DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

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ABSTRACT

A model for the simulation of the sonar process has been developed for sidescan sonar, which considers the physical phenomena underlying the process and uses ray tracing techniques to produce realistic synthetic sonar images. Initial concentration on sidescan sonar permitted many simplifying assumptions, including modelling the transducer as a point source and receiver with a directivity index and assuming that movement of the transducer between transmission and reception was negligible. The objective of producing a general sonar model capable of modelling other sonar architectures has led to the reconsideration of these assumptions. Removal of the point source assumption could lead to an astronomical increase in the number of rays to be traced in order to provide adequate coverage of the transducer. An approximation is presented, based on vector methods, which removes the necessity for tracing speculative reflected rays. This limits the unavoidable increase in computational load, in terms of the calculation of the time of travel and returned intensity for each ray, while remaining consistent with the full ray tracing method.

1. INTRODUCTION

With the current rapid developments in sonar signal processing, the use of modelling to permit testing and evaluation of such systems is of increasing importance. One such model, suitable for this, has been developed for the simulation of the sidescan sonar process [1][2]. This model is unique in that it permits the visualisation of the physical characteristics of the sonar process by creating synthetic images of the form produced by the actual sonar system, through simulation of the physical processes which result in the generation of sidescan images. However, the model is currently limited to sidescan sonar, and makes several limiting assumptions of which two of the most significant are that the transducer can be modelled as a point source and receiver of the sonar signals, and that the transducer does not move between transmission and reception of a signal. The usefulness of the model could be extended by removal of such limiting assumptions, producing a generic structure to simulate a range of sonar geometries and architectures including sector scan, forward looking, multi-beam and synthetic aperture sonar.

Such a generic model, capable of accurately synthesising sonar images and signals, could have a wide range of potential applications - from training and visualisation to complex design and testing of sonar systems. This paper addresses the limitations of the existing sidescan model and suggests a possible technique to overcome these, to produce a structure for a generic model which can accurately simulate the sonar process without limiting its applicability through computational complexity.

2. THE EXISTING MODEL

The existing sidescan model is based on the concept of ray tracing, often employed in computer graphics applications. In this, artificial images are created by tracing light rays through the propagation medium (generally air) until they interact with the solid, reflecting areas of the pre-defined scene. More relevantly, ray tracing techniques have also been used to model the response of rooms to acoustic stimuli. More complex than the image modelling technique used for a similar purpose [3], ray tracing is, however, more easily extendible to non-regular environments i.e. those with changing surface textures, with non-rectangular volumes and with signal absorption dependent on signal frequency and incident angle. Extension of this approach to the tracing of sound rays through water is, however, complicated by the non-linear underwater environment which has temperature, pressure and salinity variations contributing to changes in the sound velocity in addition to some degree of randomness in the presence of obstructions, additional noise sources and changes to the acoustic environment. It is also complicated by the time based display of sonar images compared to the spatial domain representation of visual images.

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

2.1 Model Characteristics

Sidescan sonar images are generated sequentially, line-by-line, by the emission of pulses of acoustic energy to the side of the towed sidescan device. The reflected energy returned to the transducer for each pulse is then used to create one line of the sidescan image, which is a plot of the intensity of the acoustic returns against time. To emulate this continuous-time process, the discretised model represents each acoustic pulse as a set of rays orthogonal to the expanding wavefronts of the emitted signal. These rays are emitted at pre-set angles from the transducer and their trajectories are traced until they interact with the pre-defined underwater scene. From the intensity and time values returned for each ray, the sidescan image can be generated. The water affects the path of the rays and a ray based acoustic propagation model is used to determine the actual trajectories of the rays as a result of the changing sound velocity within the water column.

There are many other propagation models available, each of which is a limited solution of the three-dimensional wave equation to produce a model which will be accurate only within certain limits. However, ray tracing provides a clear, easily understood depiction of how signals are propagated through the water environment. In addition, since the model must have the ability to deal with a variable sound speed profile, randomly rough boundaries, omni-directional propagation, and operate at high sonar frequencies, ray tracing was the preferred option. The variable sound speed profile is assumed to be horizontally stratified, a reasonable assumption for many cases which limits the increase in complexity.

