NUMERICAL METHODS FOR ACTIVE SONAR PERFORMANCE

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1. INTRODUCTION

There is a continuing need for improved performance in the application of sonar systems to underwater acoustics. Both active and passive sonar arrays are required to be ever more sophisticated with increased capacity for discrimination. This makes great demands on designers. With the pressure on reducing expensive prototype costs the need for reliable modelling of system performance is manifest. When considering complicated design issues with many stringent and potentially conflicting constraints, resorting to several initial trials can be prohibitive. These questions must be resolved using accurate predictive tools.

It is vital that the selected method of performance prediction is commensurate with the questions being asked of it. The first question to be addressed should be what specifically is of interest and is an accurate answer required? For an expensive novel design with an element of technical risk a detailed understanding as to its acoustic behaviour will be needed. However it is worth bearing in mind that the simplest solution with acceptable accuracy should always be sought in any modelling problem. This paper is generally restricted to finite element and boundary element methods which have shown great utility in calculating the dynamical response of general elastic structures and is a condensed version of [1] focusing on active sonar only. The reader is referred to this review for a comprehensive list of references. We will focus our attention primarily on how to predict quantities critical for the acceptance of a sonar array, namely, projector or receiver sensitivities and directivity beam patterns. This requires a fully coupled method yielding a relation between the electrical and acoustic quantities. Of course other system parameters are important (e.g. projector depth dependence) and the methods outlined here may be of use in their prediction. In section 2 of this review paper various numerical methods suitable for predicting sonar array response are discussed. Section 3 considers specific methods appropriate for active arrays. Further aspects of sonar array design are discussed in section 4. Brief conclusions (section 5) end this paper.

2. THEORY AND METHODS

For steady infinitesimal linear motion the partial differential equations governing the dynamic behaviour of underwater acoustic projectors and receivers are well known [2]. There are many methods for their numerical solution over the region of interest or domain. Partial wave analysis and related methods (e.g. T matrix approaches) solve the equations using separation of variables. The Helmholtz equation with its associated Sommerfeld radiation condition is

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tractable by these means. These are exact procedures provided accurate representation of the eigenfunctions specific to the particular geometry under consideration is assured. Sufficient terms must be included to achieve a converged solution. Unfortunately, coupled elasto-acoustic problems can only be addressed for simple structures as separation of variables is restricted to specific geometries. Methods exist for overcoming these difficulties at the cost of adopting more complicated numerical procedures involving non-orthogonal functions. For acoustic radiation from active arrays these methods have been used to a limited extent. Usually a Neuman-type boundary condition (specified surface normal velocity) is imposed, structural coupling being ignored [3]. This may be sufficient for simple directivity predictions.

A great deal of useful transducer and array design work is based on simple lumped parameter methods [4]. These do not attempt to solve any general field or potential problem but attempt to map the dynamics to a relatively simple equivalent circuit. To describe the behaviour of more complicated projectors over a wide operational frequency band requires an elaborate system of circuits which requires careful consideration in its fabrication. More refined approaches exist. Those based on transmission line theory have proved to be accurate and straightforward in their implementation [5]. When considering arrays these methods can become unwieldy but if the projectors have simple behaviour adopting a lumped parameter model can be fruitful. Lumped parameter and related methods are specific to known designs and their application to novel transducers is not straightforward. Nevertheless their inherent capacity to aid physical understanding will ensure that these methods will continue to be popular.

Finite element methods have been applied successfully to a number of transducer design problems [6]. Indeed the development of piezoelectric finite elements is as a result of demand by sonar designers. Using these elements a fully coupled approach is possible. Admittance loops can be predicted and more elaborate methods of assessment can be performed. Reliable material data is necessary to get accurate results when using piezoelectric elements. Depending upon the age of the ceramic, it may be appropriate to use degraded values. The straightforward finite element method is restricted to finite regions in space only. This method gives rise to positive definite matrices that are frequency independent allowing for modal analysis. The matrices are banded due to the local nature of the method; only neighbouring elements interact. These properties allow fast inversion, hence quick solution times.

