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ADAPTIVE ACTIVE CONTROL OF STRUCTURAL VIBRATION BY MINIMISATION OF TOTAL SUPPLIED POWER

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1. INTRODUCTION

Active control of vibration by minimisation of power has been suggested in the literature [1,2] as an alternative to least mean squares methods. In this practical study, an aluminium beam exposed to sinusoidal excitation was controlled at excitation frequencies ranging from 50 Hz to 500 Hz.

2. THE ADAPTIVE POWER MINIMISING ALGORITHM

The derivation of the control algorithm is analogous with the least mean squares steepest descent method [1]. However, here the cost function of the control problem is expressed as a sum of products of instantaneous forces and velocities measured at the source locations. This presentation is based on Laugesen [2] and formulated in the time domain. Let the vibration to be controlled originate from K primary sources and the control algorithm feed M secondary sources. Then the total instantaneous power $P(n)$ at time n supplied by the K+M sources can be expressed as

$$P(n) = \sum_{l=1-K}^{M} f_l(n)v_l(n). \tag{1}$$

By splitting the force $f_l(n)$ and the velocity $v_l(n)$ at source l into separate contributions from the individual sources m, (1) expands to

$$P(n) = \sum_{l=1-K}^{M} \Big(\sum_{m=1-K}^{M} f_{lm}(n)\Big)\Big(\sum_{m=1-K}^{M} v_{lm}(n)\Big). \tag{2}$$

These contributions may now be expressed as a filtered version of a reference signal $x(n)$ convolved with a structural response, i.e.

$$f_{lm}(n) = \sum_{j=0}^{J-1} g_{lmj} \sum_{q=0}^{Q-1} a_{mq}\, x(n-j-q), \tag{3}$$

$$v_{lm}(n) = \sum_{j=0}^{J-1} u_{lmj} \sum_{q=0}^{Q-1} a_{mq}\, x(n-j-q), \tag{4}$$

where g_{lmj} and u_{lmj} are the structural responses from source m to force sensor l and velocity sensor l respectively. a_{mq} represents the coefficients of the adaptive control filters for $m = 1,..,M$ and the response from the reference signal to the primary sources for $m = 1\text{-}K,..,0$. J is the number of coefficients in the structural responses and Q is the control filter length. By rearranging terms and denoting the reference signal convolved with the structural responses by $h_{lm}(n)$ and $y_{lm}(n)$, then (3) and (4) reduce to

$$f_{lm}(n) = \sum_{q=0}^{Q-1} a_{mq} \sum_{j=0}^{J-1} g_{lmj}\, x(n-j-q) = \sum_{q=0}^{Q-1} a_{mq} h_{lm}(n-q), \tag{5}$$

$$v_{lm}(n) = \sum_{q=0}^{Q-1} a_{mq} \sum_{j=0}^{J-1} u_{lmj}\, x(n-j-q) = \sum_{q=0}^{Q-1} a_{mq} y_{lm}(n-q). \tag{6}$$

Taking the partial derivative of $P(n)$ with respect to the coefficients of the control filters yields

$$\frac{\partial P(n)}{\partial a_{mq}} = \sum_{l=1-K}^{M} [h_{lm}(n-q)v_l(n) + f_l(n)y_{lm}(n-q)];\ m = 1,..,M;\ q = 0,..,Q-1. \tag{7}$$

Denoting the adaptation gain by α, the coefficients of the adaptive control filters can now be calculated by the steepest descent update formula

$$a_{mq}(n+1) = a_{mq}(n) - \alpha \frac{\partial P(n)}{\partial a_{mq}};\quad m = 1,..,M;\ q = 0,..,Q-1. \tag{8}$$

3. EXPERIMENTAL SETUP

The basic elements of the experimental setup are outlined in Fig. 1. While running the control system, the primary exciter is fed continuously with a sinusoidal reference signal. The vibration induced in the structure is then controlled by feeding the two secondary exciters with the reference signal convolved with the control filters. The control filter coefficients are updated every sampling interval by the adaptive algorithm, which adjusts the filters by knowledge of the immediate sensor measurements and by *a priori* knowledge of the response from the output of the control filters to the sensor locations. This *a priori* knowledge is presented to the algorithm as filtered versions of the reference signal, $h_{lm}(n)$ and $y_{lm}(n)$, which are based on estimates of the structural responses. These estimates are obtained by adaptive filtering, a method generally referred to as system identification. The control objective is to reduce the total power supplied by the three exciters connected to the structure. Instant measures of the velocities are obtained by integrating the accelerometer values in the digital domain. Though the adaptive algorithm is surprisingly insensitive to measurement errors, a very careful correction of bias, phase and amplitude is

