

## A SIMILARITY TECHNIQUE FOR THE PREDICTION OF AIRFLOW GENERATED NOISE IN DUCTS

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### 1. INTRODUCTION

A number of investigators have tried to devise generalised prediction techniques for air flow generated noise and this work has been summarised in a recent paper by Mak, Waddington and Oldham [1]. The authors [2] recently described a predictive technique based upon the static pressure loss due to in-duct elements which built upon earlier work of Nelson and Morfey [3]. In this paper we describe how the Nelson and Morfey work can also be employed as the starting point of an alternative technique based upon aerodynamic and acoustical similarity.

Most fluid flow situations are of such complexity that they cannot be modelled using analytical techniques. An alternative approach is to use data measured on one configuration for another configuration which is sufficiently similar for simple scaling laws to apply. For example, this is the basis of the wind tunnel testing of scale models to determine the drag characteristics of dynamic objects.

In employing similarity in this way it is necessary to identify the relevant dimensionless groupings of parameters. The Strouhal Number is such a dimensionless grouping and its most well known application is to relate the frequency of vortex shedding from a cylindrical spoiler in an air stream to the velocity of the airflow and the diameter of the spoiler. The Strouhal Number is also frequently employed to normalise spectral data relating to the noise generated by in duct flow spoilers (see, for example, Oldham and Ukpoho [4]).

In this paper we show how similarity can be used to predict the airflow noise generated by common types of duct components.

### 2. THE GOVERNING EQUATIONS

Nelson and Morfey [3] devised an expression for the sound power radiated from an in-duct spoiler subjected to the action of fluctuating drag forces arising from the flow turbulence in the vicinity of the spoiler. The noise generated by an in-duct flow spoiler was treated by replacing the spoiler and the turbulence by an equivalent acoustic distribution of dipole sources radiating into a duct filled with fluid at rest. They obtained two simple approximate expressions for the sound power of regenerated noise from the solution for the dipole source of the inhomogeneous wave equation governing the propagation of sound in an infinite hard walled duct. One expression applies to frequencies below the cut-on frequency  $f_c$  of the first transverse duct mode (plane wave propagation only), the other one applies to frequencies above the cut-on frequency  $f_c$  (multi-mode propagation).

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The sound power  $W_M$  radiated in a given bandwidth  $\Delta_f$  is given by:

for  $f_c < f_o$ ,

$$W_M \approx (1/4A\rho_o c_o) (\overline{F_3^2})_M, \quad (1)$$

for  $f_c > f_o$ ,

$$W_M \approx (\omega_c^2 / 24\pi\rho_o c_o^3) (\overline{F_3^2})_M [1 + (3\pi c_o / 4\omega_c)(a+b)/A], \quad (2)$$

where:  $A$  is the cross sectional area of the duct,  $\rho_o$  is the density of air,  $c_o$  is the speed of sound in air,  $(\overline{F_3^2})_M$  is the mean square of the fluctuating force in a given band,  $f_c$  is the centre frequency of the band of frequencies under consideration,  $f_o$  is the cut-on frequency of the duct,  $\omega_c$  is the angular center frequency,  $a$  is the cross sectional duct width, and  $b$  is the duct height.

However, as it was impossible to quantify the fluctuating drag forces Nelson and Morfey developed their predictive technique by assuming that the root mean square fluctuating force acting on the spoiler is directly proportional to the steady state drag force to yield the following equations.

For  $f_c < f_o$ ,

$$W_M = (\rho_o / 16c_o) AK^2 (St) [\sigma^2 (1 - \sigma)]^2 C_D^2 U_C^4, \quad (3)$$

for  $f_c > f_o$ ,

$$W_M = (\rho_o \pi / 24c_o^3) [1 + (3\pi c_o / 4\omega_c)(a+b)/A] (A/d)^2 (St)^2 K^2 (St) [\sigma^2 (1 - \sigma)]^2 C_D^2 U_C^6, \quad (4)$$

where  $C_D$  is the drag coefficient,  $\sigma$  is the open area ratio,  $U$  is the flow velocity,  $U_C = \frac{U}{\sigma}$  is the maximum effective velocity,  $d$  is the characteristic dimension,  $St = fd/U_C$  is the Strouhal number, and  $K(St)$  is a Strouhal number dependent constant. The drag coefficient  $C_D$  is a dimensionless quantity which will be constant if measured on geometrically similar configurations over the range of Reynold's Numbers encountered in ventilation ducts. Thus it represents the starting point for the procedure to be outlined in this paper.

