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A FRESNEL ZONE APPROACH TO THE PREDICTION OF SOUND PROPAGATION ABOVE A MULTI-IMPEDANCE PLANE

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1. INTRODUCTION

Efficient and accurate methods exist for the calculation of the sound field produced by a point source above a plane boundary of uniform surface impedance. For boundaries comprising two regions of different surface impedance, with a straight discontinuity perpendicular to the direction from source to receiver, several solutions have been presented. Enflo & Enflo [1] have derived an exact solution which is very difficult to evaluate for practical situations. De Jong [2] developed a useful semi-empirical approximate solution based on far-field diffraction theory which is easy to evaluate. The method can be applied to multiple discontinuities and is not restricted to a particular orientation of the discontinuities in the boundary. Approximate numerical methods have also been developed based on the Kirchhoff diffraction formula in which the propagating wavefront is discretised above the discontinuity [3] and the boundary element approach where the discretisation is applied is the boundary [4]. The boundary element method is potentially the most versatile and accurate but is very costly in computing resources.

In this paper, an approximate formulation is developed, based on the concept of the Fresnel zone, to predict propagation over a multi-impedance plane. Predictions using the method are compared with experimental model results.

2. PREDICTION MODEL

Consider a monochromatic point source of sound, S and a receiver, R above an infinite boundary, shown in section in Figure 1. The sound pressure at the receiver results from a combination of the direct wave and the wave reflected from the surface. The pressure of the reflected wave, p_r, can be expressed using the Kirchhoff diffraction formula as an integral over the boundary, Γ. So that

$$p_r = \frac{1}{4\pi}\iint_{\Gamma}\left\{\frac{e^{ikr_0}}{r_0}\frac{\partial}{\partial n}\left(\frac{e^{ikr_1}}{r_1}\right)-\frac{e^{ikr_1}}{r_1}\frac{\partial}{\partial n}\left(\frac{e^{ikr_0}}{r_0}\right)\right\}d\gamma \qquad (1)$$

where r_0 and r_1 are the vectors from source and receiver respectively to the surface element, $d\gamma$, k is the wave number and n is the normal into the boundary.

Referring to the co-ordinate system shown in Figure 1, the real and imaginary parts of the integrand in equation (1) along the y' axis, for a rigid boundary are shown in Figure 2. The source and receiver heights were 1m and 4m respectively, the plan distance from source to receiver was 20m and the frequency 500Hz. Over a region between source and receiver the functions change slowly but outside this region they oscillate rapidly. The oscillations are due to changes of phase arising from the increasing values of r_0 and r_1. Similar effects will be observed along any line in the surface through the specular reflection point, P. A method of solution for a two-impedance plane based on a boundary integral equation formulation [4,5] produces an integrand in the expression for the reflected wave with similar characteristics to those in Figure 2. The main significant contribution to the integral representing the reflected wave is thus localised in a region around the specular reflection point where the rapid phase changes do not occur. This region can be defined in terms of a Fresnel zone.

The path length of the specularly reflected ray ((R_1+R_2) in Figure 1) is R_3. If S' is in the image of the source in the boundary, consider points in space, P' such that

$$(S'P'+RP')-R_3 = F\lambda$$

where λ is the wavelength and F is a constant. The locus of these points comprises an ellipsoid with foci at S' and R (Figures 1 and 3). The major semi-axis of the ellipsoid along the line S'R is given by

$$a = \frac{R_3 + F\lambda}{2}$$

and the minor semi-axes are

$$b = \left(\frac{R_3 F\lambda}{2} + \left(\frac{F\lambda}{2}\right)^2\right)^{\frac{1}{2}}$$

The intersection of the ellipsoid with the boundary plane defines an ellipse, as shown in Figure 3. Choosing a suitable value for F enables the elliptical region in the surface to be defined over which the major contribution to the Kirchhoff integral occurs. The form of the ellipse will also depend upon the source and receiver heights and separation. The equation of the ellipse referred to the coordinate system in Figure 3 is

$$\frac{(x\cos\theta + c)^2}{a^2} + \frac{x^2\sin^2\theta}{b^2} + \frac{y^2}{b^2} = 1$$

where θ is the grazing angle of the specularly reflected ray and $c = R_1 - R_3/2$.

