EXACT AND APPROXIMATE METHODS FOR ACOUSTIC PROJECTORS.

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1. INTRODUCTION

There exists a wide variety in design and application of underwater sound projectors. It is usual to consider not only a single radiator's performance but also that of an array comprising two or more. This presents a considerable increase in effort. Maximum projector sensitivities and bandwidths are crucial for array acceptance and accurate prediction is demanded. A wide range of frequencies must be considered and in fine detail. This is particularly so for an array, as important interaction mechanisms may be significant over a narrow frequency band. The piezoelectric ceramic can be regarded as a component in an electric circuit and derived admittance loops required for performance assessment also need fine frequency resolution. Thus any method chosen to provide performance prediction must be efficient and fast as typically a thousand or more frequency steps may be considered for a given array design configuration. In this paper we consider various procedures for their suitability for these tasks based on the boundary element and finite element methods. We apply these methods to commonly used projectors; free flooded piezoelectric ceramic cylinders. No fully comprehensive predictive model exists for these devices and recourse to numerical means is necessary.

2. THEORY

A steady state vibrating elastic structure immersed in an infinite acoustic fluid is well described by a coupled form of the Surface Helmholtz Integral Equation (SHE). This approach is exact assuming that the fluid is inviscid and all motion infinitesimal. The form of the integral equation is such that the boundary element method can be used (via collocation or Galerkin's method) to discretise the exterior ("wet") surface of the structure and reduce it to a system of dense complex matrix equations for the unknown surface pressures and normal velocities. At each frequency the matrices must first be constructed, inverted and solved. Unfortunately, SHE fails at certain critical frequencies corresponding to eigenvalues of the internal Dirichlet problem. This is a deficiency of the integral representation rather than the physics and results in derived matrices that are ill-conditioned. This problem can be overcome in several ways such as in the CHIEF method or by the approach of Burton and Miller [1]. These improved methods resolve the non-uniqueness problem by somewhat different ways but achieve similar degrees of effectiveness at the cost of extra computation.

The fluid loading may be incorporated in an approximate manner. The Doubly Asymptotic Approximation (DAA) of Geers [2] considers the fluid integral equation as a series in the high and low frequency limits and interpolates between for intermediate frequencies. The equations describing the fluid may be derived in a more rigorous fashion from the Kirchoff retarded potential formalism [3]. Steady-state acoustic scattering or radiation may be tackled using a boundary element formulation based on the so-called DAA2c method. Here the acoustic term is based upon a curved-wave approximation. The surface is assumed to be comprised of locally spherical radiating regions and each area does not interact significantly with its neighbours. The curvature of the radiated wave associated with each region is included by using the local mean radius to first order. Consequently, the DAA method is tailored to deal with only broadly simple convex surfaces. Even for scattering by shapes of this form, poor results may occur in the deep shadow region (this is irrelevant for the radiation problems considered here). In the case of a cube say, the approximation may be poor near the edges. Complicated shaped projectors with perhaps locally concave surfaces can prove difficult to describe with the DAA method as adjacent areas may not correctly interact.

Regarding the nature of the various methods we see that the SHE is essentially global. Every part of the fluid structure boundary surface influences every other. This is the case for the low frequency part of the DAA series which assumes that the fluid is incompressible (i.e. the acoustic wavelength is very long when compared to the structure). However, for the high frequency term the curved wave approximation is of a local nature. The acoustic matrix derived is diagonal with a simple frequency dependency. This greatly enhances the potential speed of the DAA method. A further attraction of the DAA series is that it does not suffer from critical frequencies, results are reliable, albeit approximate.

Surrounding the structure with fluid acoustic elements is another method of incorporating fluid loading effects. The matrices derived for an enclosed fluid region are frequency independent but here each region of the fluid influences neighbouring regions (the matrices are banded). To account for a fluid of infinite extent correct radiating boundary conditions have to be imposed. This is a daunting task but success has been achieved using multipole expansions and employing frequency dependent "infinite" finite elements. All these purely finite element methods require relatively large fluid meshes, several wavelengths in extent.

We propose an alternative approach whereby a relatively small (much less than a wavelength) acoustic finite element fluid "bag" surrounds the structure which in turn is enveloped by, and coupled to, acoustic boundary elements. This is in contrast to a direct formulation whereby the actual "wet" surface is discretised. The fluid finite elements essentially fill in any cavities that are present and can reduce the number of boundary elements required at the expense of extra acoustic finite element degrees of freedom. Near field effects, particularly at discontinuous edges (i.e. corners) may be better described and so this method is attractive for array problems. In principle this hybrid procedure can be implemented whatever boundary element formulation is adopted, SHE, DAA2c or whatever! Provided the field quantities (in our case pressures) are consistent.

