FINITE ELEMENT ANALYSIS OF CERAMIC STACKS

D T I Francis (1), J R Oswin (2)

(1) School of Electronic and Electrical Engineering, University of Birmingham, Birmingham B15 2TT, UK
(2) BAeSEMA (Marine Division) Ltd., New Filton House, Bristol BS12 7QW, UK

1. INTRODUCTION

Finite Element (FE) analysis has been used for transducer design for over twenty years and is now becoming more widespread. Software developed in recent years (for example [1, 2]) has included the facility to model piezoelectric properties and to model acoustic loading by means of Boundary Element (BE) analysis.

Models of simple transducers can be produced with limited numbers of elements and are readily amenable to FE modelling [1]. When a much more complex transducer mechanism, such as a flextensional transducer is modelled [3], only a limited number of elements may be used to describe a piezoelectric ceramic stack which comprises a number of plates with interleaved electrodes and with insulators either end. Use of unmodified ceramic properties to describe the stack is likely to lead to misleading results. One technique previously used [4] is to model elements of ceramic only and to add extra elements to represent joints and insulators.

Providing the software can support a large enough number of nodes and degrees of freedom, it is possible to build up a detailed model of the stack including every component, but it may add significantly to running time. It may also take a long time to construct. If there are a large number of degrees of freedom, it may be a lengthy process to identify the valid mode shape amongst a large number of closely space modes. However, application of piezoelectric elements with charge applied only to appropriate nodes may reduce the number of degrees of freedom significantly.

A preferred method would be to have a library of ceramic properties which have been modified appropriately to represent the reduced performance obtained when ceramic plates are assembled into stacks, and then to use a small number of finite elements to model the stack, effectively treating it as a composite material. This paper describes the methods used to obtain the modified properties of a particular ceramic stack and compares the results with those of the stack modelled in intricate detail. Rather than use an inversion procedure, which attempts to obtain the properties from the performance characteristics directly, and which suffers from problems of ill-conditioning, an optimisation approach is adopted. In addition, the performance is considered for both unloaded ('in air') and loaded ('in fluid') conditions so as to increase the number of performance parameters available.

2. PHYSICAL PROPERTIES OF THE STACK

The stack which was modelled is shown in Fig. 1. It comprises eighteen plates of NAVY III lead zirconate titanate ceramic, with nineteen electrodes interleaved, and with steatite insulators at either end. After assembly, the stack was machined to an exact length of 97.6 mm. The electrodes were made from monel gauze, 250 mesh, 48 swg embedded in an epoxy resin according to the standard Defence Research Establishment technique. The ceramic plates had cross section 32 × 75 mm and were each 5 mm thick. Thicknesses and mechanical properties of the electrodes and insulators are given in Table 1.

Table 1. Properties of electrodes and insulators.

Property	Electrode	Insulator
Thickness, mm	0.09	2.945
Density, kg/m^3	1000	2900
Young's modulus, GPa	3	80
Poisson's ratio	0.32	0.25

By measuring the conductance and susceptance of the stack over a range of frequency around the first two observed resonances, the resonant frequencies, coupling coefficients, and mechanical and electrical Q-factors Q_m and Q_E, were obtained both in air and in castor oil. The results are given in Table 2.

Table 2. Measured performance parameters

(a) single ceramic plate

Plate thickness resonance, Hz	41900
Coupling coefficient	0.66

(b) complete stack

Parameter		Unloaded	In castor oil	Unloaded	In castor oil
C_{lf}	Low frequency capacitance (nF)	79	--	--	--
		First resonance		Second resonance	
f	Frequency (Hz)	15200	14800	20400	20100
k	Coupling factor	0.55	0.55	0.25	0.03
Q_M	Mechanical Q-factor	--	30	--	67
Q_E	Electrical Q-factor	--	0.21	--	0.18

3. STACK PROPERTIES - ONE-DIMENSIONAL APPROXIMATION

A simple one-dimensional estimate of the performance of a ceramic stack can be gained from the simple equivalent circuit illustrated in Fig. 2.