## 3. THE PREDICTIVE TECHNIQUE

### 3.1 Below the Cut-on Frequency

On examination of equation (3) it can be noted that both the open area ratio  $\sigma$  and the drag coefficient  $C_D$  are dimensionless and functions of the geometry of the noise generating element. In addition, for the conditions encountered in a normal ventilation system, the density of air  $\rho_o$  and the velocity of sound in air  $c_o$  will be constants. The remaining terms are the Strouhal number dependant constant  $K(St)$  and the constriction velocity  $U_c$  which is simply the mean duct velocity  $U$  divided by the open area ratio  $\sigma$ .

The Strouhal number is given by  $St = fd/U_c$  where  $d$  is a characteristic dimension of the noise generating element. Apart from the case of a simple duct element such as a strip spoiler, it is not readily apparent how this dimension should be defined. However, as it will be a function of the duct geometry in the vicinity of the noise generating element, the ratio of characteristic dimensions for two geometrically similar elements will be equal to the ratio of any other linear dimensions.

For the case of a given duct element the ratio of the sound power level generated at fixed Strouhal number for different air velocities is simply proportional to the fourth power of the air velocities. As  $d$  (being a function of geometry) does not change then, in order to maintain a constant value of Strouhal number, if the air velocity changes there must be a change in frequency in proportion to the change in velocity.

Thus, given the in-duct sound power level spectrum resulting from one velocity condition for a particular duct component, equation (3) can be used to obtain spectra for noise generated by the same component for different flow velocities.

For the case of a geometrically similar duct element it can be seen from equation (3) that at a given velocity and for a constant value of Strouhal Number the acoustic power generated by the two elements is simply proportional to their cross sectional areas. In this case the change in cross sectional area will result in a change in the characteristic dimension, which is in proportion to the square root of the change in cross sectional area. Thus in order to maintain the value of Strouhal Number there must again be a change in frequency in inverse proportion to the square root of the change in cross sectional area.

A change in cross sectional area will also result in a change in the cut on frequency in inverse proportion to the square root of the change in cross sectional area.

It should, therefore, be possible using of the procedures described above, to use the spectral data corresponding to a particular size of component and one airflow velocity to predict the sound power level spectrum for a different sized component for a range of air velocities.

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The resulting predictive equations are as follows:

For  $f_m < f_{po}$

$$W(f_p) = W(f_m) + 40 \log \left( \frac{U_p}{U_m} \right) + 10 \log \left( \frac{A_m}{A_p} \right) \quad (5)$$

where  $W(f_p)$  is the sound power level at frequency  $f_p$ , and

$$f_p = f_m \left( \frac{U_p}{U_m} \right) \left( \frac{A_m}{A_p} \right)^{1/2} \quad (6)$$

where  $W(f_m)$  is the measured sound power level at frequency  $f_m$ ,  $U_m$  is the measured air velocity,  $U_p$  is the prediction air velocity,  $A_m$  is the original cross sectional area,  $A_p$  is the new cross sectional area, and

$$f_{op} = f_o \left( \frac{A_m}{A_p} \right)^{1/2} \quad (7)$$

where  $f_{op}$  is the new cut on frequency and  $f_o$  is the original cut on frequency.

### 3.2 Above the Cut-on Frequency

Equation (4) is slightly more complex than equation (3) but it can be applied in a similar manner. There is, however, an apparent complication arising from the following term within the equation:  $[1 + (3\pi c_o / 4\omega_c)(a+b)/A]$ . This term effectively bridges the transition between the  $v^4$  and the  $v^6$  velocity dependant regions. It can be shown [1] that for a square section duct this expression reduces to  $[1 + 3f_o / 4f_c]$  which is again a dimensionless quantity. At the cut-on frequency this term contributes less than 2.5dB to the predicted sound power level whilst at higher frequencies its contribution becomes negligible.

From an examination of equation (4) it can be seen that for a given duct element, the ratio of the sound power level generated at fixed Strouhal number for different air velocities is proportional to the sixth power of the air velocities. As the characteristic dimension  $d$  does not change then, in order to maintain a constant value of Strouhal number, if the air velocity changes there must be a corresponding change in frequency as described in Section 4.2.

For the case of a geometrically similar duct element it can be seen from equation (4) that at a given velocity and for a constant value of Strouhal Number the acoustic power generated by the two elements is again simply proportional to their cross sectional areas. However, in order to predict the sound power level spectra, it is necessary to take into account the fact that the change in cross sectional area will result in a change in the characteristic dimension. Thus in order to maintain the value of Strouhal Number there must again be a change in frequency as described in Section 4.2.