However as it is likely to increase the volume enclosed by the finite element domain the problem of critical frequencies in the SHE method may become acute. Resort to the CHIEF or other related schemes must then be made, making this idea less attractive. An obvious feature of this approach is its potential for improving the DAA2c method: here the lack of mutual influence between remote structural surfaces is redressed and a simple exterior surface can be ensured. We assess this particular hybrid method, the hybrid DAA2c, for its suitability as a fast and accurate prediction tool for projector and array performance.

3. CYLINDRICAL PROJECTORS

Piezoelectric ceramic hollow cylinder transducers are used extensively in underwater acoustic applications. They have a number of advantages owing to their simple construction and geometry. The fact that they can operate in free flood conditions allows for great depth. The simplicity of the devices enables an axial line array to be easily constructed producing a cylindrically symmetric beam pattern. Depending on the axial length to mean radius ratio the cylinder may be classed as a ring or a tube. Unlike most bodies tubes or rings have toroidal topology and as such are not simply convex. There is a region of fluid within the confines of these structures which is open to the infinite surrounding medium. Such a system may have well defined acoustic as well as structural modes. In the case of a tube the acoustic fluid resonates as if in a nearly rigid confined space in a manner resembling an organ pipe. For a ring the behaviour of the fluid is not so easily pictured. Most usually the walls of the structure are thin compared to the acoustic wavelength while the length of the tube may be several wavelengths long. This is therefore a severe test of any numerical method based upon the boundary element method. This is particularly so for the DAA approach which ignores some long range coupling.

We compare with the work of Rogers [4] who considered two cylinders, a tube and a ring composed of radially polarized PZT8 ceramic. Rogers presents both calculations and experimental data for the transmitting current response over the range 10 - 100 kHz, however we consider only his experimental work here. The measurements were performed in a drum of "Capella" oil. As further examples we consider, in turn, a single one, and an array of two, radially polarized PZT4 ceramic DRA test rings in water. The rings are spaced to be roughly a third of a wavelength apart at the first peak frequency and strong mutual interaction effects are to be expected.

The commercially available programme PAFEC [5] has a comprehensive structural and acoustic capability and is employed here in modified form. Boundary elements based on both the SHE and the DAA2c are available as are acoustic and piezoelectric finite elements. Only axisymmetric models are considered, the projectors all being radially poled. Each cylinder is described by an assembly of piezoelectric finite elements. The exterior infinite acoustic fluid is represented by a boundary element surface mesh. More elaborate meshes are adopted for the hybrid approach. Here the interior fluid and the region in the vicinity of the opening are described explicitly using acoustic finite elements. The exterior boundary elements are coupled to the interior acoustic finite elements via axisymmetric elastic shell elements with suitable choice of material properties producing an almost transparent coupling. Obviously a higher number of acoustic degrees of freedom are present in these models. Nevertheless the overall number of boundary element degrees of freedom are not greatly exceeded and in some instances reduced. The resulting matrices are of similar size.

4. RESULTS AND DISCUSSION

Projector sensitivities (in dB. re 1μPa. per Ampere @ 1m.) calculated from the direct SHE formulation exhibit good accord (*figure.1*) with the work of Rogers [4]. In the case of the ring (Roger's transducer "A") the results are excellent: the projector sensitivities agree with Rogers' results to well within 2 dB. For the tube (transducer "B") this work agrees with Rogers' results to within 2 - 3 dB. overall away from any resonances. The predicted position of these resonances agree except at the very lowest frequency range. Here the so-called cavity resonances are important. The predicted first resonance frequency exceeds that of Rogers' by ~10%. However it must be stated that Rogers' experimental measurements at the lowest frequencies (< 15 kHz.) may suffer from systematic errors. For the single DRA ring admittance loop (*figure.2*) again the results using the SHE are excellent. Not only are the magnitudes of the admittances correct to 10% but also the frequencies corresponding to the conductance and susceptance maxima and minima are in accordance with measurement. Projector sensitivities (in dB. re 1μPa. per Volt @ 1m.) along the axial direction of the two ring array (*figure.3*) exhibit interaction at the peak resonance (~7.3 kHz.). Although not shown (*figure.3*) good accord with the limited set of measurements is seen.