In addition to calculating the propagation through the refractive water medium, and the subsequent spreading and absorption losses, the model also considers the scattering from the seabed - or objects within the scene - and the transducer directivity and motion characteristics. The seabed is defined as a grid of height values, permitting the use of genuine bathymetric data or values generated using fractal models [4]. The scattering can be calculated from complex seabed scenes, represented by height fields, using either the simple approximation of Lambert's Law, or Jackson's bistatic model [5] which also gives the opportunity to calculate specular reflection for tracing multiple reflections. The synthetic sidescan images created by the model appear realistic when compared to real sidescan images, as they can mimic the sonar artefacts. Statistical and spectral comparisons have also been undertaken to verify the output of the model [1].

3. EXTENSIONS TO MODEL

3.1 Limitations of sidescan model

The modelling process is currently limited to sidescan sonar, as a result of many simplifying assumptions used in its development. The most significant of these is the assumption that the transducer is represented as a point source and receiver. Removal of this assumption would permit the creation of a more general sonar model, capable of representing other types of sonar, by modelling the transducer realistically as a distributed array in 3D space, which would permit beamforming of the elemental outputs to provide the appropriate sonar output for any system. This necessitates that the signal can no longer be considered to be transmitted from and reflected back to the point source, and some methods must be investigated whereby the strength of signal transmitted and received across the transducer surface could be determined.

Another assumption which has been made is that the forward motion of the towfish between transmission and reflection is of little significance, and so the ray trajectories back to the transducer from the intersection with the seabed or obstruction were assumed identical to the incident paths. This is reasonable for sidescan sonar, in which the direction of travel is perpendicular to the direction of emission of the acoustic pulse, and therefore does not affect the signal returns significantly. However, if forward scan or sector scan sonar is to be modelled, the implications of this will change and it can no longer be assumed that the incident and reflected ray paths are identical. Also, it is implicit that with the removal of the point source/receiver, for each incident ray it will be necessary to trace several return rays which will increase the computational complexity significantly.

Signals coming from diverse directions can have similarly diverse phases when arriving at a particular point in space even when derived from the same signal source. The scattering effects investigated in the work thus far ignored this, and so it is possible that some synthetic reflections would exhibit amplitudes far greater than those which would be observed in practice. The phase of the signals must, therefore, be monitored during both the propagation and the reflection/scattering of the sonar signals to provide a realistic model. It is intended that in future attention will also be focused on incorporation of sea surface reverberation [6] and the effect of any stochastic noise sources in the medium, which are also currently neglected in the sidescan model.

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

3.2 Implications for Extended Model

Any developments made in extending the model must balance the effect of the change on the model's ability to produce accurate timing and phase information against the effect of the change on the complexity of the processing. There could be two possible ways of considering the array if the point source/receiver assumption were to be removed. Either it could be modelled as a plane in 3D space with a directional and positional sensitivity pattern, or each element of the array could be modelled as a separate and independent source and receiver. A problem with modelling each element as a point within the array would be that to implement the model effectively, the rays would have to be traced back to that specific point rather than just to a plane representing the array or element surface. Either of these approaches would improve the capacity for diversifying the process by introducing beam steering or beamforming, but would also have considerable impact on the complexity.

For instance, in the original model a sonar pulse was modelled as a number of rays - in the order of thousands to ensure adequate seabed coverage and to satisfy sampling conditions for the image generation. In the sidescan model, each transmitted ray would have one return ray associated with it, and since the transducer did not move, this was identical to the incident ray and so only the outward part of each ray had to be tracked. However, if a distributed array of n elements is now included, rays from each of the transmitting elements can contribute to each of the receiving elements and thus n^2 rays must be traced for every one ray of the original model. This would be increased yet further since the "trial and error" nature of ray tracing ensures that it is not possible to predict the ray between two points if the velocity and direction of the ray can change during its time in the transmission medium, and therefore the ray trajectories can only be determined in an iterative manner.

This suggests that removal of this one simple assumption will significantly increase the computation time of the model and, given the current computational complexity of the sidescan model, this basic extension could render the model impractical as a result of the execution time required. However, if some method were to be found of determining the ray paths and times of flight to each element of an array which did not require the intensive iterations of tracing each individual ray, then both the point source/receiver and the stationary towfish assumptions could be discarded. The stationary towfish assumption could safely be discarded at the same time, because if each ray is being individually traced, it would be as simple to trace rays back to a different towfish position.

