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CALCULATION OF THE THREE-DIMENSIONAL SOUND PRESSURE AROUND NOISE BARRIERS

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1. INTRODUCTION

To estimate the efficiency of noise barriers of complex geometries, numerical methods like the boundary element method (BEM) are well adapted. So Seznec [1] and Hothersall [2] [3] presented numerical results for two-dimensional diffraction problems with sound pressures created by coherent line sources. However more interesting results would be provided by the solution of the real three-dimensional problem like the pressures created by point sources or incoherent line sources which are known to be a good description of a road traffic noise.

The 3D sound pressure can also be obtained by the BEM but the problem to be solved becomes rapidly very large. To avoid such difficulties, we propose here a method using a series of solutions of simpler 2D problems in the frequency domain, both for real and complex frequencies. Then we show that a mathematical transformation will give us the 3D pressure from the 2D results obtained by the BEM. This can then be used to estimate the attenuation provided by noise barriers especially in the case of an incoherent line source.

2. 2D TO 3D TRANSFORMATION

We suppose that the barrier has a constant cross-section as the example presented in figure 1 and that it is infinite in length. The sound pressure is created by a source at point $\mathbf{r}_s$ in the fluid domain and we are interested in stationary solutions with the time behavior $e^{-i\omega t}$. To calculate the pressure around the barrier we must solve the three-dimensional problem

$$\begin{aligned}
\Delta p_{dif} + k^2 p_{dif} &= 0 \quad in\ \Omega_3 \\
\frac{\partial p_{dif}}{\partial n} + \frac{\partial p_{inc}}{\partial n} &= 0 \quad on\ \partial\Omega_3 \\
p_{inc}(\mathbf{r}) &= \frac{e^{ik|\mathbf{r}-\mathbf{r}_s|}}{4\pi|\mathbf{r}-\mathbf{r}_s|} \\
\frac{\partial p_{dif}}{\partial n} - ikp_{dif} &= o(\frac{1}{r}) \qquad (1)
\end{aligned}$$

where Ω_3 is the exterior 3D domain outside the barrier with boundary $\partial\Omega_3$. The sound pressures p_{inc} and p_{dif} are respectively the incident and diffracted fields. The ground and the barrier are supposed rigid. The last relation is the radiation condition for outgoing waves at infinity.

To avoid to solve the complete 3D problem we start from the following relation (Gradshteyn [4], formula 6.616)

$$\frac{e^{ik\sqrt{r^2+z^2}}}{4\pi\sqrt{r^2+z^2}} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{i}{4} H_0(r\sqrt{k^2-\alpha^2}) e^{i\alpha z} d\alpha \tag{2}$$

where $0 \le arg(\sqrt{k^2-\alpha^2}) < \pi$ and $r = \sqrt{x^2+y^2} > 0$ is the radial distance in the (x, y) plane between the observation point and the source. The function $\frac{i}{4}H_0(r\sqrt{k^2-\alpha^2})$ is in fact the sound pressure of a line source. We then define a series of 2D problems depending on a complex parameter μ.

$$\begin{array}{rcl} (\Delta+\mu^2)q_{dif} & = & 0 \quad in \;\; \Omega_2 \\ \dfrac{\partial q_{dif}}{\partial n} + \dfrac{\partial q_{inc}}{\partial n} & = & 0 \quad on \;\; \partial\Omega_2 \\ q_{inc}(x,y,\mu) & = & \dfrac{i}{4}H_0(r\mu) \\ + \; outgoing & wave & conditions \;\; at \;\; infinity. \end{array} \tag{3}$$

Ω_2 is the $2D$ fluid domain outside a cross-section of the barrier and $\partial\Omega_2$ its boundary. Now the solution of the 3D problem (1) can be recovered from the solutions of the previous 2D problems (3) by (for more details see Duhamel [5])

$$p(x,y,z,k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\alpha z} q(x,y,\sqrt{k^2-\alpha^2}) d\alpha \tag{4}$$

3. APPLICATIONS TO NOISE BARRIERS

We define the excess attenuation by

$$At = 20\log_{10}\left(\frac{|p_{wall}|}{|p_{free}|}\right) \tag{5}$$

This formula can be used directly for point sources or coherent line sources for which $|p_{free}| = \frac{1}{4\pi r}$ or $|p_{free}| = \frac{1}{4}|H_0(kr)|$ respectively. The 3D solution allows to make calculations for incoherent line sources parallel to the barrier. An incoherent line source is modelised as decorrelated three-dimensional point sources located on a straight line parallel to the z axis. The amplitude $\mu(z)$ has the cross correlation function

$$E(\mu(z)\mu^*(z+u)) = \nu\delta(u) \tag{6}$$

where $\nu \ge 0$. In the presence of a wall, the pressure created at point $\mathbf{x} = (x, y, 0)$ by a unit source located in the plane $z = z_0$ is

$$P(\mathbf{x}, z_0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\alpha z_0} q(x,y,\sqrt{k^2-\alpha^2}) d\alpha \tag{7}$$