The extremities of the ellipse in the boundary $x_{1,2}$ and $y_{1,2}$ are given by

$$x_{1,2} = \pm b\left(1 + \frac{B^2}{A^2} - \frac{c^2}{a^2}\right)^{\frac{1}{2}}$$

$$y_{1,2} = -\frac{B}{A} \pm \left\{\frac{1}{A} - \left(\frac{c\sin\theta}{Aab}\right)^2\right\}^{\frac{1}{2}}$$

where $A = \left(\frac{\cos\theta}{a}\right)^2 + \left(\frac{\sin\theta}{b}\right)^2$ and $B = \frac{c\cos\theta}{a^2}$.

We now assume that the form of the reflected wave is primarily determined by the region of the surface within the ellipse following the suggestion of Slutsky and Bertoni (6). Consider a uniform boundary of surface impedance, Z_1. For a given source and receiver geometry, the excess attenuation at the receiver can be calculated, EA_1. A uniform boundary of surface impedance Z_2 would produce excess attenuation, EA_2. The proposed method of obtaining the excess attenuation for a plane comprising regions of admittance, Z_1 and Z_2 is to interpolate linearly between EA_1 and EA_2 according to the proportion of the area of each type of surface within the ellipse.

3. EXPERIMENTAL RESULTS

To test the approach, a simple boundary configuration was used. This comprised a two impedance plane with a straight line discontinuity. The experimental modelling was carried out in an anechoic chamber which contained a false floor with a Formica surface, which formed the rigid boundary. The sound source comprised a loudspeaker situated beneath the floor, connected to a heavy brass tube of 7mm internal diameter, which projected through the floor. The detector was a 10mm diameter condenser microphone. The region of the boundary of finite impedance was produced by introducing a layer of felt on the rigid surface. The impedance of this surface was determined by a curve fitting technique to two spectra obtained for propagation over a uniform plane for two different source and receiver positions. It was found that the impedance of the surface could be described by assuming that it was locally reacting, that the properties of the felt could be described using the equations of Delany and Bazley, with a flow resistivity of 250,000 Nsm^{-4} and that the felt formed a hard backed layer of depth 2.9mm.

Measurements were carried out at 10kHz for source and receiver heights of 0.030m and a distance from source to receiver of 1.000m. Figure 4 shows results for excess attenuation when the discontinuity is perpendicular to the direction from source to receiver, with the rigid boundary on the source side. The measured excess attenuation is plotted against the distance from the point in the surface beneath the source to the discontinuity (d). The points show a smooth transition between the excess attenuation result for a uniform rigid plane (-5.9 dB) and that for a uniform absorbing plane (13.7 dB).

Assuming that the proportion of each surface type within the Fresnel zone ellipse are approximately given by the proportion of cover along the line in the surface between source and receiver within the zone and using a value of F = 1/3 produces the change predicted by the dotted line. The edges of the zone occur at d = 0.029m and d = 0.971m.

Figure 5 shows results for excess attenuation when the discontinuity is parallel to the line from source to receiver projected into the boundary. The distance from this line to the discontinuity is d'. When d' = 0, the discontinuity is directly below source and receiver and positive values of d' indicate rigid ground beneath source and receiver. The dotted line in Figure 5 is deduced from the proportion of each surface type along the minor axis of the ellipse. The edges of the zone occur at d' = ±0.076m. In both cases the fitting between the experimental results and the predictive method is reasonable.