To account for a surrounding fluid of almost infinite extent requires the correct acoustic radiating boundary condition to be satisfied. Simply extending the mesh ever further outward is incorrect as well as being computationally expensive. Work has been done to develop so-called "infinite finite elements" with a radially decaying frequency dependent function [7]. These are attached externally to the acoustic finite element mesh and simulate the correct frequency dependent boundary condition at infinity. Another method is to impose external damping elements around the fluid mesh with a distribution dependant on important terms of a multipole expansion [8]. This method is very efficient, particularly in predicting near-field

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pressures but requires extra computation to determine the boundary pressures necessary to calculate any far-field results. Establishing the correct Sommerfeld radiation condition at the boundary by employing spherical harmonics is an equivalent procedure relating the pressures and normal velocities at the mesh boundary. The size and shape of the acoustic fluid mesh is critical to these methods. Generally a sphere or spheroid is constructed. The radius of the sphere usually corresponds to the edge of the nearfield $\sim D/\lambda^2$. Hence for large aperture (D) linear arrays and high frequencies the acoustic mesh rapidly becomes excessive. Recently, work has been forthcoming in the form of wave envelope elements [9]. These are a refinement on the infinite element concept which suffers from integrating the oscillatory decaying shape function out to infinity. Wave envelope elements avoid this at some extra cost in slight computational complexity and in non-symmetric matrices. In their basic form these purely finite element can give rise to results that are origin dependent. This is not a difficulty for radiation problems. Much development work has been done based on the above and useful codes are available.

The boundary element method converts the associated integral equation into a set of algebraic equations relating points to others on the domain surface only. Thus a 3D problem is reduced to an equivalent 2D one. The method approximates the surface by a contiguous assembly of boundary elements or patches. The integral equation is then evaluated for each patch determining its influence on itself as well as on all the others. The first of these gives rise to a singular integral. After a suitable choice of shape functions a system of matrix equations arises. The form of these matrices differs from those obtained from the finite element method. They are dense, complex and frequency dependent and do not have the form to lend themselves to an efficient solution method. The procedure solves for unknown quantities at the surface only. All other results off this surface require post-processing of the solution via a surface integration.

The integral equation describing the surrounding acoustic fluid is the Helmholtz equation. Here the Green's function implicitly describes the influence of the surrounding infinite fluid. Unfortunately, the external Helmholtz integral equation fails at certain critical frequencies. These frequencies are the eigenvalues of the internal Dirichlet problem and their number increases as the frequency of interest is raised. This is a deficiency of the integral representation rather than the physics resulting in a lack of a unique solution. This gives rise to ill-conditioned matrices. This uniqueness problem can be overcome in several ways. The oldest of these methods is the CHIEF method of Shenck [10] which introduces extra interior points and yields an over-determined system of matrix equations to be solved. The selection of the interior points is somewhat arbitrary and care must be taken. Improved CHIEF methods have been developed which try to ensure that the choice of interior points is optimal. More elegant methods exist. The method of Burton and Miller [11] has received much attention. They proposed solving a linear combination, with arbitrary complex coefficients, of the acoustic integral equation and its normal derivative form. This involves evaluating integrals with a hyper-singularity. Various ways for doing this have been developed. The results should not be dependent on the choice of

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the coefficients required for the linear combination. However the user must be aware of any possible sensitivity of his results to changes in these values. Some recommendations for parameter choice are available [12]. The boundary element method is applicable to uncoupled external acoustics, both radiation and scattering. It has been applied to structural dynamic problems and coupled elasto-acoustic formulations based purely on boundary elements have been proposed. However a more attractive scheme is to adopt a finite element description of the structure and a boundary element formulation for the surrounding infinite acoustic fluid.

These procedures (both boundary element and finite element) are exact, within the terms of the numerical implementation. They are low to moderately high frequency methods. It is important to realize that they require considerable computational effort which increases markedly as the wavelength becomes much shorter when compared to a typical surface length scale, L. An important factor is the degree of fineness of the surface mesh. The minimum number of boundary element degrees of freedom along a length should be kL collocation points. This is applicable to both boundary element and finite element meshes, where k is the acoustic wavenumber. A similar criterion exists for the minimum number of terms in a partial wave expansion in elementary quantum scattering. The size of the resulting matrices from a coupled boundary element and finite element approach is a crucial consideration when solving a practical problem numerically. The time for their construction is of the order of the number of surface degrees of freedom squared and their inversion time during solution is proportional to that number cubed. For moderate jobs the matrix construction time is dominant. This must be done at each frequency considered. Interpolation of the frequency dependent matrices should be beneficial when considering problems with wide bandwidths and few resonances.

Provision for direct coupling between acoustic boundary elements and acoustic finite elements can reduce the critical surface area by filling cavities and smoothing out irregularities that may be present in the actual surface of the structure. Hardie [13] has proposed a combined finite element and boundary element method exploiting an approximate formulation of the Helmholtz integral equation, the doubly asymptotic approximation (DAA2c). The idea is to use the DAA2c boundary element as a general boundary condition for the finite element mesh akin to the pure finite element approaches. Here no specific shape of finite element mesh is needed, except that it be convex. The requirement to describe the whole of the array's nearfield is also relaxed. Only a small acoustic finite element mesh is needed. For an active array of test rings no distinction is seen with an exact treatment. Further advantages over conventional boundary element approaches are that the resulting matrices have a simple frequency dependence requiring to be constructed only once and that no critical frequencies exist. Thus a wide band analysis can be performed very quickly. This is of particular interest when considering fine frequency steps as in predicting admittance loops. This method needs further investigation and possible refinement to put it on a rigorous foundation.