The coupling coefficient, k may be estimated from a definition of k^2 as the motional energy divided by the total energy stored:

$$k^2 = \frac{C_m}{C_e + C_m} \tag{1}$$

then

$$\frac{C_m}{C_e} = \frac{k^2}{1-k^2} = R \tag{2}$$

where R is defined as the mechanical to electrical energy ratio.

Estimating the mechanical energy stored in the stack enables a value of coupling coefficient to be determined. For a given displacement, the mechanical energy stored is proportional to the stiffness.

Ceramic manufacturers generally quote compliance, but in this approximation stiffness is used and is taken as the inverse of the compliance s_{33}. Note that in this case only mechanical energy is being considered, so the open circuit value, s_{33}^D is used.

The simple equivalent circuit of Fig. 2 is used for ceramic plate or ceramic stack. For a ceramic plate, the catalogue value of s_{33}^D may be used. An alternative value needs to be calculated for the ceramic stack. Firstly, the stiffness of the stack needs to be calculated by adding the stiffness of the components in series.

$$\frac{l_{st}}{Y_{st}} = \frac{l_c}{(1/S_{33}^D)} + \frac{l_{ins}}{Y_{ins}} + \frac{l_{el}}{Y_{el}} \tag{3}$$

where Y = stiffness, l = length, and subscripts *st, c, ins,* and *el* refer to the stack, ceramic, insulator, and electrode respectively.

Y_{st} is inverted to provide a compliance S_{st}. Then

$$\frac{S_{st}}{S_{33}^D} = \frac{C_{m.st}}{C_{m.c}} = \frac{R_{st}}{R_c} \tag{4}$$

In this case, k_c is taken as k_{33} of the ceramic.

Strictly speaking, the electrical energy is only developed over the length l_c whereas the mechanical energy has been averaged over the full length, l_{st}.. The energy ratio R_{st} should be modified:

$$R_{st}' = R_{st} \times \frac{l_{st}}{l_c} \tag{5}$$

The coupling coefficient may then be calculated by an inversion of Eq. (2) :

$$k_{st} = \sqrt{\frac{R'_{st}}{1 + R'_{st}}} \tag{6}$$

An estimate of resonant frequency of the stack may also be obtained, using the general formula:

$$f_r = \frac{c}{\lambda} = \frac{1}{2l}\sqrt{\frac{Y}{\rho}} \tag{7}$$

where Y is the composite stiffness calculated in Eq. (3) and ρ is a composite density calculated in a similar fashion. The resonant frequency of a stack is lower than that of an equivalent length of ceramic.

Using the values quoted in Table 1, and the measured coupling coefficient of 0.66 for the ceramic plate quoted in Table 2, a value of Y_{st} of 71 GPa is obtained, which provides a calculated value of 0.57 for k_{st} and a resonant frequency of 15.9 kHz.

Values obtained by these equations are sufficiently realistic to provide confidence for the use of the one-dimensional approximation in simple parametric calculations and the method is particularly useful for observing the effect of variation of ceramic plate coupling coefficient on stack performance, but it is certainly not adequate for use in three-dimensional finite element modelling of complex transducers, where the interrelation of effects in all axes must be considered.

4. FINITE ELEMENT TREATMENT

The Finite Element (FE) method [1] uses the piezoelectric equations of state in the form

$$T = c^E S - e_t E \tag{8}$$

$$D = eS + \varepsilon^S E \tag{9}$$

where the notation of reference [7] is adopted. A nominal charge density applied to the electrode surfaces, together with external forces as appropriate, leads to a solution for the strain and field at any specified frequency, from which displacements, conductance, and susceptance can be determined; by repeating the solution over a suitable frequency range, the resonant frequencies and conductance/susceptance response can be obtained. The mechanical Q-factor, Q_M, and coupling factor, k, are obtained from standard formulae [7]:

$$Q_M = \frac{f_S}{f_2 - f_1} \tag{10}$$

$$k^2 = 1 - (f_S / f_p)^2 \tag{11}$$

where f_S is the series resonance frequency, at which the conductance is a maximum, f_p is the parallel resonance frequency, which occurs where the ratio of conductance to susceptance is equal to that at f_S, and f_1 and f_2 are the frequencies at which the conductance is half of its value at f_S.

For the in-fluid case, the FE equations are coupled, via the external force terms, to equations relating the surface acoustic pressures to the surface velocities, obtained using a form of the Boundary Element (BE) method [8], so that full account is taken of the fluid-structure interaction.