As expected with the simple direct application of the SHE formulation no problematic critical frequencies were seen for these models over the chosen frequency ranges. Even so, in providing predictions with fine frequency resolution the SHE approach can be exhaustive. Sonar engineers require quick reliable results when considering a variety of designs. Unfortunately, the very fast DAA2c method produces only qualitatively correct results (*see figures*). The projector sensitivities for a ring derived using the direct DAA2c are similar to the SHE calculations in terms of broadband behaviour. However the radiation damping is over predicted at resonance due to the lack of local interaction between neighbouring vibrating surfaces. The acoustic near field is not well represented and the admittance loops are too large.

For the case of a tube the simple DAA2c is extremely poor. The behaviour of the fluid in the vicinity of the device is complicated, with a subtle interplay between structural motion and cavity modes. The neglect within the DAA2c of mutual coupling between parts of the surface is very detrimental as to its accuracy here. When considering an array of simple rings the DAA2c results are fair for the radial direction but poor in the axial direction, again indicating that interaction effects using the direct DAA2c method are not well described.

Results derived using the hybrid DAA2c approach with an acoustic fluid bag show similar level of agreement with experiment to that found with the direct SHE method. Calculations of pressure amplitude based upon either a boundary element with the pure SHE formulation or the hybrid DAA2c are almost indistinguishable. This is exactly the case when comparing the pure SHE with a hybrid approach using the SHE formulation i.e. the hybrid SHE. For this reason the hybrid SHE is not plotted in the figures. It is worth noting that the hybrid SHE suffered from some ill-conditioning at the highest frequencies and extra acoustic degrees of freedom needed to be incorporated in the manner of the CHIEF method.

The DAA2c method's accuracy has been improved considerably even for the long tube case by adopting a hybrid finite element and boundary element scheme at the cost of increasing computation. It appears that the presence of acoustic finite elements used to describe the near field surrounding the radiating projector's structure account for the correct local interaction, providing accurate phase relations for the outgoing waves. These inferences are likely to be restricted to radiation problems only. It remains to be seen whether scattering problems can be addressed by the hybrid DAA2c with a similar degree of improvement in accuracy.

5. COMPUTATIONAL CONSIDERATIONS

Numerical solution of boundary element problems require the construction and solution of matrix equations. Post-processing is necessary to calculate results not readily available at solution. In comparison with the SHE, the DAA2c method is faster but its speed benefits depend on a number of crucial factors. If the total matrix dimension (i.e. the job size) is small then the matrix construction time dominates. This will be frequency dependent if the numerical quadrature is adaptive (i.e. for a given accuracy more Gauss points will be required as frequency increases). Unlike the SHE, the DAA2c constructs its matrices effectively only once, save for a simple scalar multiplication and matrix addition. There is no sensitivity of the matrix construction to frequency. When many frequencies are required the DAA2c is significantly faster than the SHE by a factor of ~100 (see tables 1 and 2) as when solely predicting admittance loops. Here we are considering only the electric field within the structure which is determined by the surface displacements and hence derived directly at the solution phase.

This computational advantage is reduced if many exterior field points are to be derived as is the case for projector sensitivities in several directions or in predicting near field pressures contours. These require post-processing of the surface solution. Its computational effort is common to all methods and is determined solely by the number of boundary element degrees of freedom. At best, any speed increase is of the order of the ratio of the time required to evaluate the exterior field point pressures over the matrix construction time at a single frequency. In the case of the pure DAA2c, the post-processing time is relatively large and is a considerable overhead.

The hybrid method employing the DAA2c has similar features to the pure DAA2c except here the solution time is more significant owing to the increase in acoustic degrees of freedom. Typically the hybrid method using DAA2c is roughly 4 times faster than that using SHE (the hybrid SHE) and is up to 10 times faster than the direct SHE method. The hybrid SHE is faster than the pure SHE here, since for these topologies there is considerable reduction in the number of boundary elements when incorporating a fluid finite element "bag". These observations are restricted to small jobs with matrix sizes not exceeding a thousand. As more efficient matrix inverters are adopted solution times will become smaller and so larger jobs will benefit from this hybrid approach.

6. CONCLUSIONS

Acoustic radiation from cylindrical underwater projectors has been presented. Rings and tubes with free flooding regions or cavities were considered. The boundary element formulation based on the direct SHE produces good results but is intensive and may be unreliable near certain critical frequencies. The DAA2c is very fast but is not accurate, especially for long tubes and arrays. The hybrid method is attractive for non-convex projectors and in combination with the DAA2c very efficient, faster than the simple SHE, and should not fail at any frequency. This is provided the discretisation is sufficiently fine for the frequency range of interest.

Consideration of large arrays fitted with sonar domes (free flooding acoustically transparent windows) will benefit from adopting the efficient hybrid DAA2c approach. Here the surrounding structure must be explicitly described along with the internal fluid region [6]. Work on large three dimensional problems is in progress to determine the utility of this procedure. As a final throw away comment, the relatively simple nature of the total system matrices derived from the hybrid DAA2c may lend themselves to approximate inversion methods.

