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ABSTRACT

This paper discusses the use of a penalty function method (PFM) for reducing weight jitter, whilst maintaining low sidelobes, for use in adaptive array processing. It is shown that, in some straightforward situations, the method is equivalent to the commonly used method of diagonal loading. However, by means of simulation, we show that diagonal loading can adversely affect the SNIR performance of arrays disproportionately when weak mainbeam jammers are present. These effects are shown to be greatly reduced if the full flexibility of the PFM is used.

1. INTRODUCTION

Adaptive algorithms for array processing are being considered and used for many sonar, radar and communication applications. Using the method of linearly constrained adaptive beamforming it is possible to provide a particular gain in a given direction whilst automatically suppressing jamming or interference in directions away from the look direction. However, in real working systems other aspects of algorithm design must be considered. In this paper we are primarily concerned with the reduction of sidelobe jitter whilst maintaining low sidelobes.The relevance of these concepts to detection and measurement have been discussed elsewhere within a sonar context[1].

It is helpful at this point to introduce the notation to be used. We assume a narrowband beamformer. The vector

$$x = \{x_1(t), x_2(t), x_3(t) \ldots x_N(t)\}$$

represents a snapshot at time t at the N sensors. Snapshots are processed by a set of weights

$$w = \{w_1, w_2, w_3 \ldots w_N\}$$

so that the output of the beamformer is $w^H x(t)$ where H denotes the hermitian transpose. The beamformer output power is then $w^H M w$ where M is the $N \times N$ covariance matrix given as the expectation of xx^H. We assume that $x(t)$ is widesense stationary and that expectations can be approximated by time averages.

Usually it is required that the beamformer output power be minimized (equivalent to maximizing the SNR when a constraint is present). If we assume a single look direction gain constraint is applied of the form

$$c^H w = \mu \qquad (1)$$

where c is termed the constraint vector and μ is a scalar, it is well known[2] that the weight vector which maximises SNR whilst satisfying equation 1 is given by

$$w_{opt} = \frac{M^{-1} c \mu}{c^H M^{-1} c}. \qquad (2)$$

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The problems inherent in using this expression are best explained by reference to a specific example. Figure 1 shows the beampatterns formed by a series of 5 simulations carried out using equation 2. The system simulated is a 16 sensor linear array with a half wavelength sensor-sensor spacing. One jammer is present at 45°with a power of 30dB relative to the noise at each sensor. The noise is white, spatially uncorrelated and Gaussian. The covariance matrix is constructed using 32 snapshots of data The look direction gain constraint is 0dB at 0°.
The array is producing a null in the direction of the jammer and satisfying the look direction constraint. It is clear, however, that there is a large variation in the sidelobe patterns from one noise sample to the next, so-called *sidelobe jitter*. Also, the sidelobes are very often high. It is, of course, generally desirable to keep sidelobes low. These problems can be overcome by a simple modification of the algorithm and using a large number of snapshots of data. This, however, is not always possible in practical situations and an alternative approach is required. In the body of this paper we discuss a penalty function method (PFM) as a solution to the problem.