The requirement is to determine the material properties of the stack, treated as consisting of a single uniform material, so as to produce performance characteristics closely matching those of the original stack. Because of symmetry, only one octant of the original stack (Fig. 1) was modelled in detail, with mirror symmetry taken in the three co-ordinate planes. This model was run both 'in air' (unloaded condition) and fully immersed in castor oil, the latter case providing an acoustic load to increase the influence of the piezoelectric properties on the performance. Table 3 shows the performance parameters predicted for the detailed model. Compared with the measured values, the first resonance is somewhat lower; A similar discrepancy was found between the predicted and measured frequency of the thickness resonance for a single ceramic plate (Table 2), suggesting that the difference is mainly due to variation in the ceramic properties from their assumed values. The predicted performance parameters have been taken as the 'target' for the performance of the stack when treated as consisting of a single material with uniform properties.

Table 3. Predicted performance parameters for detailed stack model.

Parameter		Unloaded	In castor oil	Unloaded	In castor oil
C_{lf}	Low frequency capacitance (nF)	77	n.a.		
		First resonance		Second resonance	
$f_S^{(1)}, f_S^{(2)}$	Frequency (Hz)	14248	13718	20043	19732
G	Conductance (mS)	703	109	50.8	28.7
B	Susceptance (mS)	6.02	21.9	4.80	30.2
Q_M	Q-factor	374	13.78	573	55.5
k	Coupling factor	0.513	0.532	0.137	0.172
$a_{1,x}$	x-displacement at centre of narrow side face (μm per V)	0.716	0.029	0.070	0.010
$a_{2,z}$	z-displacement at centre of top face (μm per V)	0.307	0.012	0.096	0.0051
		Higher resonances			
$f_S^{(3)}, f_S^{(4)}$	Frequency (Hz)	24090	--	29661	--
$f_S^{(5)}, f_S^{(6)}$	Frequency (Hz)	38519	--	41601	--

A simplified model of the stack, consisting of a just two elements representing the octant of the stack, was used to predict the performance under the same two conditions as above. It is of course be necessary to perform a multiplication to modify the electrical values in relation to the numbers of plates in the original and simplified structures. For the models described, the number of plates has been reduced from 18 in the original stack to 4 in the simplified model, which requires that the original capacitance, conductance, and susceptance be multiplied by $(4/18)^2$, and the displacements by 4/18 to obtain the corresponding target values for the simplified model.

Excluding the mean density, which can be calculated to be 7201 kg/m^3, there are thirteen property values (variables) to be determined; these are listed in Table 4, together with their original values for Navy III type ceramic. By modifying these variables, the effect of such changes on the performance of the simplified model can be assessed; and changes made to improve agreement with the performance of the full model. To assist this process, an optimisation procedure was adopted as follows. A value function is defined by

$$v = \sum_{i=1}^{n} w_i \left(\frac{p_i - t_i}{t_i} \right)^2 \tag{12}$$

where p_i, $i = 1..n$, are the performance parameters for the simplified model, t_i the corresponding target values, and w_i weights assigned according to the perceived importance of each parameter. The aim is to minimise the value of v. If this function is evaluated for three values of one of the variables, the other variables remaining constant, a quadratic curve may be fitted, and a minimum value sought. The location of this minimum, if it exists and does not involve excessive extrapolation beyond the interval defined by the three values of the variable, can then be used as the starting value of that variable for the next iteration, otherwise the best of the three values is chosen.

Table 4. Material property coefficients, with their values for Navy III ceramic.
Matrix coefficients not listed are zero.