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3. ACTIVE ARRAYS

Active arrays radiate acoustic energy and the quality of the sound field is characterised by its directivity and source level. Their operational bandwidth is usually limited. Generally, for low frequency sonars, only a few acoustic projectors are involved as individual sources are expensive and of appreciable size. The small number of structures and low relative frequency of operation (typically, $kL < 5$) makes the modelling of complete active arrays numerically feasible. Whether there are strong interaction effects present is of fundamental importance to the function of an active system. This is increasingly an issue of importance as greater emphasis is made towards low frequency and therefore long range systems. Sonar arrays are constrained to be attached to or be deployed from the platform vessel. The arrays are made to be as compact as possible. Consequently the projector separation within the array may be significantly less than a wavelength at the lowest part of the operating frequency band.

Piston stack Tonpilz projectors are commonly used acoustic sources. Their simple operation enables successful modelling using a variety of techniques. Much work has been done using finite element techniques to describe the behaviour of Tonpilz transducers. The compliance of the stack dominates the resonant character of a single device and care must be made to include the effects of the joints between the active ceramic pieces. When deployed in an array, these devices normally require a baffle to perform efficiently. This increases the computational cost. If the baffle is rigid, planar and of large extent this can be reduced by using a combined finite element and boundary element approach with appropriate symmetry conditions. Good agreement with an exact result is seen for a single piston in an infinite rigid plane. The structural part of the problem can be reduced further by employing a mixed method adopting a transmission line approximation for the Tonpilz stack. Audoly [14] used a coupled boundary element approach to quantify the interaction between adjacent Tonpilz transducers in a planar rigid baffle of finite size. A full finite element treatment of the transducers allowed for consideration of the piston head velocities when a constant voltage is applied to the array. The nearfield is strongly affected by mutual interaction for closely packed arrays with spacing $\sim \lambda/4$, but the directivity of the beam appears to be less sensitive. Only by adopting a fully coupled method can these issues be addressed adequately.

A novel multihead array of Tonpilz projectors arranged in a cylindrical fashion has been studied by Bernard et al. [15]. The array is self baffling with the projectors in free flood. Different configurations based on different spacings and with or without air-filled compliant tubes. A purely finite element code was used to predict the array's performance when amplitude or phase shading was imposed. The resulting mesh is very complicated even when exploiting symmetry to the full. A large radius part sphere was constructed of acoustic finite elements. Calculations show good agreement with measurement with recommendations for maximising the source level.

Flextensional projectors are popular devices for compact active arrays. Many successful types of flextensional exist such as the barrel-stave designs and the more common class IV

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configurations. The operation of flextensionals is based on the flexing of a thin shell of simple form. Adopting a finite element approach is therefore attractive. Indeed most theoretical work on flextensional projectors is based on this technique. Arrays of these projectors have been modelled. The compact nature of the projectors makes them behave as almost simple sources, at least around the first resonance. For certain array design questions treating the devices as acoustic monopoles may be sufficient. However for detailed analyses (e.g. quantifying interaction effects) recourse to fully coupled numerical methods must be made. Rynne and Gillete [16] consider an array of flextensionals for an application requiring low power but wide bandwidth. The relatively high frequencies of interest (kL ~ 10) demand that the directional nature of the higher acoustic modes be accounted for. Including interaction implicitly in the method gave rise to a significant computational task which was reduced by adopting a simplified 2D model.

Free flooding rings are attractive acoustic sources for high power active systems. Simple theoretical descriptions of their dynamical behaviour are available, at least with regard to classifying the various acoustic resonances. Sherman and Parke [17] used a partial wave approach with Neuman boundary conditions to derive directivity beam patterns for line arrays. Despite these successes it is necessary to resort to numerical means to predict completely the performance of even a single ring. Gallaher [18] predicted the acoustic field radiating from a line array of radially poled piezoelectric rings using a combined boundary element and finite element method. Comparing with the series expansion of Sherman and Parke for the directivities revealed great disparities. Experiment showed somewhat better accord with the calculations of Sherman and Parke for compact arrays with less than $\lambda/2$ spacing. This was surprising as Sherman and Parke ignored any mutual interaction between the rings. The actual rings were waterproofed with a polymer coating and this appears to dampen the motion and hence reduce mutual interaction. Accounting for structural damping by a simple term in the combined boundary element and finite element model resulted in excellent accord with measurement both for source levels and beam patterns. These findings indicate that, unlike a baffled Tonpilz array, the directivity of a compact free flooding ring line array can be strongly affected by mutual interaction effects.