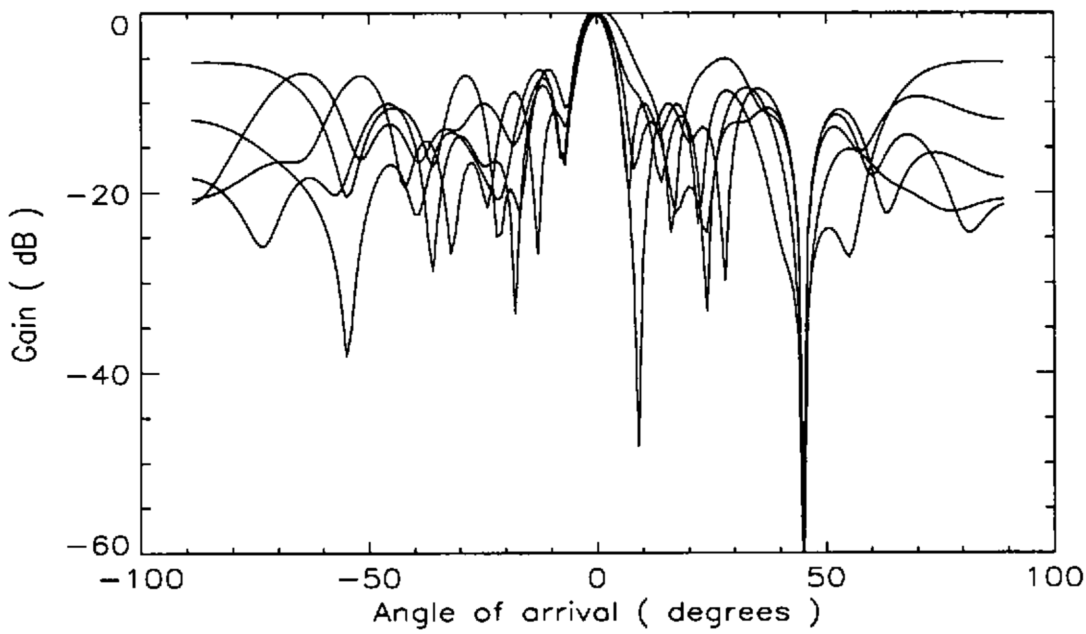

Figure 1: Beampatterns for 5 independent samples of data showing the effects of weight jitter. A jammer of 30dB is present at 45°.

2. THEORY

When the PFM is used the output SNIR of the array is maximised subject to a set of secondary conditions which need only be satisfied approximately. The degree to which these constraints are satisfied can be thought of as being determined by a set of user defined parameters (A soft constraint approach).

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We begin by considering a constraint which requires the beampattern obtained after adaptive beamforming to lie close to a desired or quiescent pattern. In the rest of this text, for concreteness, we consider a uniform linear half-wavelength spaced array. The quiescent pattern is assumed known and defined by a weight vector w_q. Similarly the adapted pattern can be thought of as being defined by an adapted weight vector w. The difference in the two beampatterns at an angle θ, then, can be sensibly defined as

$$e(\theta) = \left|s^H(\theta)w - s^H(\theta)w_q\right|^2 \qquad (3)$$

where $s(\theta)$ is the steering vector appropriate to the direction θ. Now, we are actually interested in the total error taken over the range of look directions. For generality we introduce a weighting function $k^2h(\theta)$ where k is a scalar weighting and $h(\theta)$ is an angularly dependent weighting function defined in the range $|\theta| \le \frac{\pi}{2}$. The weighting function $h(\theta)$ gives emphasis to parts of the beampattern at values of θ at which $h(\theta)$ is large. The total error E can, then, be written in the form

$$E = k^2(w - w_q)\left[\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} h(\theta)s(\theta)s^H(\theta)d\theta\right](w - w_q), \qquad (4)$$

The terms in square brackets define an $N \times N$ matrix which will be denoted by Z. We also wish to impose a look direction gain constraint as defined in equation 1. For convenience, this constraint will be introduced as part of the penalty function and given a weighting λ^2. The overall function to be minimized is therefore

$$w^HMw + k^2(w - w_q)^HZ(w - w_q) \quad + \lambda^2(c^Hw - \mu)^*(c^Hw - \mu). \qquad (5)$$

It is straightforward to show that the weight vector w which minimizes expression 5 is

$$w_{opt} = [M + k^2Z + \lambda cc^H]^{-1}[k^2Zw_q + \lambda^2\mu c]. \qquad (6)$$

For simplicity of notation a new matrix $A = M + k^2Z$ is defined and the matrix inversion lemma is used to expand the first square bracket. This gives

$$w_{opt} = \frac{\lambda^2\mu A^{-1}c}{1 + \lambda^2c^HA^{-1}c} + k^2A^{-1}Zw_q - \frac{k^2\lambda^2A^{-1}cc^HA^{-1}Zw_q}{1 + \lambda^2c^HA^{-1}c}. \qquad (7)$$

