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REDUCTION OF NON-LINEAR DISTORTION IN CONDENSER MICROPHONES BY USING NEGATIVE LOAD CAPACITANCE

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1. INTRODUCTION

Condenser measurement microphones have very wide dynamic ranges, typically 140 dB. At low sound levels the range is limited by inherent noise of the microphone and/ or preamplifier. At high levels it is generally limited by a non-linear distortion which is proportional to the sound pressure and is produced by the microphone itself. This distortion has been analysed in theory and in practice for some frequently used types of microphone. The good agreement which was found between measured and calculated results verifies the derived distortion formulae and points clearly at the load capacitance and mode of diaphragm displacement as being the dominating reasons. The formulae were used for calculation of the capacitive loading which would lead to the lowest possible distortion. This appeared to be negative and to be a function of the ratio between the backplate and diaphragm diameters. Tests made with an experimental preamplifier with negative input capacitance gave promising results.

2. THEORETICAL DISTORTION ANALYSIS OF TRANSDUCTION

General. The operation of most condenser measurement microphones is based on the application of a constant electrical charge stored on the active microphone capacitance and on its parallel (stray) capacitance. The constant charge may either be supplied from an external voltage source via a resistor (typ. $10^9\ \Omega$) or by a build-in electret. The formulae below describe the transduction of capacitance variation to voltage:

$$E \cdot (C_a + C_p) = Q_0 = E_0 \cdot (C_0 + C_p) \quad \text{or} \quad E = E_0 \cdot \frac{C_0 + C_p}{C_a + C_p} \qquad (1)$$

E : Voltage across capacitances with diaphragm displaced by sound pressure
E_0 : Voltage across capacitances with diaphragm at rest position
C_a : Active diaphragm-backplate capacitance (varies with sound pressure)
C_0 : Active diaphragm-backplate capacitance with diaphragm at rest position
C_p : Parallel capacitance (passive)
Q_0 : Constant charge stored on the active and passive capacitances

As the charge (Q_0) is kept constant the voltage (E) will vary with the variation of the active capacitance (C_a) which is caused by the sound pressure. The voltage produced depends on the microphone configuration and on the diaphragm deflection mode.

Flat Diaphragm Displacement Mode. If the microphone diaphragm is considered to be parallel to a circular backplate and displaced like a flat piston, then the active capacitance (C_a) and its rest capacitance (C_0) can be expressed by the equations:

$$C_a = R_S \cdot \frac{\varepsilon \cdot \pi \cdot R_b^2}{D+d} = C_0 \cdot (1+y)^{-1}$$

$$C_0 = R_S \cdot \frac{\varepsilon \cdot \pi \cdot R_b^2}{D}$$

where $R_S = \frac{A_b - A_h}{A_b}$ and $y = \frac{d}{D}$

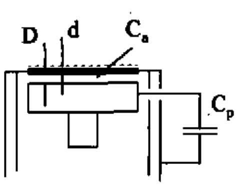

Fig. 1 Microphone with flat diaphragm displacement mode and parallel capacitance.

ε : Dielectric constant of air
R_b : Radius of backplate
D : Rest distance, backplate to diaphragm
d : Displacement of diaphragm
R_s : Ratio of effective and total backplate area
A_b : Total backplate area (includes area of holes)
A_h : Area of holes in backplate (a uniform hole distribution is considered)
y : Relative diaphragm displacement

Insertion of the above expressions into Equation (1) leads to Equation (2) which defines the output voltage of a microphone with a flat diaphragm displacement mode:

$$E_{fm} = E_0 \cdot \frac{C_0 + C_p}{C_0 \cdot (1+y)^{-1} + C_p} \qquad (2)$$

Series expansion of equation (2) leads to:

$$E_{fm} = E_0 + E_0 \cdot \frac{C_0}{C_0 + C_p} \cdot \left(y - \left(\frac{C_p}{C_0 + C_p} \right)^1 \cdot y^2 + \left(\frac{C_p}{C_0 + C_p} \right)^2 \cdot y^3 - \cdots\cdots \right) \qquad (2a)$$

For a sinusoidally varying diaphragm displacement with time y, y^2 and y^3 become:

$$y = y_m \cdot \sin \omega t ; \quad y^2 = \frac{1}{2} \cdot y_m^2 \cdot (1 - \cos 2\omega t) ; \quad y^3 = \frac{1}{4} \cdot y_m^3 \cdot (3 \sin \omega t - \sin 3\omega t)$$

where y_m : Maximum value of relative diaphragm displacement.