Investigating the acoustic behaviour of segmented rings continues. Their large size requires an elaborate structural design which gives rise to potentially computationally intense models. Bonin et al. [19] calculated the projector sensitivity of a single large segmented free flooding ring. They compared numerical schemes based on a pure finite element method and the combined boundary element finite element method. The pure finite element model is axisymmetric using generalised piezoelectric elements and effective properties while the mesh is 3D exploiting full symmetry. Little difference between the sets of results is apparent over the main frequency band. Similar levels of agreement are seen in comparing different methods on other segmented ring designs.

In addition to steady state, constant frequency methods as described above, mention must be made of time-dependent methods. A recent study [20] of ring transducers has demonstrated the utility of this approach. These methods based on the Lagrangian frame of reference with explicit time integration (called hydrocodes) have been previously applied to shock and impact problems and were aimed at transient non-linear dynamics. They are very efficient requiring no matrix inversion. Hence memory requirements are not excessive. However there is a possibility of instability in the simplest schemes employing a finite element description of the Lagrangian space. Care must be taken to ensure that the explicit time integration is stable and that no "hour glass modes" arise, due to reduced integration for the element matrices which result in spurious oscillations. The size of the mesh is determined by the duration of the transient response considered (via the sound speed). There is the possibility that, in combination with infinite elements that this condition could be relaxed. A fast fourier transform is required to calculate the steady-state source level at a given frequency.

4. FURTHER ASPECTS

Sonar arrays can be fitted with acoustically transparent structures or domes. Their function is to isolate the array from the flow past the vessel. In passive sonars flow noise can be a severe limit to the array's ability to detect acoustic sources and the high wavenumber components of the flow noise are attenuated by the standoff distance provided by the dome. For high frequencies the refraction due to the solid dome can disrupt array directivity response. A sonar dome cannot be made fully acoustically transparent. A coupled finite element and boundary element calculation was used to determine the sound field at the hydrophone sites [21]. It is important to note that a thin shell finite element description of the dome structure is not generally sufficient. To include all possible structure-borne waves a full elastic description is necessary. When considering interaction effects in a domed array wherein elastic effects couple the projectors a more elaborate numerical scheme has been proposed [22]. Good results are achieved with a combined 2D and 3D finite element and boundary element approach.

Potentially, optimisation procedures can be used to aid array design. A number of commercial codes contain some capability to optimise designs. The optimisation process is described in terms of design variables, which are the input parameters (e.g. Shell thickness, elastic modulus etc.) subject to change, and response variables which are chosen to assess the design. Resonance frequency and overall weight are possible responses. The whole procedure attempts to maximise the multi-dimensional evaluation function having formulated an initial design. Obviously the closer the initial design to an agreed optimum the better. These facilities enable the transducer engineer to automate his design procedures. More specifically, proposals for optimising acoustic performance of active arrays have been investigated [23]. Macey [24] has proposed a procedure using a combined boundary element and finite element method to design a volumetric active array with the maximum possible source level over a selected frequency band. Here the variables are the relative distances between projectors. These methods are computationally intense and require a careful consideration of the sonar engineer to see whether the design issues can be resolved by simpler means and good judgment.

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The introduction of new active materials promises greater performance for future sonar systems. The currently commercially available ceramics such as PZT4 are ferroelectric with small active domains rather than truly piezoelectric as are quartz or barium titanate crystals. The desire for higher power densities for acoustic projectors has heightened interest in magnetostrictive and electrostrictive materials. The inherently non-linear response of these materials poses no problem for the finite element technique provided the amplitude of the motion does not exceed the applied bias field [25]. A feature of rare-earth magnetostrictive devices are their poor material permeability. Hence in a closely packed active array stray magnetic fields can constitute severe interaction between neighbouring projectors. Finite element codes for predicting magnetic and electric fields are available and will allow for detailed scrutiny of possible magnetic interaction effects of a particular array design. For the case of electrostrictive devices, the active material employed can display very non-linear behaviour even when the amplitude is not excessive and it may be expedient to adopt a time dependent hydrocode approach to fully describe the acoustic response.

5. CONCLUSIONS

A brief review of numerical methods used for sonar array performance has been given. Specifically finite element and boundary element methods are considered. Active arrays generally have few projectors and not a large frequency bandwidth. Pure finite element or combined finite element and boundary element approaches have proved to be very useful in predicting acoustic performance.

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