3.3 Vector Approximation

Therefore, the development of this general model whereby these assumptions are removed, relies on the less limiting assumption that the outward and return rays will follow paths which are similar (but not identical) and almost (but not quite) straight - a reasonable approximation if the sound velocity profiles have no significant discontinuities. The latter means that modelling an "equivalent vector" for each ray is feasible as a means of determining the position of the transducer at the instant of ray intersection with any interface, and by extension, at that instant when the reflected ray returns to the transducer. The position of each element can therefore be estimated at the return instant for a ray emitted from any given transmitter, and an impulse response can be calculated for each element using the return times. By this method it will be possible to use beam steering and beamforming methods rather than assume a single point with a beam pattern, which will be extendible to different sonar architectures. It will also be possible to incorporate non-flat transducer planes e.g. if the transducer array is on a cylindrical or spherical surface, since the method depends on knowing only the original positions of the elements, the direction and speed of travel, the orientation of the towfish and the direction and speed of ray travel, all of which are user-controlled.

The incident and reflected rays paths, and hence the times of flight, will be very similar for each element-to-element path with only slight differences in position and velocity of the rays contributing to minute differences in the times of flight. The method developed to take advantage of this is henceforth known as *vector approximation*, and will be described in detail. The aim has been to use this vector approximation method in order to calculate the time for a ray to return to each element of a 2 or 3 dimensional array of transducers. The approximations will allow a reduction in the computation time, while maintaining accuracy in the time of flight results.

With this technique, a ray is accurately traced out from a point of transmission as before, through the refractive medium to intersection with the sea bed, as illustrated in Figure 1. The "point receiver" assumption was removed before the "point source" assumption to permit the verification of the procedure, therefore within the following discussion the source of the rays is still assumed to be the point at the geometric centre of the transducer array. At the time of intersection of the ray

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

with the seabed, or any pre-defined object, the following parameters are known: the incident time of flight t_i, the origin position O (assumed for the moment to be the centre of the array), the intersection position I, the vector $\underline{l}$ which connects these two points, the final angle and speed of the ray at intersection, the speed v_t and direction of towfish motion. From the speed and direction of the motion and t_i, it is simple to determine the position of the towfish H (centre of the array) at the intersection time by breaking down the towfish velocity v_t into three orthogonal components v_{tx}, v_{ty} and v_{tz}.

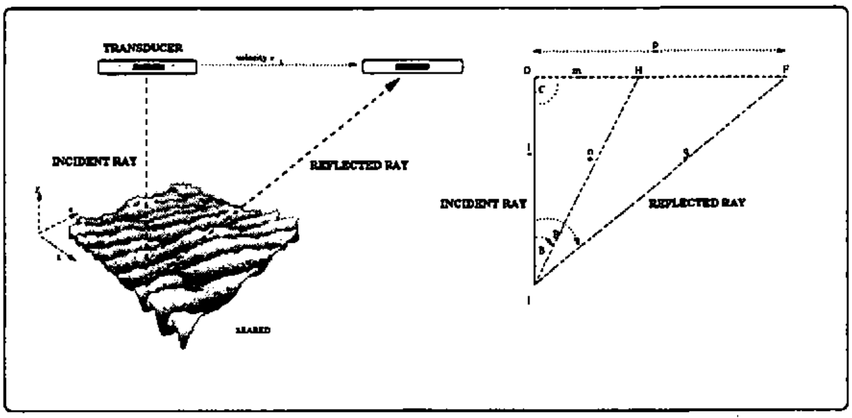

Figure 1: Sonar ray paths with equivalent vectors between ray start and end points

Any three points in three dimensional space must be coplanar, and so the three points O, I and H can now be used to define a plane, and the angle (C) between vectors $\underline{l}$ and $\underline{m}$ will be known since the direction of both towfish and ray motion is pre-defined. The magnitudes of the vectors $\underline{m}$ and $\underline{n}$ can be determined from the differences in the displacement in three dimensions and the angle (B) between the incident and return vectors $\underline{l}$ and $\underline{n}$ can then be calculated from the sine rule as

$$B = \sin^{-1} \left((|\underline{m}| / |\underline{n}|) \, . \, \sin C \right) \tag{1}$$

With reference to figure 1, the vector representing the difference between I and the final towfish position F can be determined by doubling this angle B to give B', since the angle of incidence of the return vector within this plane will be equal to the angle of reflection, and the path of the return vector will be the incident vector reflected about an axis represented by $\underline{m}$. The magnitude of the return vector $\underline{q}$ can be obtained using the sine rule again since B is known and C is unchanged, if it is assumed that the direction of travel of the towfish will not change significantly during the transmit/receive period. The final position of the centre of the array F can then be obtained using the magnitude and direction of vector $\underline{q}$ and the intersection position I.