This leads to the following second (D_2) and third (D_3) harmonic distortion components of the output voltage (relative to the fundamental frequency component):

$$D_2 = \left(\frac{1}{2} \cdot y_m \cdot \frac{C_p}{C_o + C_p} \right)^1 \cdot 100 \ \% \quad \text{and} \quad D_3 = \left(\frac{1}{2} \cdot y_m \cdot \frac{C_p}{C_o + C_p} \right)^2 \cdot 100 \ \%$$

The dominating second harmonic component (D_2) is proportional to the diaphragm displacement and thus to the sound pressure while the third harmonic (D_3) is proportional to the square of the pressure. Notice, that the distortion decreases with the parallel capacitance and that it becomes zero if this capacitance becomes zero ($C_p = 0$).

Parabolic Diaphragm Displacement Mode. Generally condenser microphones use foil diaphragms with a high internal mechanical tension which gives the diaphragm its required stiffness and determines the displacement mode at lower frequencies. At higher frequencies the air damping and the foil mass do also influence the mode. This discussion covers only the low frequency mode which can be considered to occur up to a frequency which is 0.2 to 0.5 times the diaphragm resonance frequency. The displacement mode of an ideal circular diaphragm, which is purely stiffness controlled, is defined by the formula below and illustrated in Fig. 2:

$$d(r) = d_0 \cdot \left(1 - \frac{r^2}{R_d^2}\right)$$

where

d_0 : centre displacement

r : distance to centre

R_d : Radius of diaphragm

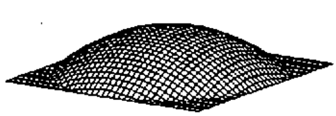

Fig.2
Calculated foil diaphragm displacement mode

To verify this the displacement mode of a one inch microphone Type 4144 was measured. This was done by scanning its diaphragm along a diameter. The scanning was made with a small microphone (Type 4138, 1.8 mm backplate diameter) with its own diaphragm dismantled. This microphone was moved in a fixed distance in front of the large diaphragm in such a way that it detected the local displacement of the larger diaphragm while this was exposed to sound pressure. The measured and the calculated displacement magnitude were found to be in very good agreement; see Fig. 3.

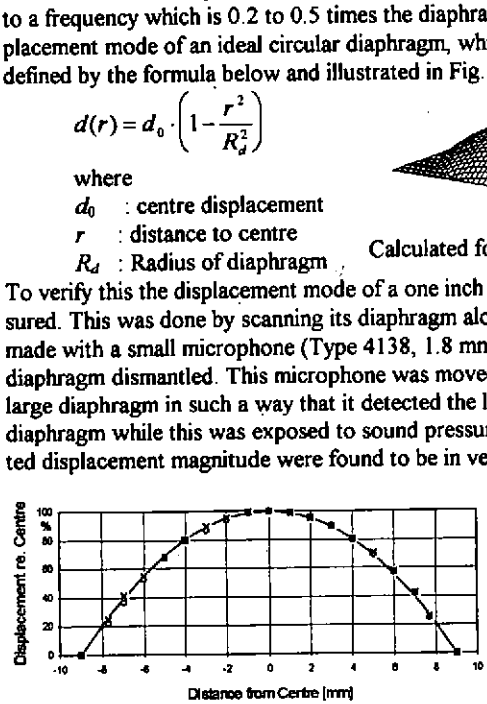

Fig. 3. Diaphragm displacement as a function of distance to centre for a one inch microphone with a diaphragm of 18 mm diameter. The points are measured at 40 Hz and at 130 dB and 150 dB SPL. The curve is calculated.

Considering the above displacement mode the active capacitance (C_a) and its rest capacitance (C_0) may be expressed by the equations below:

$$C_a = R_S \cdot \int_0^{R_b} \frac{\varepsilon \cdot 2 \cdot \pi \cdot r}{D + d_0 \cdot \left(1 - \frac{r^2}{R_d^2}\right)} dr \qquad C_0 = R_S \cdot \frac{\varepsilon \cdot \pi \cdot R_b^2}{D} \qquad \text{where } R_S = \frac{A_b - A_h}{A_b}$$

Simplification and integration of the first equation leads to:

$$C_a = R_S \cdot \frac{\varepsilon \cdot \pi \cdot R_b^2}{D} \cdot R_b^{-2} \cdot \int_0^{R_b} \frac{2 \cdot r}{1 + y_0 \cdot \left(1 - \frac{r^2}{R_d^2}\right)} dr = C_0 \cdot R_b^{-2} \cdot \int_0^{R_b} \frac{2 \cdot r}{1 + y_0 \cdot \left(1 - \frac{r^2}{R_d^2}\right)} dr$$