4. CONCLUSION

A simple model has been developed to describe propagation of sound above a boundary comprising regions with two different values of surface impedance. The limited comparisons between the experimental model results and the predictive method show reasonable agreement. The approach potentially allows prediction of sound propagation over surfaces with complex, two dimensional distributions of surface impedance. The only other method offering a solution to this general problem is a three dimensional boundary element approach which, at present, is too expensive in terms of computer requirements.

Although the method involves a high degree of approximation, it is probably quite appropriate for predictions of outdoor sound propagation, where octave or 1/3-octave bands are considered and it is very difficult to obtain an accurate, detailed description of the site surface conditions and topography .

Atmospheric conditions could be incorporated in the model by distorting the path of the reflected ray and the shape of the ellipse to allow for curvature of the ray paths due to wind and temperature effects.

5. REFERENCES

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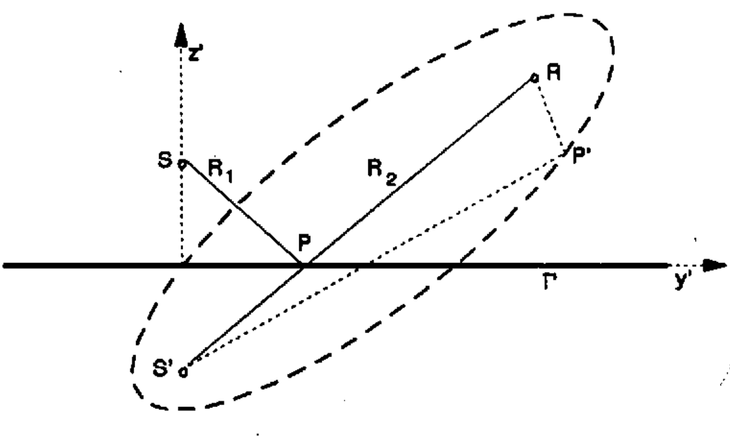

Figure 1. Source and receiver geometry.

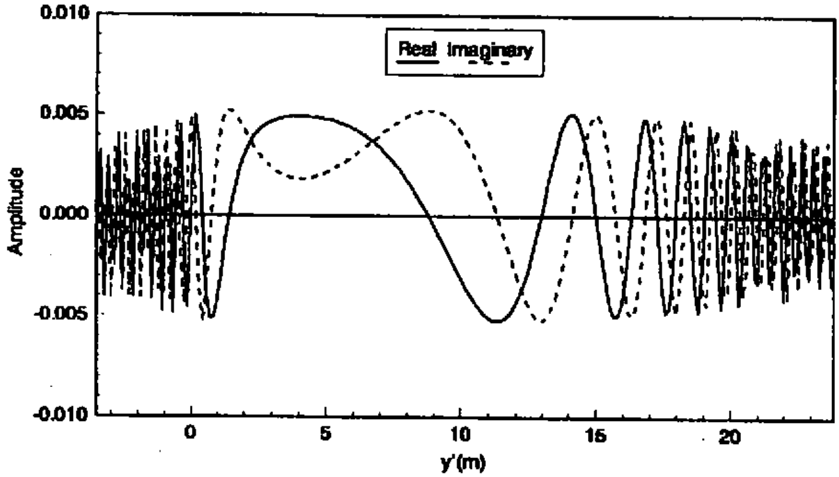

Figure 2. Real and imaginary parts of the integrand in equation (1) along the y' axis. The boundary is rigid.

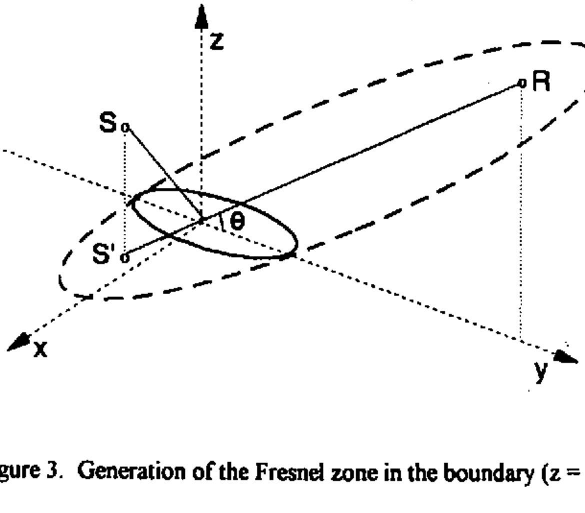

Figure 3. Generation of the Fresnel zone in the boundary ($z = 0$).