Since the orientation of the transducer in all three dimensions and the position of each element relative to the centre of the array will have been specified, then the position of each element at the time when the centre of the array has reached this final position can be determined. It can also be assumed that the array will not move significantly between a ray reaching one element and a neighbouring ray reaching the next element, since the velocity of the rays will be very much greater than that of the towfish.

This method considerably reduces the number of actual rays which must be incrementally traced through the complex medium, which is the most time-consuming and computationally intensive section of the model. As was previously discussed, it would not be possible to know the precise direction in which to trace rays back in order for them to reach their intended destination at the array. Therefore, it was entirely possible that many unnecessary rays would have to be traced to assist further development of the system. Another significant advantage of the vector approximation technique is that it calculates *only* the return times for the rays to each element, and should therefore prove advantageous in this respect in terms of computation time and the avoidance of redundancy.

3.4 Effective Velocity of Vectors

This vector approximation method relies on accurate determination of an *effective velocity* for each vector representing a returning ray between the seabed interface and the transducer. Determination of an effective velocity for the incident ray is a relatively simple procedure, since the start and endpoints will be determined by the ray tracing software as will the

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

time of flight - from these the effective velocity of the incident vector can be obtained.

In the first stage of development, it was assumed that the effective velocity of the return vectors could be considered approximately equal to the effective incident velocity. However, this assumption is invalid since the elements of the array will be distributed in space. Since a horizontally stratified sound speed profile is employed, if the elements vary in height, then the velocity of the returning rays will differ for each element. If the transducer is moving in the vertical plane between transmission and reception of the signal, the effective velocity will be similarly affected. Therefore, it was necessary to develop a method to determine the effect of this variation in vertical positioning on the effective return velocity values.

If the medium were isovelocity, then the vectors would represent the exact ray paths and the times of flight could be determined using the constant velocity value specified. However, this trivial case is unrepresentative and in practical environments the incident time of flight t_i along a non-linear ray path will be calculated incrementally as it crosses each new layer of medium of a particular velocity gradient. The first significant assumption comes in the determination of an *effective velocity* for the incident vector, $\underline{l}$. The magnitude of $\underline{l}$ is easily calculable, and if this is divided by the incrementally estimated incident time of flight t_i, then the quotient can be regarded as the effective incident velocity ev_i for this vector. This can be assumed to be close in value to the effective velocity for the return vector $\underline{q}$, if the motion of the towfish between transmission and reception is fairly small in both the vertical and the horizontal plane, as will be the case with standard towfish towing speeds. A different effective return velocity ev_r can be determined for the return path to each of the array elements, with a correction factor added to the return velocity which depends on the difference in height between the points O and F and the velocity gradient between those points. If it is assumed that the array is smaller than one velocity gradient layer (which are frequently only recorded at 1m intervals), then the assumption of a constant linear velocity gradient between points O and I is reasonable.

A formula based on this linear relationship assumption could therefore be derived, by determining the actual effective velocity for a number of rays traced back to the transducer plane. From the time and displacement figures determined during the ray tracing process, the effective velocity for each return ray could be calculated and the variation of these with the height of the ray's endpoint (each commencing from the same intersection point) could be noted.

Graphs were plotted of the effective velocity against the height difference between transmission and reception for a range of greatly exaggerated test profiles and for actual profiles [7], and in each case it was found that the relationship appeared to be linear. An average coefficient of 0.503 was obtained for all cases. Therefore, a formula for determining the effective return velocity ev_r for a ray returning to a point with height h_r, when the incident ray (with effective velocity ev_i) started from a height h_i and the velocity gradient between these two heights is given by g, could be

$$ev_r = ev_i + (0.503g\,(h_r - h_i)) \qquad (2)$$

Assuming that the effective velocity of signals returning to points of equal height will be identical regardless of the horizontal range seems reasonable, since height is the only factor contributing to a change in velocity. For the cases where the SVP has a constant velocity gradient, then, estimating the effective return velocities using the above formula is simple and accurate. Using this formula with more variable SVPs such as those obtained from the Clyde Estuary [7] - in which the gradient will change depending on the SVP layers a ray enters, may be less accurate. However, the velocity profiles and height differences were exaggerated for this experiment in order to obtain accurate values of the parameters and to confirm the reliability of the method. In practical cases, it is unlikely that the elements will be so widely spaced, nor the velocity profiles so pronounced, and the accuracy of the values should be adequate.