$$C_a = C_0 \cdot \left(\frac{R_b^2}{R_d^2}\right)^{-1} \cdot y_0^{-1} \cdot \ln \frac{1 + y_0}{1 + \left(1 - \frac{R_b^2}{R_d^2}\right) \cdot y_0} = C_0 \cdot (1 - k)^{-1} \cdot y_0^{-1} \cdot \ln \frac{1 + y_0}{1 + k \cdot y_0}$$

where $k = 1 - \frac{R_b^2}{R_d^2}$ and $y_0 = \frac{d_0}{D}$

Insertion of C_a and C_0 in Equation (1) gives the equation valid for parabolic mode:

$$E_{pm} = E_0 \cdot \frac{C_0 + C_p}{C_0 \cdot (1-k)^{-1} \cdot y_0^{-1} \cdot \ln \frac{1+y_0}{1+k \cdot y_0} + C_p} \qquad (3)$$

Series expansion of Equation (3) leads to:

$$E_{pm} = E_0 + E_0 \cdot \frac{k+1}{2} \cdot \frac{C_0}{C_0 + C_p} \cdot \left(y_0 + F_2 \cdot y_0^2 + F_3 \cdot y_0^3 + \cdots \right) \qquad (3a)$$ where

$$F_2 = - \frac{C_0 \cdot (k^2 - 2 \cdot k + 1) + 4 \cdot C_p \cdot (k^2 + k + 1)}{6 \cdot (C_0 + C_p) \cdot (k+1)}$$ and

$$F_3 = \frac{(C_o^2 + 4 \cdot C_o \cdot C_p) \cdot (k^2 - 2 \cdot k + 1) + 6 \cdot C_p^2 \cdot (k^2 + 1)}{12 \cdot (C_0 + C_p)^2}$$

The factors F_2 and F_3 are functions of the parameter k and thus of the ratio between the backplate and diaphragm radii. The larger the backplate becomes, the higher becomes the distortion. Equation (2a) indicates that this effect was to be expected as area added along the outer circumference represents a less active capacitance which loads the more active capacitance located at the diaphragm and backplate centres.

Notice, that the smaller the backplate becomes in comparison with the diaphragm, the more flat does the active part of the diaphragm become. Therefore, for k equal to '1', Equation (3a) becomes equal to Equation (2a) which is valid for flat diaphragm mode. In practice , distortion should be defined as a function of Sound Pressure (p) rather than relative diaphragm displacement at the centre (y_0). The relation between displacement and sound pressure can be obtained from the following equations:

$$e_1 = E_0 \cdot \frac{k+1}{2} \cdot \frac{C_0}{C_0 + C_p} \cdot y_0 \quad \text{and} \quad p = e_1 \cdot \left(S_0 \cdot \frac{C_0 + C_p - C_i}{C_0 + C_p} \right)^{-1}$$

e_1 : Microphone output voltage according to Equation (3a) (1st term with y_0)
p : Sound pressure
S_0 : Measured open circuit sensitivity

For a sinusoidal diaphragm displacement with time, the ratios (D_2 and D_3) between the second and third harmonic components and the fundamental component become:

$$D_2 = \left(\frac{y_m}{2} \right)^1 \cdot F_2 \cdot 100\% \quad (4) \quad \text{and} \quad D_3 = \left(\frac{y_m}{2} \right)^2 \cdot F_3 \cdot 100\% \quad (5)$$

where $y_m = \sqrt{2} \cdot p_{RMS} \cdot \frac{S_0}{E_0} \cdot \frac{C_0 + C_p - C_i}{C_0} \cdot \frac{2}{k+1}$

3. COMPARISON OF CALCULATED AND MEASURED DISTORTION

Calculated and measured distortion data were evaluated by a comparison. See input data and distortion results for some commonly used microphones in the tables below.

B&K Type No.		4144	4133	4165	4135	4190	4192/93
Dimension		1/1"	1/2"	1/2"	1/4"	1/2"	1/2"
S_0	mV/Pa	50	12	50	4.0	50	12.5
E_0	V	200	200	200	200	200	200
R_d	mm	9.1	4.6	4.6	2.1	4.6	4.6
R_b	mm	6.65	3.60	3.65	1.75	3.45	3.45
D_0	µm	23.5	21.0	22.2	18.0	24.5	19.0
A_h	mm²	20.3	1.70	4.24	0	3.96	5.50
C_e	pF	2.1	0.5	0.9	0.2	0.8	0.8
C_h	pF	2.1	1.1	1.8	1.6	1.6	1.5

B&K Type No.		4144	4133	4165	4135	4190	4192	4193
Load Capacitance	pF	0.2	0.2	0.2	0.2	0.2	0.2	100
Sound Pressure Level	dB	140	150	140	160	140	150	150
2. harm. calculated	%	0.51	0.49	0.97	1.52	0.97	0.63	3.0
2. harm. measured *	%	056	0.50	1.08	1.55	0.90	0.54	3.4
Ratio - Calc. re. HB-data		**0.91**	**0.97**	**0.98**	**0.98**	**1.08**	**1.17**	**0.88**
3. harm. calculated	%	0.01	0.01	0.02	0.04	0.02	0.01	0.09

*Sources: B&K's blue Microphone Handbook and the Falcon Handbook . The data applied for Types 4133/65 originates from measurements performed in January 1994.