Density, ρ, kg/m^3	7600		
Mechanical, GPa		Piezoelectric, C/m^2	
$c_{11}^E = c_{22}^E$	137	$e_{31} = e_{32}$	-4.0
$c_{12}^E = c_{21}^E$	69.7	e_{33}	13.8
$c_{13}^E = c_{31}^E = c_{23}^E = c_{32}^E$	71.6	$e_{15} = e_{24}$	10.4
c_{33}^E	124	Dielectric, nF/m	
$c_{44}^E = c_{55}^E$	31.4	$\varepsilon_{11}^S = \varepsilon_{22}^S$	7.95
c_{66}^E	33.7	ε_{33}^S	5.15
$\tan \delta_M$	0.00083	$\tan \delta_E$	0.004

The optimisation process was carried out in two stages. The mechanical stiffness coefficients c^E almost entirely determine the unloaded resonant frequencies. The first stage therefore consisted of optimising these variables using only the resonant frequencies $f_S^{(1)}..f_S^{(6)}$ as the performance parameters in Eq. (10). However, the coefficient c_{66}^E has very little influence on these frequencies, and could not be reliably optimised. This coefficient represents shear around the z-axis, and the symmetry of the stack inhibits modes of vibration with such shear. Repeating the process without the symmetry conditions increased the influence of the coefficient on the resonant frequencies for this case, but not sufficiently to give a reliable result. The figure obtained for this coefficient must therefore be regarded as provisional. This uncertainty is unlikely to be detrimental in transducer applications, as the stack mode of vibration will generally lack shear about the z-axis, and the effect of an inaccurate value of c_{66}^E on the performance will be marginal.

The second stage used the performance parameters for the first two resonant frequencies (Table 2) to optimise the values of the remaining variables.

The resulting values of the variables are listed in Table 5. The performance characteristics with the modified values are shown in Table 6; these can be compared with the required characteristics listed in Table 3. The percentage differences are shown in brackets. The agreement between the two sets of values tends to reflect the weighting assigned to each variable in the optimising process, and the difficulty of obtaining reliable values in some cases - for example, the susceptance around resonance varies rapidly, increasing the uncertainty in the value determined.

Table 5. Modified values of the material property coefficients

Density, ρ, kg/m^3	7201		
Mechanical, GPa		Piezoelectric, C/m^2	
$c_{11}^E = c_{22}^E$	117	$e_{31} = e_{32}$	-3.7
$c_{12}^E = c_{21}^E$	50.5	e_{33}	11.5
$c_{13}^E = c_{31}^E = c_{23}^E = c_{32}^E$	44	$e_{15} = e_{24}$	10.5
c_{33}^E	78	Dielectric, nF/m	
$c_{44}^E = c_{55}^E$	19	$\varepsilon_{11}^S = \varepsilon_{22}^S$	13
c_{66}^E	21	ε_{33}^S	7.5
$\tan\delta_M$	0.0025	$\tan\delta_E$	0.0054

Table 6. Performance of the simplified stack model with the modified properties. Quantities marked * have been scaled to allow for the differing number of plates, as described in the text.

Parameter		Unloaded	In castor oil	Unloaded	In castor oil
C_{lf}	Low frequency capacitance (nF)*	87 (+13%)	n.a.		
		First resonance		Second resonance	
$f_S^{(1)}, f_S^{(2)}$	Frequency (Hz)	13870 (-3%)	13430 (-2%)	19430 (-3%)	19130 (-3%)
G	Conductance (mS)*	748 (+6%)	110 (+1%)	38.7 (-24%)	6.67 (-77%)
B	Susceptance (mS)*	5.3 (-12%)	17.6 (-20%)	5.21 (+9%)	29.4 (-3%)
Q_M	Q-factor	394 (+5%)	13.0 (-6%)	384 (-33%)	18.3 (-67%)
k	Coupling factor	0.49 (-3%)	0.491 (-8%)	0.135 (-1%)	0.23 (+35%)
$a_{1,x}$	x-displacement at centre of narrow side face (μm per V)*	0.784 (+9%)	0.031 (+7%)	0.059 (-15%)	0.007 (-27%)
$a_{2,z}$	z-displacement at centre of top face (μm per V)*	0.301(-2%)	0.012 (-2%)	0.062 (-35%)	0.007 (47%)
		Higher resonances			
$f_S^{(3)}, f_S^{(4)}$	Frequency (Hz)	23980 (-0.5%)	--	29858 (+0.7%)	--
$f_S^{(5)}, f_S^{(6)}$	Frequency (Hz)	39090 (+1%)	--	43281 (+4%)	--

5. CONCLUSIONS

Use of a simplified model for a ceramic stack in FE modelling reduces computing time and power requirements. It simplifies the analysis and provides for more accurate computation. Values for the mechanical and electrical properties of the stack can be calculated from the known mechanical properties of the stack and a revised piezoelectric property matrix can be derived. There are some discrepancies between either FE model and measurement, but the simple FE stack model gives results which are sufficiently consistent with the full stack model to allow its application in modelling the stacks of complex transducers.