The above formula is an attempt to approximate a non-linear change in the velocity of a returning ray by a linear modification of the incident value. This approximation seems to hold for a wide range of heights, as long as the intersection point and incident transit time are carefully determined. Any slight inaccuracies in the velocity values obtained by using the formula given above will also be negligible in their effect on the final result. For instance, consider a transmission height of 9.5m and a reception height of 9.382419m. At a depression angle for the transducer of around 40°, the range of the return signal to the latter height is 11.494591m, indicating that the total displacement of the ray in all three dimensions is 14.83763487m. The effective velocities and travel times for a ray returning to this latter height, obtained for one particular exaggerated SVP are illustrated in table 1.

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

	Ray Traced	Formula
Effective Velocity (m.s^{-1})	**1475.859495**	**1475.856104**
Travel Time (s)	**0.010053555**	**0.010053578**

Table 1: Comparison of effective velocities and travel times for a sample ray

Therefore, the difference between the two methods for the return time to the furthest extreme considered - which was also the point of greatest divergence from the incident velocity value - was only 23ns. Even at a frequency of 500kHz, this will only represent around 1/87 of the period of the waveform. This is for a height difference of over 0.1m, or 100mm, which is representative of that expected for a practical transducer, and in a region of linear velocity gradient of -5, which is far greater than many which will be experienced in practice. This particular calculation was repeated using an actual profile from the Clyde Estuary and the difference in the travel times was reduced to 3ns, which is even less significant.

4. RESULTS

Initial results verified that the model was capable of reproducing the type of results generated by the original, simpler model. Figure 2 shows an artificially generated sinusoidal height field, and the impulse response obtained by transmitting rays in a knife edged beam perpendicular to the ripples. Several anticipated effects can be observed in the impulse response - for example, the peaks of the height field and their increased separation due to increasing distance from the transducer. The artificially high returns sometimes associated with this modelling method, due to a large number of rays contributing to a limited number of samples, can also be seen on the first two peaks.

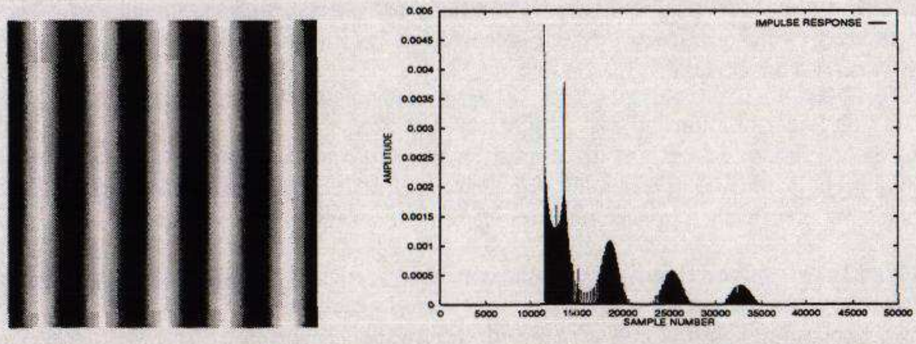

Figure 2: (left) synthetic sinusoidal seabed (right) impulse response for a line of synthetic sonar image

Thus, with the verification of the vector approximation concept and the derivation to a sufficient accuracy of the formula for the effective velocity, the model was then extended to remove the assumption of a point transmitter. This then resulted in accurately tracing rays from all elements and then approximating their return to every element of the array using the vector approximation technique, discussed in the previous section. To illustrate the resulting phase and time differences obtained when using a distributed array, Figure 3 shows an impulse response for a scene incorporating a flat seabed, with a gap of approximately 22° between neighbouring rays. For each set of plots, the left hand graph is the full impulse response, and the centre and right hand plots are details of this impulse response, concentrating on the third set of peaks from the original, with an expanded y-axis to highlight the significant features. The transducer has been defined as moving parallel to the z-axis, with a depression angle of 45° and otherwise oriented parallel to the z-axis.