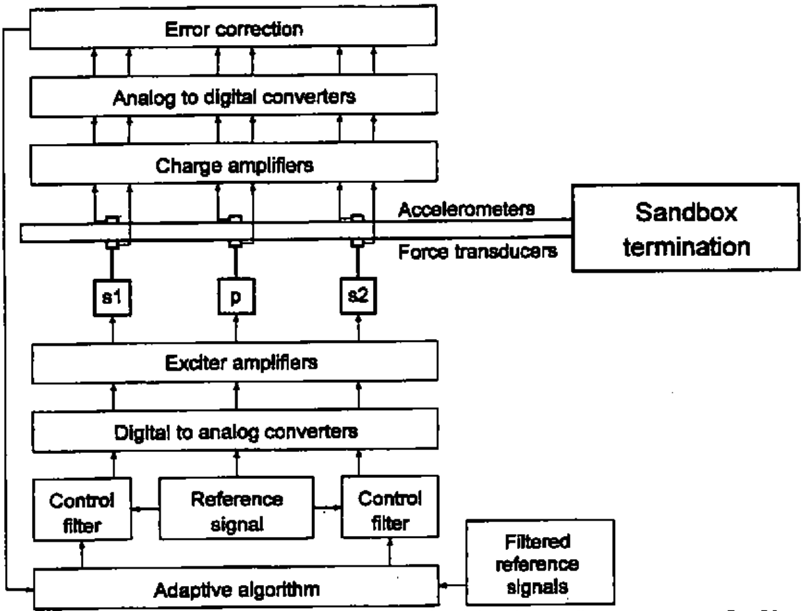

Fig. 1. *Outline of the experimental setup. The aluminium beam dimensions were 5 x 50 x 2050 mm. Exciters were located 779 mm, 950 mm and 1151 mm from the free beam end.*

necessary. Otherwise, the algorithm may easily converge to a state, where the measured averaged total supplied source power becomes negative. Thus, small measurement errors can lead to apparently meaningless results.

4. RESULTS

The averaged total supplied source power before control and after 58 seconds of control is shown in Fig. 2. The applied adaptation gain was found empirically, and an upper limit, beyond which the algorithm would diverge, turned out to be frequency dependant. The power attenuation was ranging from 0.4 dB at 363 Hz to 23.5 dB at 394 Hz. At 363 Hz, convergence was not obtained after 58 seconds of control, while at 275 Hz, convergence was obtained within 10 seconds of control. In general, the best results were obtained at excitation frequencies close to the structural resonances. This may not be obvious from Fig. 2, since the primary exciter input was reduced near the resonances. This was necessary to obtain a reasonable signal to noise ratio in the measurements at all excitation frequencies. By running the control system with only a single active secondary source, it was observed that whenever at least one source could obtain control individually, so would the combined application of the two secondary sources. The worst case result at 363 Hz corresponds to a situation were none of the individual

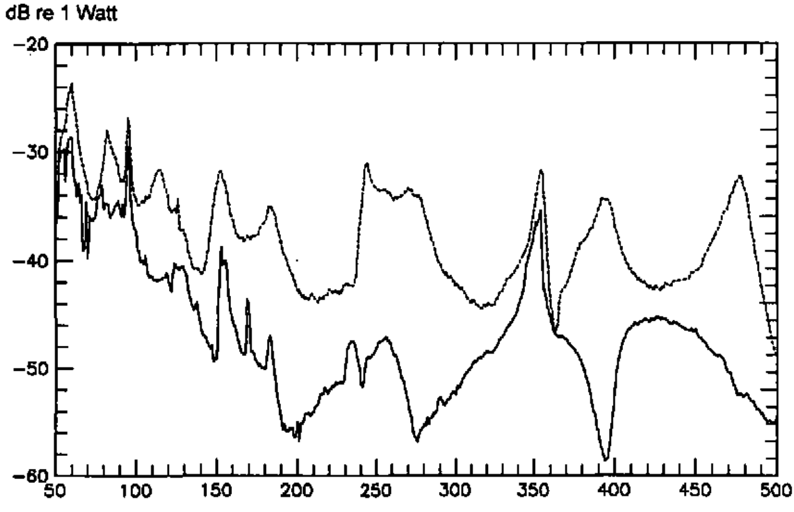

Fig. 2. *Measured averaged total supplied source power. The dashed line represents the power supplied before control, the full line the power supplied after 58 seconds of control.*

secondary sources could control the structure alone. By interchanging the primary source and the leftmost secondary source, the power supplied at 363 Hz could be reduced 7 dB. Thus, the efficiency of the control strategy depends on the source geometry as well as the structural properties.

5. CONCLUSION

A practical study of the power minimising adaptive algorithm has shown its applicability in controlling structural vibration over a wide range of sinusoidal excitation frequencies. Though the requirements to the precision in measurements are more demanding with the power method, the algorithm turns out to be as robust as least mean squares methods.

6. ACKNOWLEDGMENT

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7. REFERENCES

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