6. ACKNOWLEDGEMENTS

The first author has been supported by the European Communities under contract MAS2-CT91. The second author was supported by BAeSEMA Ltd. The authors wish to thank members of the Acoustics and Sonar group at the University of Birmingham for helpful discussions during the course of the work.

7. REFERENCES

[1] S S JARNG, D T I FRANCIS, J R DUNN & B V SMITH, 'Optimisation of Sonar Transducers Using a Coupled FE-BE Method', *Proc IoA*, 12 (4), pp57-65 (1990)

[2] P C MACEY, 'Analysis of Transducers using Piezoelectric Finite Elements and Acoustic Boundary Elements', *Proc IoA* 12 (4), pp66-75 (1990)

[3] J R OSWIN & D T I FRANCIS, 'Three-Dimensional Vibration Patterns and Electroacoustic Performance of Class IV Flextensional Transducers. Comparison of Modelling and Measurement', 1994 (*to be published*).

[4] R BRIND, 'Finite Element Modelling of the ARE Low Frequency Flextensional Transducer', *Proc IoA* 10 (9) pp105-128 (1988)

[5] D A BERLINCOURT, D R CURRAN & H JAFFE, 'Piezoelectric and Piezomagnetic Materials and their Function in Transducers', in W P Mason (Ed), Physical Acoustics, 1(a) Academic Press (1964)

[6] Vernitron Ltd, 'Five Modern Piezoelectric Ceramics', *Bulletin* (1976).

[7] D STANSFIELD, 'Underwater Electroacoustic Transducers', Bath University Press and IoA, pp45-46 (1990)

[8] D T I FRANCIS, 'A Gradient Formulation of the Helmholtz Integral Equation for Acoustic Radiation and Scattering', *J.Acoust.Soc.Am* 93 (4), pp1700-1709 (1993)

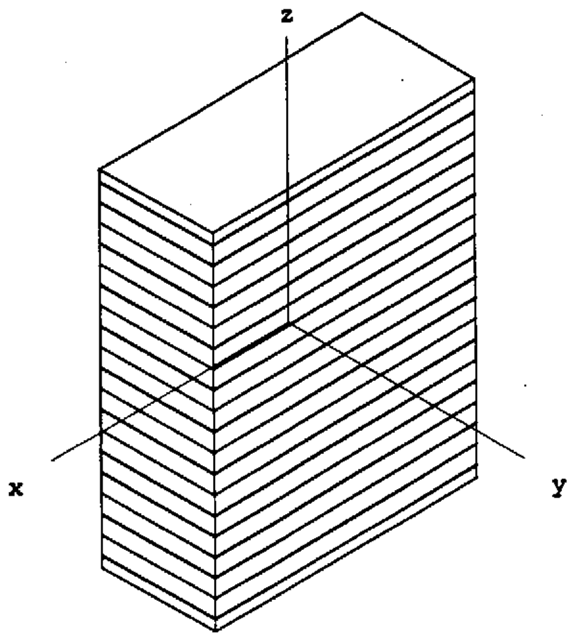

Figure 1. Piezoelectric stack modelled and measured.

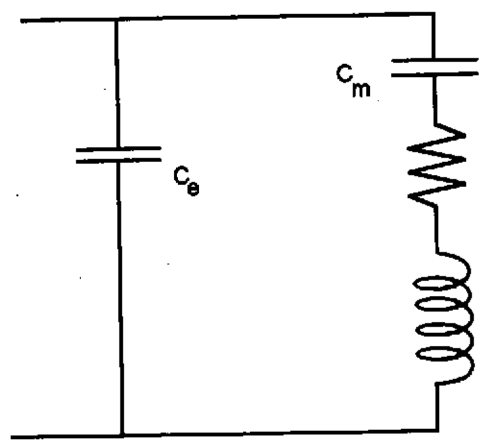

Figure 2. Simple equivalent circuit