For the single element case depicted in the upper row, each ray contributes a single impulse to the impulse response. However, if there are three elements transmitting, as in the middle and lower examples, then there will be 3 incident rays rather than 1, and each incident ray will have 3 return rays associated with it i.e. there will be 9 returns, resulting in a total of 27 impulses compared to the 3 of the single element case; an increase of n^2 as anticipated.

The distance between the elements has been specified as 100mm. In the second case, where the elements are distributed vertically, the depression angle results in a height difference between adjacent elements of 70.7mm. The differing heights and consequent differing effective velocities result in slight differences in both the incident and the return times - a phase change of around 260 sample periods is evident for the response of any one receiver to a change in transmission height, and a change of around 20 samples is observed between returns to different reception heights from the same transmitter.

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

However, if the elements are distributed horizontally (in either the x and/or z plane) as in the third example, there is no significant change in the effective velocities and so the differences in incident and return times are much less significant, depending only on the element separation. The nine returns for each set of three rays can therefore be seen to be contributing to the same sample in all bar one case, which contributes to the immediately neighbouring sample. The result is an impulse response which is apparently increased in intensity, because for n elements there are now n^2 impulses contributing to the intensity of response at each sample, with the occasional contribution to a neighbouring sample.

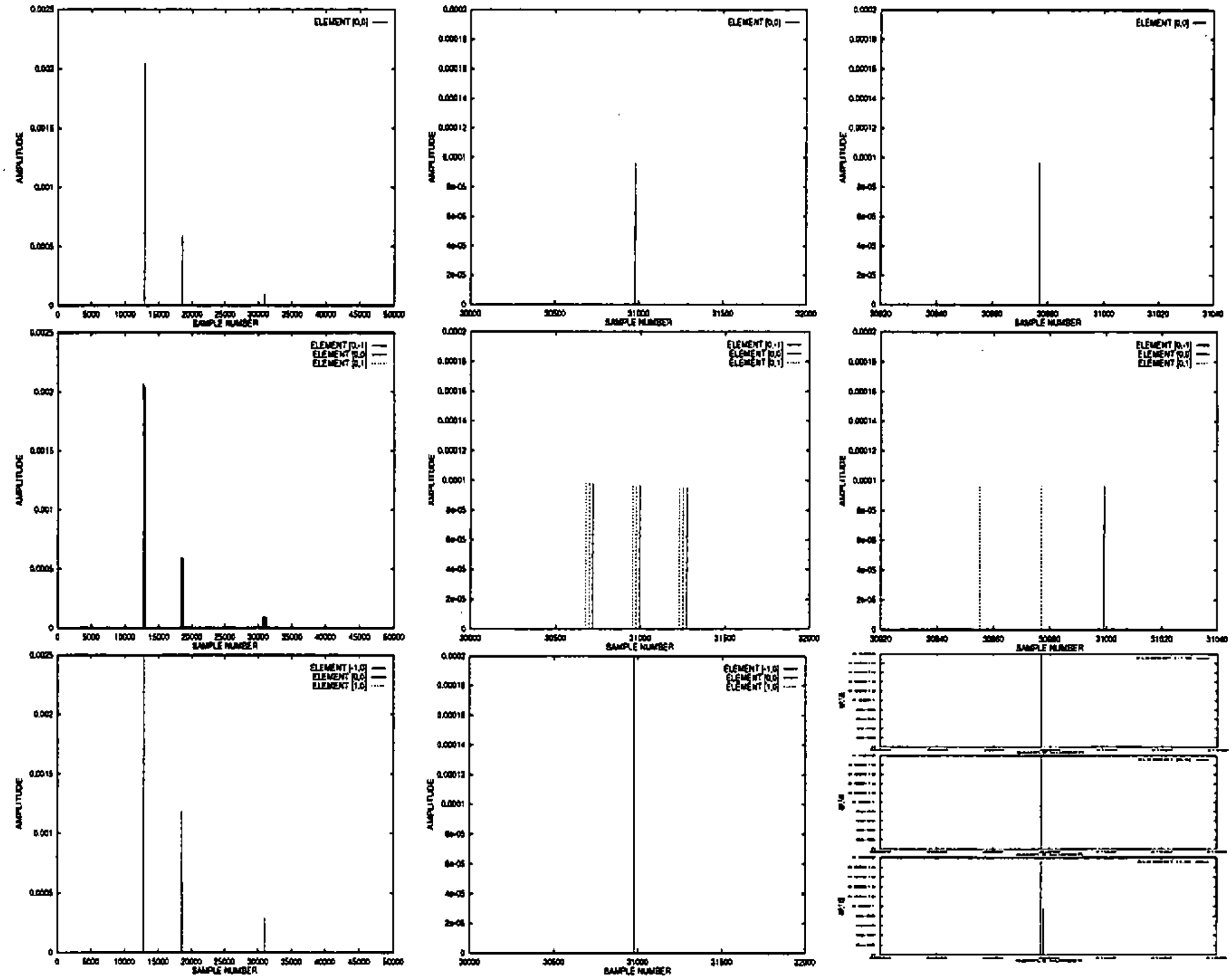

Figure 3: Example impulse responses for a flat seabed using (top) single element transducer, (middle) three vertically distributed elements, (bottom) three horizontally distributed elements

Clearly exhibited in these graphs, therefore, are the ability of the newly-developed model to estimate phase change relationships between diverse elements of the transducer, and the potential effect of a sound velocity profile with variations dependent on height on those phase changes. The phase changes are dependent in part on the separation of the elements, and in part on any difference in ray velocity due to that separation. This second component of the change is more significant, and so in a horizontally stratified medium, vertical separation between elements produces more significant changes in phase between those elements than would a corresponding separation in a horizontal direction.

5. CONCLUSIONS AND FUTURE DIRECTIONS FOR RESEARCH

A significant development in the revision of the sonar model has been the introduction of the vector approximation technique, which allows the estimation of the times of flight of rays representing sonar pulses, while avoiding having to

DEVELOPMENT OF A GENERAL SONAR SIMULATION MODEL BASED ON RAY TRACING TECHNIQUES