7. ACKNOWLEDGEMENTS

The author is grateful to Dr. P. C. Macey of PAFEC Ltd. for writing the software and providing invaluable support.

8. REFERENCES

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Table1: Ring Transducer - Rogers' A.

Process	*SHE*	*DAA2c*	*Hybrid SHE*	*Hybrid DAA2c*
Construction & Solution.‡	47.9 - 63.4	0.96 - 0.98	22.2 -22.3	7.23 - 7.27
Post Processing.‡	4.46 - 7.35	4.46 - 7.35	3.21 - 3.25	3.21 - 3.25
Admittance Loop. †	1	0.015 - 0.02	0.35 - 0.47	0.11 - 0.12
Source. Level. †	1	0.12 - 0.17	0.41 - 0.61	0.15 - 0.21

Table2: Tube Transducer - Rogers' B.

Process	*SHE*	*DAA2c*	*Hybrid SHE*	*Hybrid DAA2c*
Construction & Solution.‡	42.9 - 103	0.23 - 0.24	22.4 -45.2	5.91 - 6.62
Post Processing.‡	3.21 - 23.1	3.01 - 21.7	3.45 - 7.16	3.61 - 7.41
Admittance Loop. †	1	0.0006 - 0.002	0.43 - 0.53	0.064 - 0.14
Source. Level. †	1	0.072 - 0.174	0.39 - 0.57	0.11 - 0.21

‡ notional CPU times for lowest and highest frequencies considered.
† normalised to a single frequency step using the pure SHE method.

Figure 1. Rogers' transducers A & B.