The resulting predictive equation is given by:

For  $f_m > f_{po}$

$$W(f_p) = W(f_m) + 60 \log \left( \frac{U_p}{U_m} \right) + 10 \log \left( \frac{A_m}{A_p} \right) + 10 \log \left[ \frac{\left( 1 + \frac{3f_{po}}{4f_m} \right)}{\left( 1 + \frac{3f_o}{4f_m} \right)} \right] \quad (8)$$

## 4. SOUND POWER LEVEL DATA

The predictive techniques described above were applied to three different sizes of circular ductwork with long radius bends. Data has been supplied by Atkins Noise and Vibration of Epsom, UK obtained as part of a comprehensive series of measurements undertaken several years ago. The sound power level spectra were determined from sound pressure levels measured into a reverberation chamber into which the duct containing the test element fed [5].

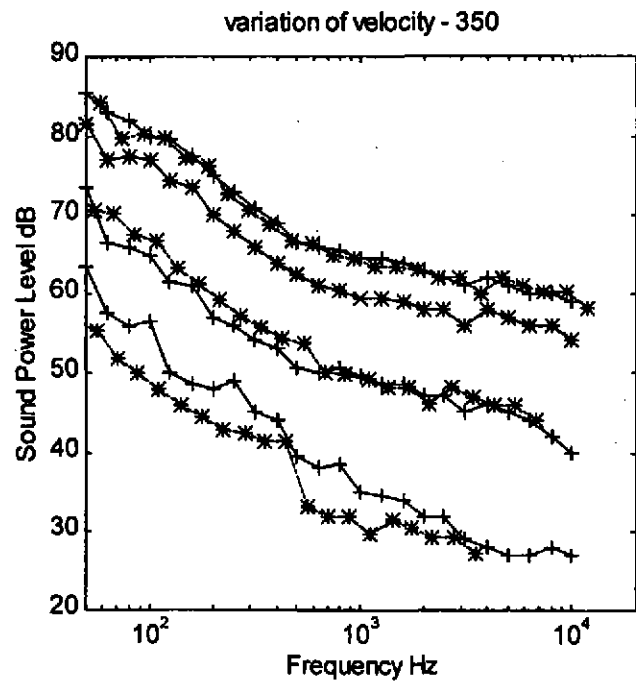
### 4.1 Component with Different Air Velocities

An example of the application of the above procedure can be seen in figures 1 and 2 which show in-duct spectra corresponding to a long radius bend in 350mm and 200mm diameter circular ductwork for a range of different flow velocities. The second spectrum has been employed to predict spectra for four other air velocities. These are shown with corresponding experimental data. It can be seen that the agreement between the predicted and experimental data is generally good.

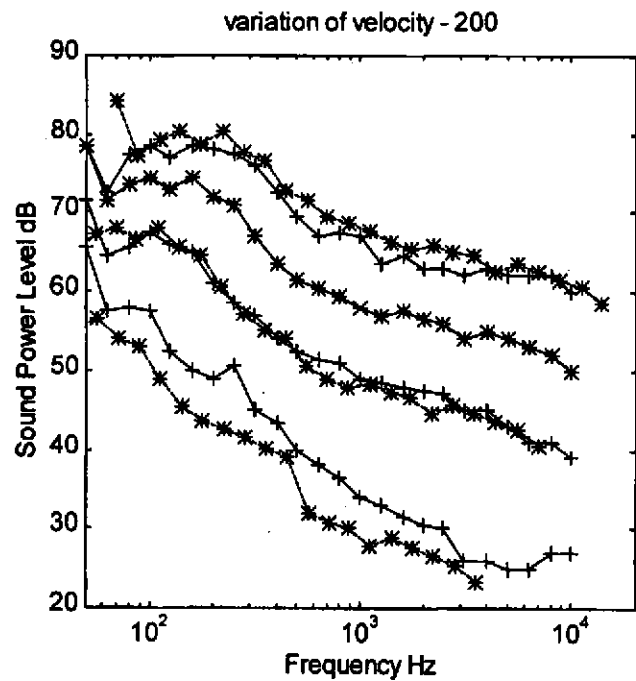
As the predictive curves have been produced using experimental data, they have the same degree of uncertainty as the experimental curves. Although no information is available regarding the actual scatter associated with the measured data, it is generally accepted that measurements of sound power level made using the reverberation chamber method involve uncertainties of  $\pm 2-3\text{dB}$  at the lowest and highest frequencies and  $\approx \pm 1\text{dB}$  at the mid frequencies [3, 4].

The agreement is least good for the spectra corresponding to the lowest air velocity. This may be due to the low levels of noise generated at this velocity which are comparable to the reported system background noise present during the measurement programme [5]. In addition, the experimental uncertainties associated with the measurement of low air speeds are greater than those associated with higher speeds and these will affect the noise predictions for low air velocities.