trace an enormous and unwieldy number of rays back and forth though the medium on a trial and error basis. This has been achieved by assuming that the velocity of the return signals can be estimated with reference to the velocity of the outgoing signal, the change in position between the transmission and reception points of the signal, and the sound velocity profile of the medium. This has been demonstrated to produce adequate estimates of the times of flight.

The removal of the point source/receiver assumption will also permit more detailed pre- and post-processing of the signals than the simple implementation of a weighted 3D beam pattern. By use of weights and delays on each element of the transducer, the signals can be steered prior to transmission or combined after reception in a beamformer. The potential now exists for each transmitting element to contribute in some way to each receiving element. In essence, the model computes an impulse response to and from each element of the array for every line of the image. The impulse responses for each receive element can be convolved with a signal representing the transmitted pulse in order to produce a stream of data representing the signals received by each element of the transducer array.

The impulse responses are also under investigation using a technique whereby the impulses are modelled using the low-pass filtered method of Peterson [8] rather than the shifted impulse method, in which each impulse is centred around the nearest sampling instant of the impulse response. It is anticipated that such a modification will produce more accurate estimates of the phase difference between elements, and may reduce the effect of the artificially sharp caustics associated with ray tracing methods. A second avenue which has been explored is the possibility of tracing only a small number of incident rays when there are a great many transmitting elements, and interpolating between the ray endpoints obtained. This could minimise the inevitable increase in computational complexity associated even with the amended version of the ray tracing technique presented in this paper, albeit possibly at the expense of reduced resolution on the synthetic images produced.

Further thought must also be given to the number of rays which must be used in modelling of an individual sonar pulse. The distribution of the rays used in the ray tracing method must be sufficient to satisfy the Nyquist criterion for sampling, which has hitherto been addressed by ensuring that the times of flight are equally distributed in the time domain for rays incident on a perfectly flat seabed. As the seabed will invariably not be flat, the distribution of the return times for any given incident ray will not be regular and there may be more or fewer rays incident on a particular area of the seabed than would be expected. This could produce a sonar record with excessively dark or light patches, and this lack of regularity in the distribution will become more pronounced with a rougher surface, a vertical region on the seabed or an object obstruction in the ray path. Enhancing the generality of the model further will therefore require that this problem be given considerable thought.

ACKNOWLEDGEMENTS

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