

A METHOD FOR PREDICTING THE SOUND ABSORPTION OF PERFORATED FACINGS WITH BACK CAVITY

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1. INTRODUCTION

Sound absorber systems composed of a perforated panel with absorptive materials and air cavity backed by a hard wall have been widely used in meeting the requirement of architectural acoustics and noise control problems. As for the theoretical investigation, many works have been reported since the pioneering work of Bolt[1] in 1947. These studies, however, seem not to give any reliable predicted results over the whole frequency range of practical use. The method for analyzing the system used in these works is based on the idea of Bolt, i.e., the impedance of one hole is converted into a simple averaged value corresponding to the fraction of perforated open area. So, many works[1]-[10] focus on one point, that is how to simulate the acoustic impedance of one hole with a back cavity. Recently, more detailed and generalized theory is presented by Allard *et al.*[9],[10], and more recently Ingard[11] provides a program for calculating the sound absorption of the system. However these studies, unfortunately, do not give enough investigation about the accuracy of the method.

In these past studies, the effect of interaction between one hole and another part of the facing on the reflected sound field, has been disregarded. It is well known that any discontinuities in the surface impedance cause particular wave diffraction phenomena[12]. Thus the treatment in a simple averaged sense does not include this effect, and seems not to give sufficiently good agreement with measurements.

It is the original point of the present study to treat the problem as a wave scattering from a boundary surface with impedance discontinuities, and to calculate the absorption coefficient from the scattered waves.

2. SOUND ABSORPTION FROM WAVE SCATTERING THEORY

Consider a scattered sound field with a boundary surface composed of two kinds of acoustic properties. The top surface of a perforated panel is lying in the plane $z = 0$ as shown in Fig.1, which is here regarded as a periodically arranged flat surface with two parts of different specific admittance A_1 and A_2 corresponding to the hole and another part of the facing, respectively. The hole of perforated

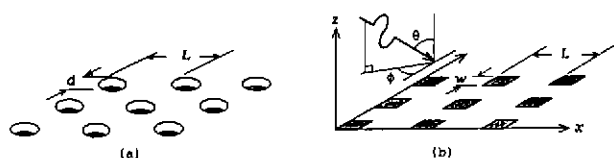


Fig.1 Geometry of (a) a perforated facing, and (b) its analytical model.

panels is generally a circle, which is here assumed to be a square with same area, with the fraction of perforated open area $\sigma (= \pi(d/2L)^2)$.

Fundamental of the analysis for finding the solution of the scattered field has been developed by Rayleigh[13]. From this theory, the absorption coefficient depending on both angles of incidence is given by

$$\alpha_{\theta\phi} = 1 - \sum_{\text{Re}(\gamma_{mn})} \frac{\gamma_{mn}}{\gamma_0} |\Psi_{mn}|^2, \quad (1)$$

where the sum runs over both integer m and n such that γ_{mn} is real, and where $\alpha_m = \alpha_0 + 2m\pi/L$, $\beta_n = \beta_0 + 2n\pi/L$, and $\gamma_{mn} = \{k^2 - (\alpha_m^2 + \beta_n^2)\}^{1/2}$ and $\alpha_0 = k \sin \phi \sin \theta$, $\beta_0 = k \cos \phi \sin \theta$, $\gamma_0 = k \sin \theta$ with $k = \omega/c$ the acoustic wavenumber, ω the angular frequency, and c the speed of sound. The time factor $e^{-i\omega t}$ is suppressed throughout. The unknown coefficients Ψ_{mn} can be obtained by solving the following equation:

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} G_{mn}^{m'n'} \Psi_{mn} = H_{m'n'}. \quad (2)$$

The coefficients appeared in this equation are given by

$$G_{mn}^{m'n'} = \frac{1}{L^2} \int_0^L \int_0^L (\gamma_{mn} + Ak) e^{-i2\pi \frac{(m'-m)x + (n'-n)y}{L}} dx dy, \quad (3)$$

$$H_{m'n'} = \frac{1}{L^2} \int_0^L \int_0^L (\gamma_0 - Ak) e^{-i2\pi \frac{m'x + n'y}{L}} dx dy. \quad (4)$$

The integration of this type can be done easily in the explicit closed form. Generally, the admittance A in eqns(3) and (4) depends on both the angle of incidence θ and the variable x and y , in the present case that is given by

$$A = \begin{cases} A_1 : & \text{part of the hole} \\ A_2 : & \text{another part of the facing} \end{cases} \quad (5)$$

If these values are determined, then the solution can be obtained by solving eqn(2) with respect to the unknown coefficients Ψ_{mn} and by substituting them into eqn(1).

3. ADMITTANCE OF THE BOUNDARY SURFACE

The factors belonging to the admittance A_2 are the acoustic properties of the panel surface and the effect of panel vibration. As for the former, the surface of perforated panels used as the ordinary building material can be assumed to be acoustically rigid, and for the latter, it has been found (from an experimental

investigation beforehand) that the effect of panel vibration on the performance of the system can be disregarded. Then it is here assumed that $A_2 = 0$.

The specification of the admittance A_1 that corresponds to the entrance of a hole can be made by consideration of impedance boundary-conditions at each interface. The result is given by

$$A_1 = \left[\frac{\sigma Z_L / \rho_0 c - i \tan kh'}{1 - (\sigma Z_L / \rho_0 c) i \tan kh'} + \frac{R_F}{\sigma \rho_0 c} \right]^{-1}, \quad (6)$$

in which h' is the panel thickness including the end correction of an open tube. This equation also includes an airflow resistance regarding both the flow in the tube and flow distortion at the entrance.