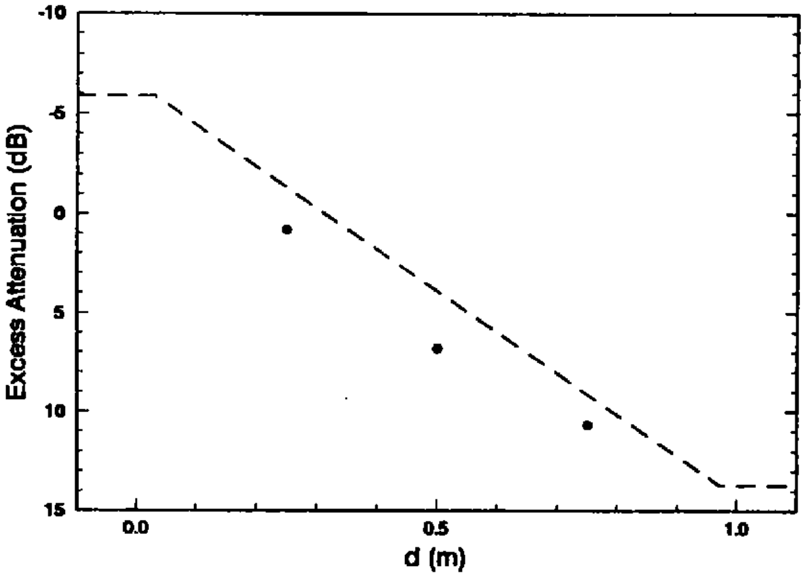

Figure 4. Excess attenuation against the position of the discontinuity for a two-impedance plane. The discontinuity is perpendicular to the direction from source to receiver.

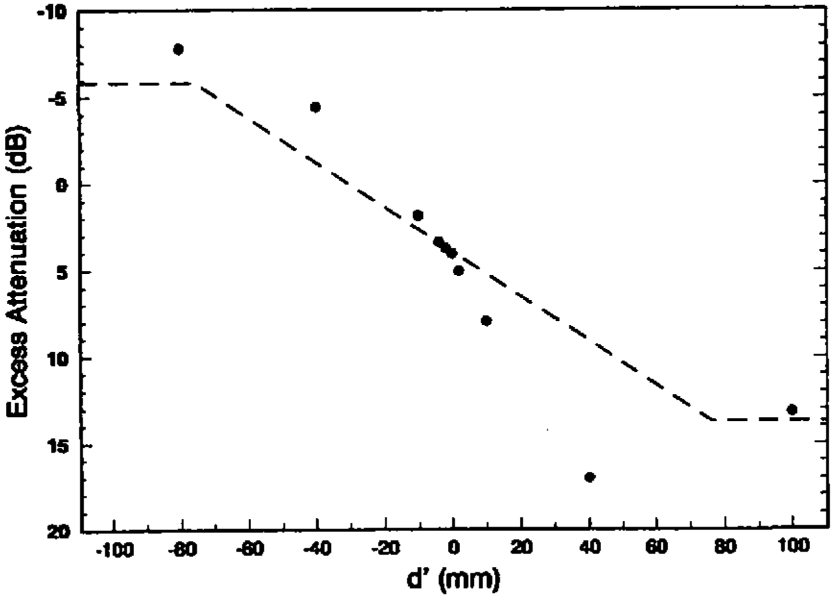

Figure 5. Excess attenuation against the position of the discontinuity for a two-impedance plane. The discontinuity is parallel to the direction from source to receiver.