\varsigma) \qquad (2)$$

The solution of above equation can be written in the form

$$u(z)=\frac{u_L[f(z)+G'f(\varsigma)f(z-\varsigma)H(z-\varsigma)]}{f(L)+G'f(\varsigma)f(L-\varsigma)} \qquad (3)$$

where G'=G/T, and H(z) is the unit step function.

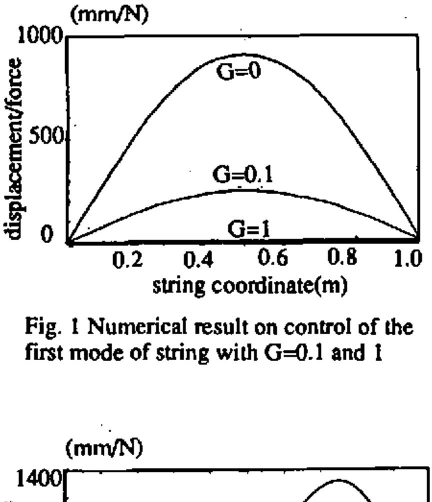


Fig. 1 Numerical result on control of the first mode of string with G=0.1 and 1

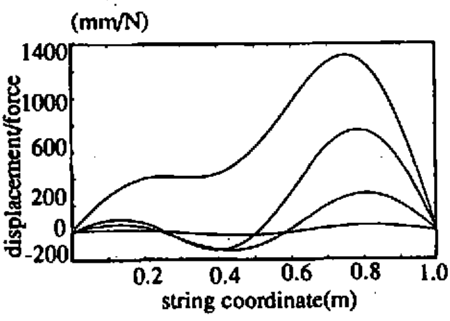


Fig. 2 Numerical results on control of first three modes of string with G=0.1 and 1

Numerical results of the first mode control with different gains G' of a string of L=1m and wave velocity v=66.2 m/sec are shown in Figures 1. In Figure 2 we show the result of the excitation force with frequencies of first, second and third natural modes superposed. Numerical results of the transverse vibration of the string by calculating waves from both directions due to the harmonic excitation force at z=L, the control force $f^c{}_\zeta=-Gu(\zeta,t)$ at z=ζ, and reflection waves from the boundary are shown in Figure 3. The time domain vibrational signals of the string is excited under its second natural frequency at different control gains.

A successful implementation of the wave absorbing concept of control would ordinarily require precise mathematical modeling of the physical state of vibration, and accurate solution of the integral equation for estimating the traveling wave of vibration. These two arduous tasks, however, can be circumvented by using two similar electro-mechanical transducers, one as the sensor to detect the traveling wave of vibration at a point and the other as the actuator to apply the secondary wave. When these two transducers are placed colocately onto the structure and are connected to an inverting amplifier, they form

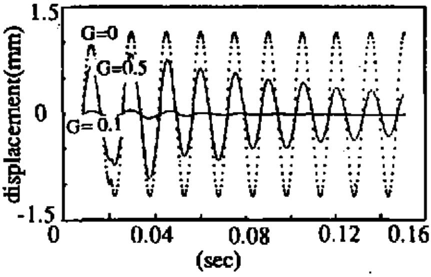


Fig. 3 Numerical results of second natural freq. vibration of string with G=0.1 and 1

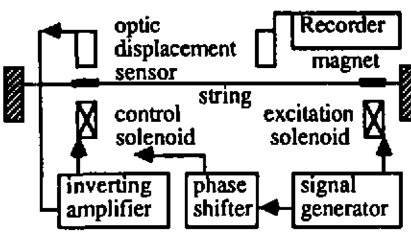


Fig.4 Experimental set-up for wave absorbing control of a string

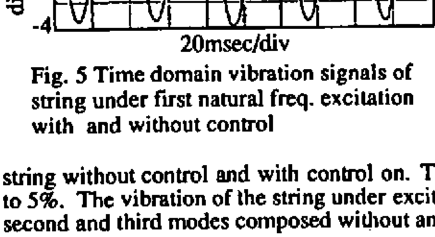


Fig. 5 Time domain vibration signals of string under first natural freq. excitation with and without control

a closed-loop feedback system. The sensed signal of the traveling wave of vibration at a point is inverted and amplified then fed to the actuator to generate the control wave.

To test the concept of wave attenuation experiment is performed on a one meter long cotton string as shown in Figure 4. A small pieces of magnet is attached next to one end of the string which is used to excite the string through the solenoid coil. Another piece of magnet is attached at a distance 60 cm away from the exciter is used for generating the control motion. A harmonic siganl from the siganl generator is applied to the excitation solenoid to excite the string. This signal is inverted and delayed a period equals to the traveling time of the excitation wave to arrive the control point. Therefore the phases of the excitation and control waves are exactly out phase. The resultant motion of the string is measured with an optical sensor and is shown in Figure 5.

Experimental set-up for vibration control of the string with the control system is also shown in Figure 4. An optical displacement sensor placed colocately with the magnetic actuator is used to measure the vibration of the string. The measured signal is inverted and amplified then fed to the magnetic actuator to generate the control motion.

In Figure 6 we show the time domain vibrational signals of the string under excitation of the first (24.8 Hz) natural frequency of the string without control and with control on. The vibration amplitude is suppressed to 5%. The vibration of the string under excitation force with frequencies of first, second and third modes composed without and with control is shown in Figure 7.

## 3. CONCLUSION

Applications of a simple electro-mechanical control system based on the concept of destructive interference between the traveling primary vibration wave and the control wave are reported. Numerical and experimental results on vibration control of a string showed that the traveling vibration wave is destructively interferenced by the control wave. The effectiveness of the control system is determined by the amplitude and phases of the excited travelling wave and the control wave[4]. The simple electro-mechanical control system can be applied to various elements of a structure and a number of the systems can be used combinally to suppress the vibration of the entire structure[5].

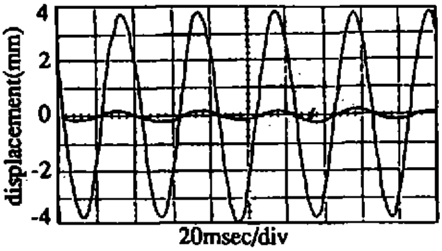


Fig. 6 Time domain vibration signals of string under first natural frequency excitation with and without control

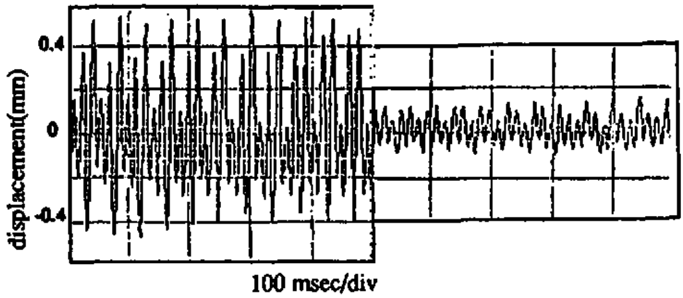


Fig. 7 Time domain vibration signals of the string under excitation composed with first three natural frequencies with and without control

## ACKNOWLEDGEMENT

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