The admittance Z_L can be obtained by analyzing the response of a layered structure under an incident plane wave at angle θ , which is, for example, given by

$$Z_L = \frac{\rho_0 c_0 \zeta \tanh q_2 d_2 + 1}{Q_2 \zeta + \tanh q_2 d_2}, \quad \zeta = \frac{Q_1 Q_1 \tanh q_1 d_1 + A_B}{Q_2 Q_1 + A_B \tanh q_1 d_1}, \quad (7)$$

for a two-layered structure, where $Q_j = q_j \rho_0 c / Z_j \gamma_{jz}$, $q_j = \gamma_{jz} \sqrt{1 + (k \sin \theta / \gamma_{jz})^2}$ with $\text{Re}\{q_j\} \geq 0$, $\text{Im}\{q_j\} \leq 0$, and $j = 1, 2$. In these expressions, A_B is the specific admittance of the back wall (normally $A_B = 0$), γ_{jx} and γ_{jz} are the propagation constants regarding the x and z direction, respectively, and Z_j is the characteristic impedance. If the j -th layer is an air, the two quantities can be given as: $\gamma_{jx} = \gamma_{jz} = -ik$ and $Z_j = \rho_0 c$, respectively.

The second term in eqn(6) represents the resistance induced from both a change in flow velocity at the hole entrance and the frictional effect in the tube. The flow resistance is defined as: $R_F = \Delta p / u$, in which Δp is the pressure difference between both sides of the tube and u is the flow velocity in the tube. For estimating this value, an experiment using an ordinary flow-resistance apparatus was carried out for the specimens of perforated acrylic panel with 90 samples. The results are shown in Fig.2, in which the straight line represents the results from Poiseuille's law of steady flow in a narrow tube. From this result, although a degree of scatter increases with the parameter $h/(d/2)^2$ decreases, the Poiseuille's law can be applied to the present case, that is $R_F = \frac{8\eta h}{(d/2)^2}$ with the air viscosity $\eta (= 1.8 \times 10^{-5} \text{ kg/m-sec in the normal condition})$.

4. NUMERICAL RESULTS IN COMPARISON WITH EXPERIMENTS

The calculated results in comparison with the existing experimental data[14],[15] are shown in Fig.3, in which theoretical results are represented as the field-incidence-averaged absorption coefficient. Almost all results in these figures, which include wide variation of the related parameters, show fairly good agreement between theory and experiments. Then the presented analytical method will give an effective tool for prediction of perforated absorber systems.

5. CONCLUSIONS

A new theoretical model including the diffraction phenomenon for predicting the sound absorption of the perforated absorber systems was formulated and discussed in comparison with the experimental data. Thinking of the complexity in the absorption mechanism, the presented analytical method gives fairly good prediction for sound absorption characteristics over the whole frequency range of interest.

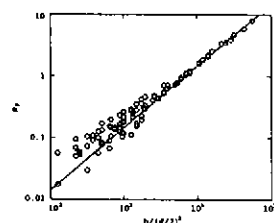


Fig. 2. Flow resistance of a hole.

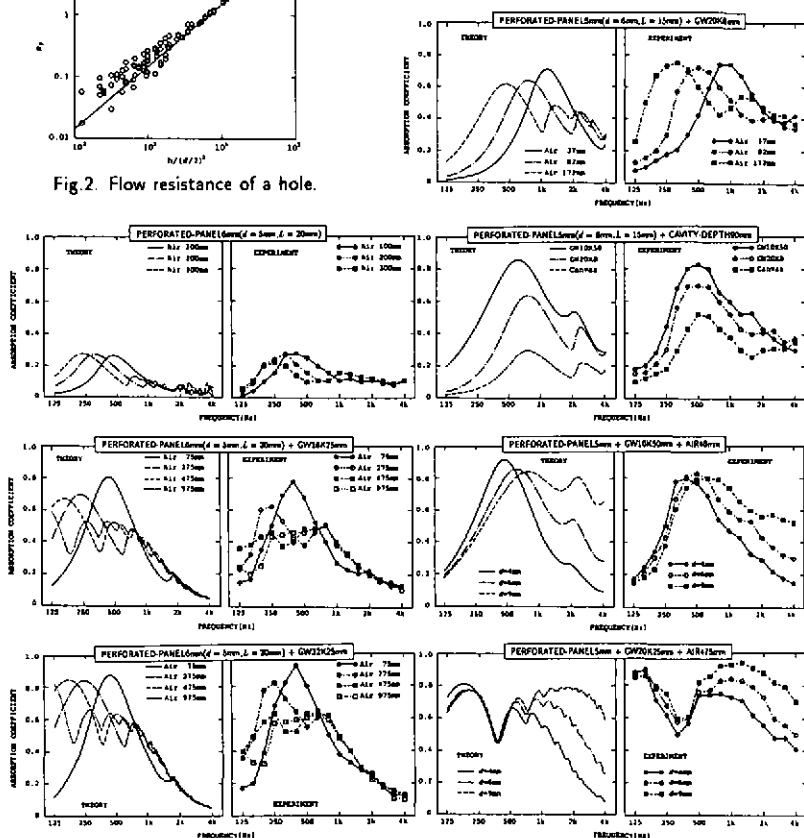


Fig. 3. Sound absorption characteristics of perforated systems.

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