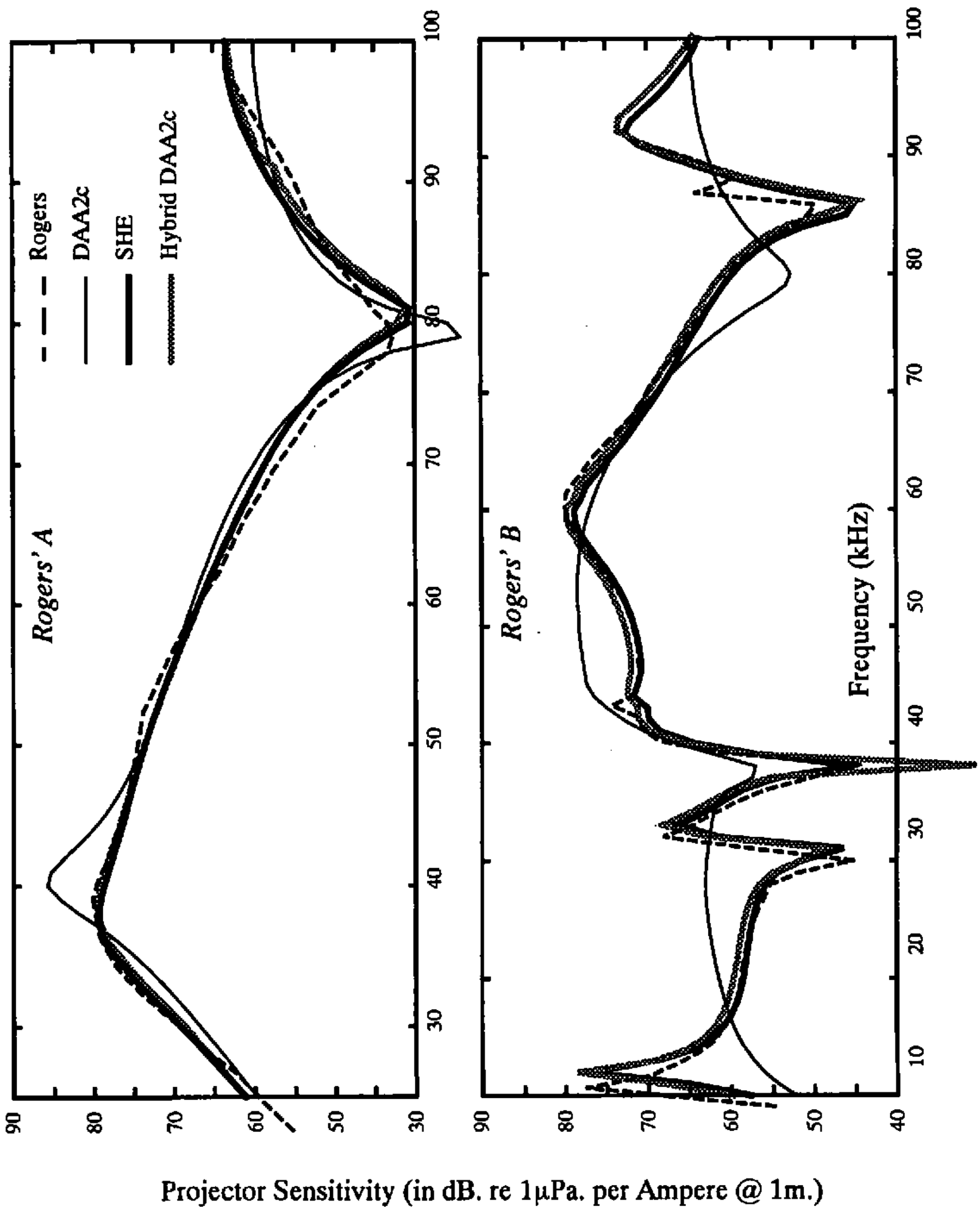

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Figure 2. Admittance Loop for DRA Ring.

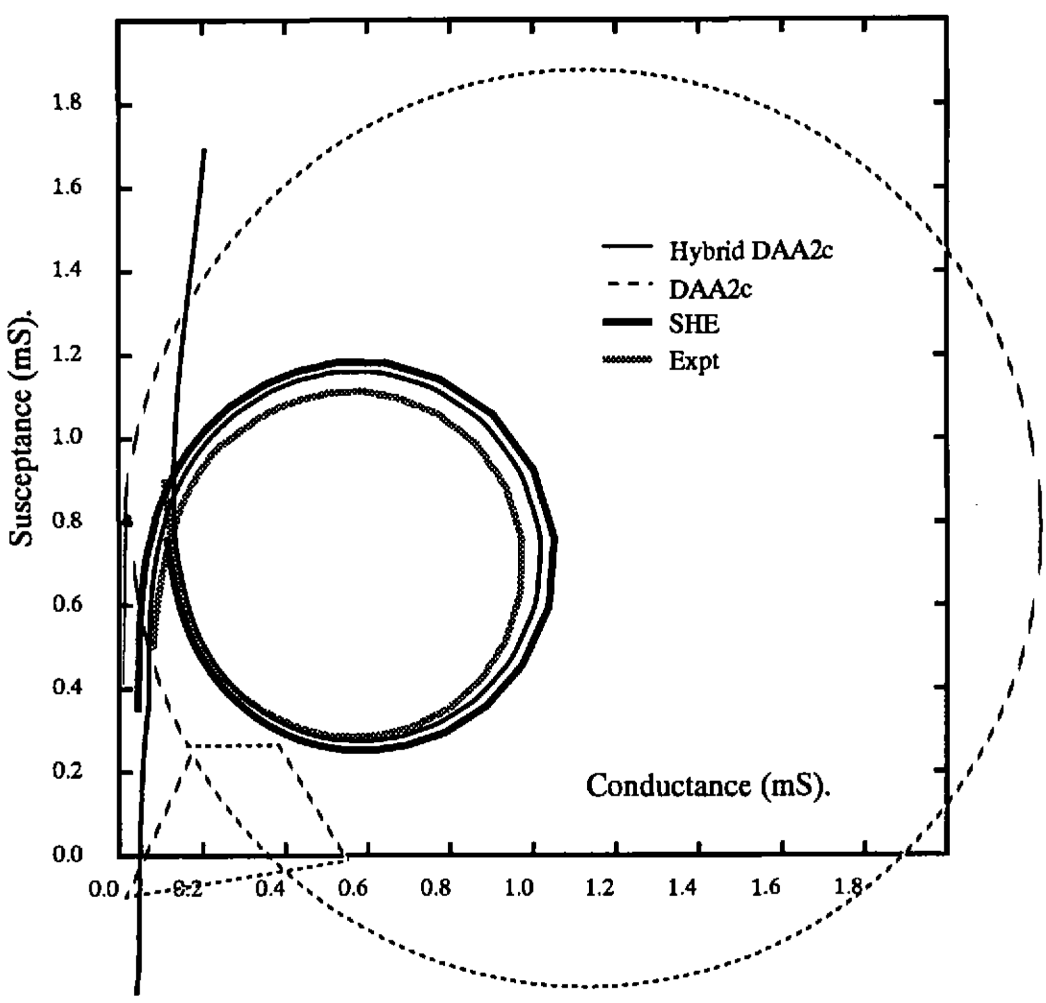

Figure 3. Array of Two DRA Rings.