In the limit as the look direction gain constraint weighting becomes a hard constraint (corresponding to $\lambda \to \infty$)

$$w_{opt} \to \frac{\mu A^{-1}c}{c^HA^{-1}c} + k^2A^{-1}Zw_q - \frac{k^2A^{-1}cc^HA^{-1}Zw_q}{c^HA^{-1}c}. \qquad (8)$$

This result, for a hard look direction gain constraint, could have been obtained by including a penalty function to constrain the beampattern whilst imposing the look direction gain constraint ex-

actly by means of a Lagrange undetermined multiplier.
As can be seen, from equation 4, the form of the Z matrix is determined by the weighting function $h(\theta)$. For a half wavelength spaced array, assuming $h(\theta)$ is an even function of θ we can write

$$Z = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} h(\theta)\cos(\pi(i-j)\sin\theta)d\theta. \qquad (9)$$

It is instructive, at this point, to consider if a form of $h(\theta)$ exists which makes Z diagonal. It is straightforward to show that a weighting function $h(\theta) \propto \cos\theta$ gives the correct form.(This simple analytic form is a direct consequence of the array being used, for more complicated arrays the function would be more involved, however in general more weighting is always given to the mainlobe).
Using this diagonal form for Z leads to an obvious simplification of equations 6,7 and 8. It is easily shown that the form of equations are exactly what would be obtained for a system in which the SNR is minimized whilst a look direction gain constraint $c^H w - \mu = 0$ is imposed with a weighting λ and a set of constraints $w = w_q$ are imposed separately with a weighting k. In other words the constraints controlled by the parameter k force the adapted weights to lie close to the quiescent weights as opposed to forcing the adapted beampattern to lie close to the quiescent beampattern as discussed above. It is also important to notice that a diagonal form of Z means that the appended covariance matrix is now of the form $A = M + k^2 I$. This, of course, is the very transformation that would be carried out in diagonal loading. This gives geometrical insight into the effect of diagonally loading the covariance matrix whilst maintaining low sidelobes: it is equivalent to constraining the beampattern to lie close to the quiescent pattern more strongly around the mainlobe and tapering off (as $\cos\theta$) for increasing values of θ. A similar interpretation has been discussed by Carlson but where he considers the constraining influence to be due to omnidirectional jamming[3].

3. SIMULATIONS

The explanation given above, and in particular the expression for the optimum weight vector given in equation 8 provide a straightforward method of solving the sidelobe jitter problem as discussed in the introduction. For demonstration purposes we have simulated a half wave-length spaced linear array with 16 sensors and 32 snapshots used in the construction of the covariance matrix. The beampattern is constrained to lie close to the Tchebychev beampattern, with a sidelobe level of -30dB for $\theta > 0°$ and unconstrained for $\theta < 0°$. The look direction gain constraint is set, in this example, at 0°. The weighting function is chosen as

$$h(\theta) = 0.0 \qquad \theta < 0°$$

$$h(\theta) = \cos\theta \qquad \theta \geq 0°$$

and from equation 4 Z takes the form

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$$Z_{ij} = \left[\frac{2\sqrt{-1}}{\pi(i-j)}\right], \text{for } (i-j) \text{ odd,}$$
$$Z_{ij} = 1, for\ i = j \qquad (10)$$
$$Z_{ij} = 0, \text{otherwise.}$$