For a line source with the amplitude $\mu(z)$ we get

$$p(\mathbf{x}) = \int_{-\infty}^{+\infty} P(\mathbf{x}, z)\mu(z)dz \tag{8}$$

The expectation of the density of acoustic potential energy is thus

$$e_{wall} = \frac{1}{4\rho c^2}\int_{-\infty}^{+\infty} |P(\mathbf{x}, z)|^2 \nu dz = \frac{\nu}{8\pi\rho c^2}\int_{-\infty}^{+\infty} |q(x, y, \sqrt{k^2 - \alpha^2})|^2 d\alpha \tag{9}$$

using the Parseval's relation for the last equality. The attenuation provided by the barrier is then given by

$$At = 10\log_{10}(\frac{e_{wall}}{e_{free}}) \tag{10}$$

We will now use the previous results to compare the efficiency of noise barriers of different shapes. So the attenuations for the cross-sections of pictures 2 and 4 are calculated with a sound source at $50cm$ above the ground and $1.9m$ on the right of the wall at point $(2, 0.5)$. This source can be a point source, a coherent or an incoherent line source. The observation point is $2m$ behind the wall and $50cm$ above the ground at point $(-2, 0.5)$. The two points are on the same cross-section (same z). The origin is at point O.

The figures 3 and 5 show that a point source and a coherent line source give almost the same result. The attenuation is a complex curve especially for the case B because of interferences between the waves diffracted by the barrier and reflected on the ground. An incoherent line source gives, on the contrary, an attenuation increasing with the frequency and a much lower average value. There is no interference phenomenon in this case. So the attenuation seems to be less dependent on a particular choice for the frequency or the position. This could provide a better estimate of the efficiency of the wall. Figure 6 compares the attenuations for an incoherent line source in the cases A and B.

4. CONCLUSION

The proposed method allows an efficient determination of excess attenuations of noise barriers of constant cross-sections. We only have to solve two-dimensional problems for real and imaginary frequencies for instance by the BEM with a very simple one-dimensional mesh. We can get the pressure for a point source and for an incoherent line source. Comparisons can be made between different geometries in this interesting practical case.

References

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[3] **Hothersall D. C., Crombie D. H. and Chandler-Wilde S. N.** Applied acoustics, 32:269–287, 1991.

[4] **Gradshteyn I. S. and Ryzhik I. M.** Table of integrals, series and products. Academic press, 1980.

[5] **Duhamel D.** L'Acoustique des problèmes couplés fluide-structure: Application au contrôle actif du son. PhD thesis, Ecole Nationale des Ponts et Chaussées, 1994.

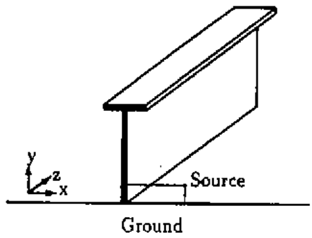

Fig. 1: Structure of constant cross-section.

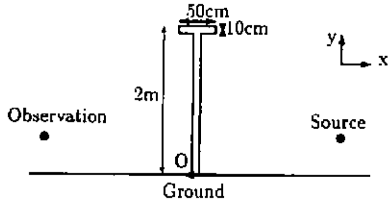

Fig. 4: T shape wall. Case B.

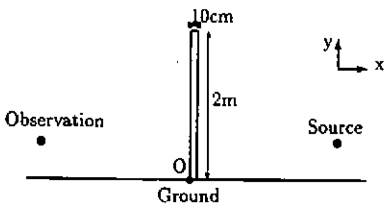

Fig. 2: Straight wall. Case A.

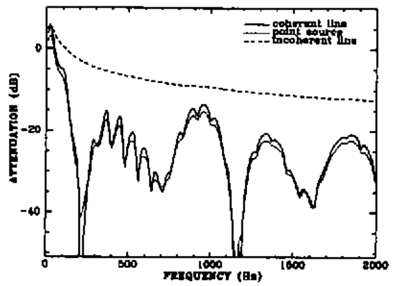

Fig. 5: Excess attenuation. Case B.

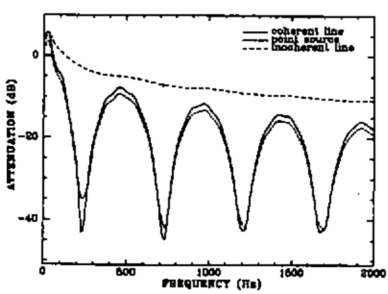

Fig. 3: Excess attenuation. Case A.

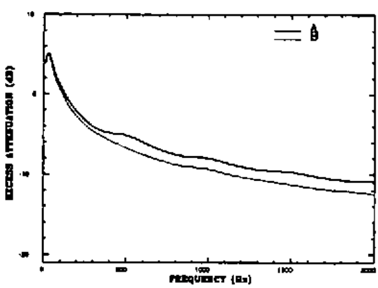

Fig. 6: Comparison of attenuations for an incoherent line source.