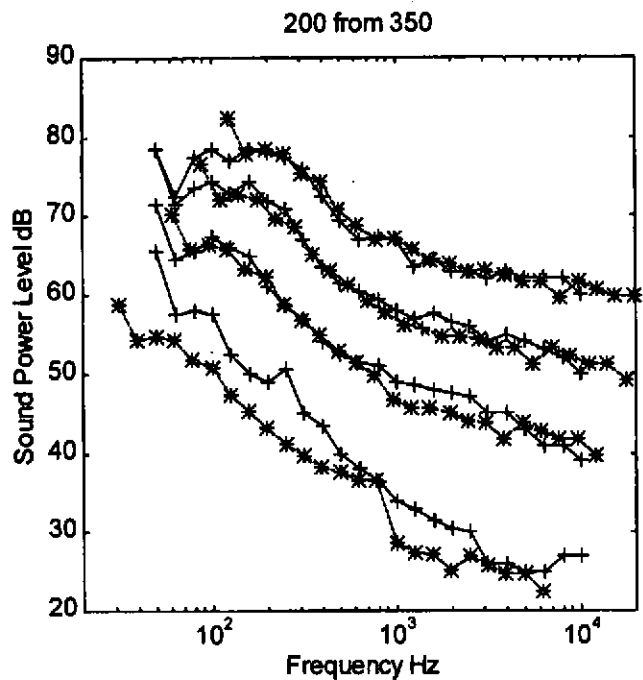
The data does indicate the potential limitation of the technique due to the limited bandwidth of the measured spectrum. This results in there being; (i) no data from which to extrapolate at the low frequency end of the spectrum when considering a velocity lower than that of the measured data, or (ii) from which to extrapolate at the high frequency end of the spectrum for velocities higher than that of the measured spectrum. Thus as one attempts to predict noise levels for velocities which differ by greater and greater amounts from that used as the basis of the prediction then the degree of uncertainty will increase.



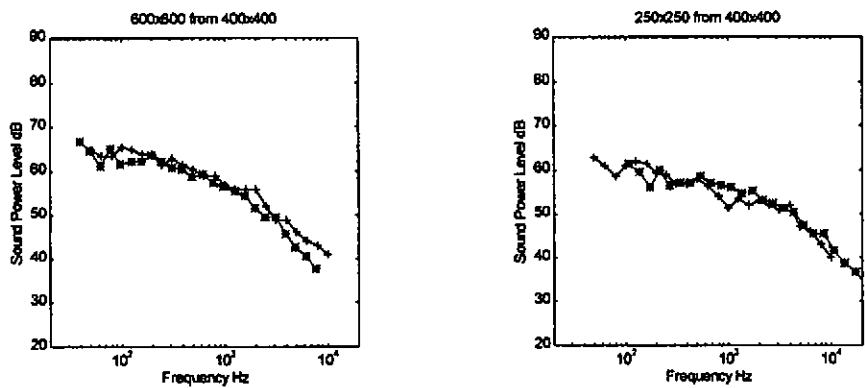
**Figure 1:** Comparison of Measured (+) and Predicted (\*) Flow Noise Spectra for Long Radius Bend in 350mm diameter Circular Duct for Different Air Velocities.



**Figure 2:** Comparison of Measured (+) and Predicted (\*) Flow Noise Spectra for Long Radius Bend in 200mm diameter Circular Duct for Different Air Velocities.



**Figure 3:** Comparison of Measured (+) and Predicted (\*) Flow Noise Spectra for Long Radius Bend in 200mm diameter Circular Duct from Data Measured on 350mm Diameter Duct.



**Figure 4:** Comparison of Measured (+) and Predicted (\*) Flow Noise Spectra for Mitred Bend in 600mm and 250mm Square Ducts from Data Measured on 400mm Square Duct.

### 4.2 Geometrically Similar Components with Different Air Velocities

Figure 3 shows the in-duct spectra corresponding to a long radius bend in 200mm diameter circular ductwork for a range of different flow velocities. The spectra displayed are predicted by the technique described above using experimental data for the 350 mm diameter duct. Also shown are the data obtained from experimental measurements.

The agreement between the predicted and measured data is generally good apart from at the lowest air velocities. As discussed above, the low velocity data is likely to be adversely affected by background noise and experimental uncertainties associated with the measurement of low air velocities. The data again indicates the potential limitation of the technique due to the restricted bandwidth of the measured spectrum. Figures 5 and 6 show a comparison of predicted and measured data for mitred bends in 250mm and 600mm square section ductwork. The predicted values have been calculated from measured data on 400mm square section ductwork. The agreement can again be seen to be good.

## 5. CONCLUSIONS

It has been shown that by consideration of similarity, the Nelson and Morfey equations can be employed as the basis of a prediction technique for the noise generation due to the interaction of airflow and duct elements. Although in principle data from a single measurement can be employed for a large range of air velocities and duct dimensions, the uncertainty associated with the predictions increases for situations which differ greatly from those used as the basis for the predictions.

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