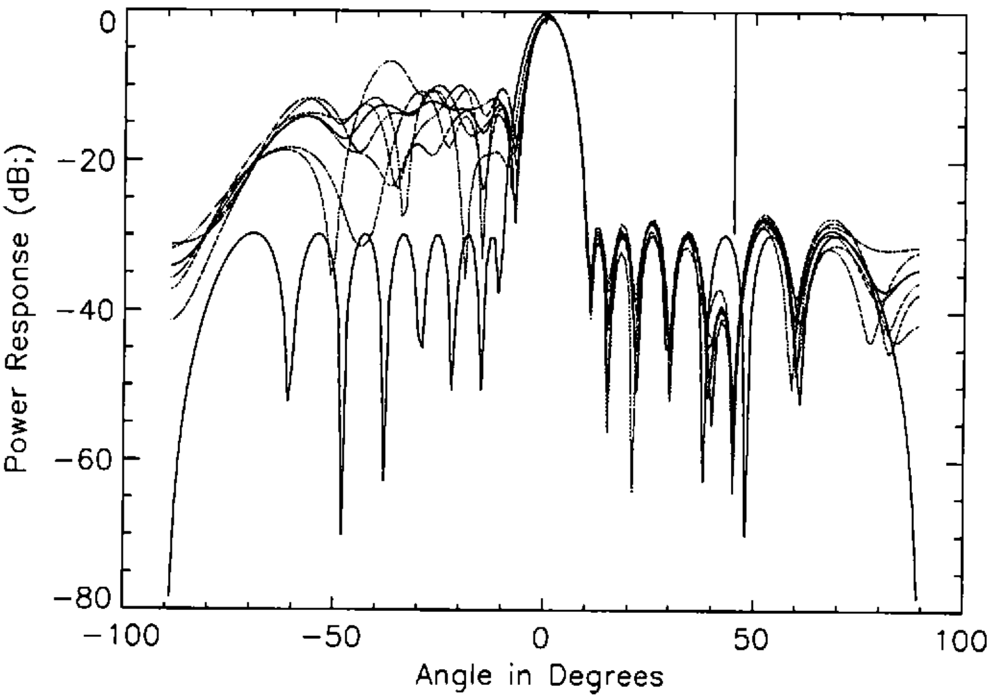

Figure 2: Asymmetric beam showing how flexibility of PFM allows a particular region to be constrained (using equation 10).

Figure 2 shows the effect of using this form for the $\mathbf{Z}$ matrix in equation 8 using k=30.0 and with a 30dB jammer present at 45°. The noise scenario is as used for figure 1. It is clear that the constraints for $\theta \geq 0°$ have a profound effect on the beampattern in that region. In particular the weight jitter can be seen to be drastically reduced whilst the sidelobes are flattened close to the Tchebychev levels.Furthermore, the array is still forming a null in the direction of the jammer. However, it is clear that for $\theta < 0°$, where the beampattern is unconstrained, high and jittery sidelobes are displayed.
The efficacy of this method, using $Z = I$, has been investigated, in some detail, elsewhere[4]. It was shown that quantitative measures of sidelobe jitter and sidelobe suppression behave well for a range of jammer scenarios and for a range of strength parameters k.

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4. MAINLOBE JAMMERS

In the above analysis it was pointed out that choosing $Z = I$ was equivalent to diagonal loading or equivalently to choosing a weighting function, $h(\theta)$ which gives preferential weighting to the look direction. This means that the PFM lays more stress on the main beam than on constraining the sidelobes. This is also partly because the mainbeam response is larger than for the sidelobe (For the Tchebychev case discussed below the ratio of the mainbeam peak to sidelobe level is 30dB). This interpretation raises the question of how well the modified diagonal loading described above would cope with mainlobe jammers since the strongly constrained mainlobe will find it difficult to place a null in the direction of the jammer since this would produce significant deviation from the quiescent pattern.

To investigate this we have carried out a set of simulations on a 16 sensor linear half wavelength spaced array. The desired pattern is Tchebychev with -30dB sidelobes. All simulations are carried out with 32 snapshots and a jammer of 30dB at 45°. A signal of 0dB is present on adaption. Noise is taken to be white, Gaussian and uncorrelated from sensor to sensor as used in previous simulations but now a mainbeam jammer of 0dB is present at 3.9° (corresponding to the 3dB point). We consider the output SNIR of the array for varying mainlobe jammer powers. We use two algorithms, both using a value for the strength parameter of $k = 15$.

Algorithm 1: The method described by equation 8 using $Z = I$.