There is a very good agreement between the calculated and the measured distortion data. The calculated ratio (see lower table) is very close to one. As the calculations only account for the transduction itself this seems to be the only source of low frequency distortion which is of importance for the analysed types of microphone.

4. DISTORTION REDUCTION BY NEGATIVE CAPACITANCE LOADING

Theory. Equation (4) shows that the 2. harmonic component is proportional to the factor F_2. It is a function of the ratio between the active and the passive parallel capacitances as well as of the ratio between the backplate and diaphragm radii. For a certain radii ratio (i.e. a certain value of 'k') F_2 and the second harmonic distortion become zero if the microphone is loaded with a proper negative capacitance. Calculated distortion is shown for two extreme microphone configurations in Fig.4.

Typical microphone/preamplifier combinations have a C_p/C_0-ratio of +0.1 to +0.4. Fig. 4 shows that the optimum with respect to distortion is between '-0.25' and '0'. The ideal C_p/C_0-ratio is defined by the following equation:

$$\left(\frac{C_p}{C_0}\right)_{ideal} = -\frac{1}{4}\cdot\frac{k^2-2\cdot k+1}{k^2+k+1}$$

Negative Capacitance. In principle, negative load capacitance may be created by a circuit like that shown in Fig. 5. A capacitor (C) is connected between the input and output of the microphone preamplifier whose gain (> +1) can be adjusted to give the proper input capacitance (C_i); see the formula:

$$C_i = \frac{e_1 - e_2}{e_1}\cdot C = (1-A)\cdot C$$ where e_1: input voltage, e_2: output voltage.

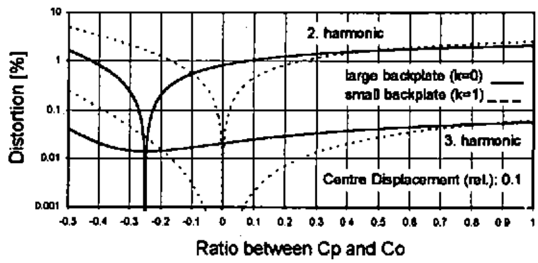

Fig. 4 Harmonic distortion calculated as a function of the ratio between the passive and active capacitances.

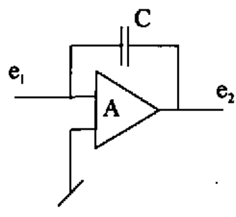

Fig. 5
Principle of negative input capacitance circuit.

Experimental Results. An experimental preamplifier with adjustable negative input capacitance was designed and tested with three different microphones at 100 Hz. For each microphone the capacitance was adjusted to give the lowest possible 2. harmonic distortion. The results are shown in the table below:

Initial Experimental Results obtained with Negative Input Capacitance	Unit	Type 4165	Type 4133	Type 4135
Sound Pressure Level	dB	134	146	156
Nominal 2. harmonic distortion	%	0.61	0.32	0.98
Minimised 2. harmonic distortion (negative cap.)	%	0.10	0.16	0.18
Reduction factor	-	**6.1**	**2.0**	**5.4**
Measured increase of sensitivity (approximate)	dB	2.0	1.5	4.3
Calculated increase of sensitivity	dB	2.0	1.4	3.6
Optimal input capacitance (calculated)	pF	-3.7	-2.6	-2.2

The idea (patented) of using negative input capacitance for distortion reduction seems to work well in practice but further experiments have to be made to clarify all aspects of its use. The B&K High Pressure Calibrator Type 4221 was used for the distortion measurements.

5. CONCLUSION

Harmonic distortion of condenser microphones using constant electrical charge has been analysed for the frequency range where the diaphragm displacement is stiffness controlled. Distortion formulae have been derived for the transduction from diaphragm displacement to output voltage. The formulae were applied for calculating distortion of some commonly applied types of measurement microphone. The results were compared with data supplied by the manufacturer and a very good agreement was found. This verified the formulae and pointed at parallel capacitance and displacement mode as being the dominating reason for the distortion. The distortion analysis indicated the possibility of reducing harmonic distortion by loading the microphone with negative capacitance. This was confirmed by experiments.
Further work has to be done to analyse the practical possibilities which in addition to distortion reduction might be improvement of high level peak measurements and extension of the applicable dynamic range of condenser microphones.