Algorithm 2: A method which decreases the influence of the constraint in the mainlobe. An obvious way to do this is to use a weighting function of the form

$$h(\theta) = 0, |\theta| \le \theta_0$$

$$h(\theta) = \cos\theta, |\theta| > \theta_0.$$

Consequently for a region around 0° the mainbeam shape is not constrained. $2\theta_0$ defines the region over which the pattern is not constrained and is chosen to be slightly greater than the extent of the mainbeam. This choice of $h(\theta)$ produces a modified form for Z of the form

$$Z = 2 - 2\sin\theta_0, \text{ for diagonal terms}$$

$$Z = -2\frac{\sin(\pi(i-j)\sin\theta_0)}{\pi(i-j)}, \text{ for off-diagonal terms.}$$

can be inserted into equation 8.

Figure 3 shows the effect of varying the mainlobe jammer power between -20 and + 30dB. The solid line shows the effect of using algorithm 1. It is evident that as the mainlobe jammer power increases the output SNIR falls rapidly. This is due to the inability of the mainlobe to null in the direction of the jammer so that the residue of the mainlobe jammer becomes large. At higher values the SNIR performance begins to improve as the system begins to place a null in the direction of the mainlobe.

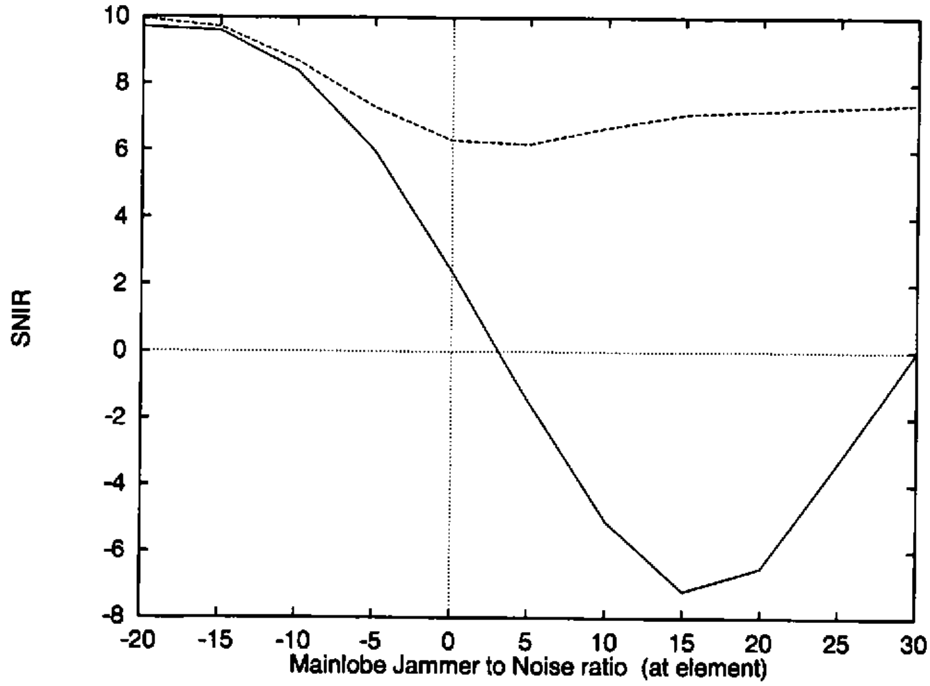

Figure 3: Comparison of signal to noise plus interference ratio for algorithms (in dB) 1 and 2. The solid line shows results for algorithm 1 and the dashed line for algorithm 2 in which the mainlobe is given freedom to adapt.

The dashed line shows the output response of the second algorithm, in which the mainlobe is allowed to adapt. Here also the SNIR reduces as the mainlobe jammer power increases. However, the reduction in performance is relatively small. In fact, the use of the modified $h(\theta)$, in the most severe region shows an improvement of approximately 15dB over algorithm 1.
As further evidence of the arguments given above for the poor performance of algorithm 1 (i.e. the inability to produce adequate nulls in the direction of mainlobe jammers). Figure 4 shows the beampatterns for the two algorithms with a 0dB jammer present at the 3dB point. Figure 4a shows the beampattern for algorithm 1. Although the jitter is much reduced (for instance, relative to figure 1) and the system is nulling at 45° there is no discernible adaption in the mainlobe. Algorithm 2 however has produced a significant null in the direction of the mainlobe jammer and consequently a superior SNIR (from figure 3 this can be seen to be approximately 4dB). At the same time it is clear that allowing the null to form in the mainlobe has distorted the overall beampattern. In particular the mainlobe peak has been displaced. However, the sidelobes are still depressed to below -25dB, the null in the direction of the jammer at 45° has formed and the jitter is small.
It is interesting to observe that these effects occur for what might be termed intermediate jammer powers as opposed to very powerful jammer powers as may have been imagined *a priori*.(In reality, of course, there would be calibration error and element mismatch effects which would reduce the SNIR for higher jammer powers). If these intermediate jammer powers are likely to be encountered it seems likely that algorithm 2 should be used. It is worth noting that in this case, for practical use where the sonar must be steered in many directions, a separate modified $\boldsymbol{Z}$ needs

to be calculated for each beam.

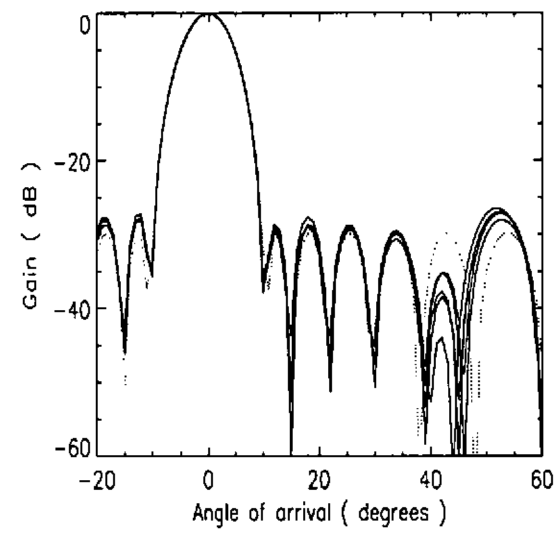

Figure 4a. Adapted beampatterns for algorithm 1 (shown by solid line). A 0dB jammer is present at the 3dB point as well as a jammer at 45°. The dashed line shows the Tchebychev quiescent pattern.

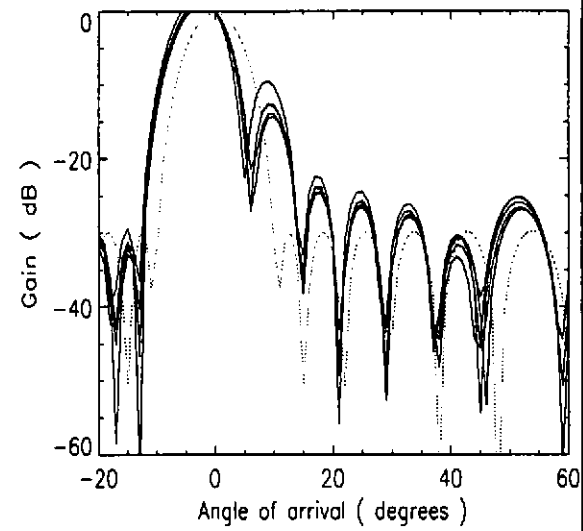

Figure 4b. Adapted beampatterns for algorithm 2 (shown by solid line). The jamming and noise scenario is as for figure 4a. Here, however, a null is formed in the mainlobe and a subsequent improvement of approximately 7dB is accrued.

5. CONCLUSIONS

We have described a penalty function approach to adaptive array processing. In certain situations the method reduces to a modified diagonal loading. Diagonal loading is found to work well for reducing weight jitter in many situations. However, it has been shown that for the situation when mainlobe jammers are present diagonal loading can produce a large decrease in output SNIR whereas using the penalty function to remove the shaping constraint from the mainlobe region reduces the effect of the mainlobe jammer drastically.

6. REFERENCES

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