inter•noise 96
25th Anniversary Congress – Liverpool

INCE: 47.3

ACOUSTIC PROPAGATION IN A SEMI-INFINITE CYLINDER TREATED WITH ISOTROPIC HOMOGENEOUS POROUS MATERIAL

E Lesquins (1) & D Lafarge (2)

(1) Snecma Villaroche, France, (2) Université du Maine, Le Mans, France

1. INTRODUCTION

The specific consumption decrease by increasing the by-pass ratio and reducing the nacelle, will involve the development of civil turbofan engines with an important low-frequency content. On account of the bulk, locally reacting acoustic treatments used in aeronautics will be associated with non-locally reacting porous materials more efficient. There has been much studies of sound transmission in ducts with locally reacting walls and bulk reacting liners [1] [2] [3]. In recent years, a simple physical modelling of sound propagation in porous media has been developed [4] [5] [6]. This work deals with an application of this new description of sound propagation in porous materials to the problem of sound transmission in ducts with bulk reacting liners.

2. THEORETICAL BACKGROUND

Rigid porous materials are two-phases air-solid media, characterized by 6 parameters, which are the porosity ϕ (air volume fraction), the tortuosity α_∞ (related to the relative inertia increase for inviscid incompressible flow through the porous network), the viscous characteristic dimension Λ (first defined in a precise manner by Johnson and al.[4], it characterises viscous losses in a high frequency limit), the thermal characteristic length Λ' (defined in analogy with Λ [5], it describes thermal losses in a high frequency limit), the flow resistivity σ (which describes viscous losses in the low frequency limit, $\sigma = \eta / k_0$ where η is the air viscosity, and k_0 the Darcy permeability), and a last parameter k_0' defined in analogy with k_0 [6], which governs thermal exchanges in a low frequency limit. The material is assimilated with an equivalent fluid with density $\tilde{\rho}(\omega)$ and compressibility $\tilde{K}(\omega)$, taking into account the frequency dependent inertial/viscous and thermal effects. As discussed by Johnson and al. [4], a simple scaling function provides a reasonably accurate description of the frequency dependent tortuosity $\tilde{\alpha}(\omega)$:

$$\tilde{\alpha}(\omega)=\frac{\tilde{\rho}(\omega)}{\rho_0}=\alpha_\infty\left[1+\frac{1}{j\tilde{\omega}}\left(1+j\frac{M}{2}\tilde{\omega}\right)^{\frac{1}{2}}\right] \qquad (1)$$

where the reduced pulsation $\tilde{\omega}$ is given by: $\tilde{\omega}=\omega\frac{\eta\alpha_\infty k_0}{\rho_0\phi}$

and M is a geometrical form factor: $M=\frac{8\alpha_\infty k_0}{\phi\Lambda^2}$.

A description of the frequency dependent $\tilde{K}(\omega)$ in terms of a scaling function similar to (1) has recently been obtained [6] :

$$\tilde{K}(\omega)=\frac{K_0}{\gamma-(\gamma-1)/\tilde{\alpha}'(\omega)} \quad \text{and} \quad \tilde{\alpha}'(\omega)=1+\frac{1}{j\tilde{\omega}'}\left(1+j\frac{M'}{2}\tilde{\omega}'\right)^{\frac{1}{2}}$$

where : $\tilde{\omega}'=\omega\frac{Pr\,\rho_0 k_0'}{\eta\phi}$, $M'=\frac{8k_0'}{\phi\Lambda'^2}$

with γ specific heats ratio and P_r Prandtl number.

3. APPLICATION

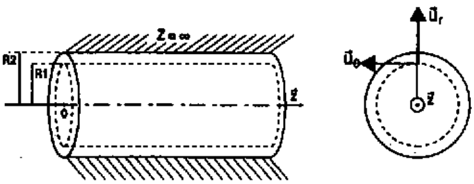


Figure 1: The cylindrical duct geometry

The equivalent fluid model is applied to an isotropic homogeneous rigid porous material set on the wall of a semi-infinite cylinder with radius R_1 and axis $0\vec{z}$ (Fig.1). An acoustic field propagates along the duct, being partially reflected on the treatment and transmitted in a compression wave in the porous material.

Viscous and thermal losses in the duct being negligible, the solutions of the wave equation $\Delta\varphi+k^2\varphi=0$ with Δ the cylindrical laplacian and k the wave number in the duct, are:

$$\varphi(r,\theta,z)=\sum_m A_m J_m\left(k_{r_m}.r\right)e^{jm\theta}e^{-jk_{z_m}z} \qquad (2)$$

where φ is the velocity potential $\vec{v}=\vec{\nabla}\varphi$. The wave vector components k_{r_m} and k_{z_m} verify $k^2=k_{r_m}^2+k_{z_m}^2$.

The propagation in the porous material is described by the wave equation:

$$\Delta\varphi^p + k^{p^2}\varphi^p = 0$$

where k^p is the wave number in the treatment: $k^p = \frac{\omega}{c_p} = \omega\sqrt{\frac{\tilde{\rho}(\omega)}{\tilde{K}(\omega)}}$

The wave equation's solutions are:

$$\varphi^p(r,\theta,z) = \sum_m \left[B_m H_m^+\left(k^p_{r_m}.r\right) + C_m H_m^-\left(k^p_{r_m}.r\right)\right] e^{jm\theta} e^{-jk_{z_m}z} \qquad (3)$$

where φ^p is the velocity potential $\vec{v}^p = \vec{\nabla}\varphi^p$. The wave vector components $k^p{}_{r_m}$ and $k^p{}_{z_m}$ verify $k^p{}_{z_m} = k_{z_m}$ (Snell-Descartes) and $k^{p^2} = k^{p^2}_{r_m} + k^{p^2}_{z_m}$

The continuity conditions, in $r = R_1$, are:

- the pressure continuity : $p(R_1) = p^p(R_1)$ (4)
- the flow continuity : $v_r(R_1) = \phi v^p{}_r(R_1)$ (5)

On the duct wall, in $r = R_2$, the velocity normal component is equal to 0:

$$v^p_r(R_2) = 0 \qquad (6)$$

The equations (2) to (6) can be rewritten: $\frac{p_m(R_1)}{v_{r_m}(R_1)} = Z_m\left(k^p_{r_m}\right)$

where Z_m is a modal impedance:

$$Z_m\left(k^p_{r_m}\right) = -j\frac{\omega\tilde{\rho}(\omega)}{\phi\rho_0 c_0}\frac{1}{k^p_{r_m}}\frac{\left[J_m\left(k^p_{r_m}R_1\right)N'_m\left(k^p_{r_m}R_2\right) - N_m\left(k^p_{r_m}R_1\right)J'_m\left(k^p_{r_m}R_2\right)\right]}{\left[J'_m\left(k^p_{r_m}R_1\right)N'_m\left(k^p_{r_m}R_2\right) - N'_m\left(k^p_{r_m}R_1\right)J'_m\left(k^p_{r_m}R_2\right)\right]}$$

Considering the wave propagation in the treatment, the surface impedance p_m / v_{r_m} depends on radial and azymutal order of the mode taken into consideration. The propagation modes' radial components $k_{r_{mn}}$ are computed numerically with Runge-Kutta method, by solving the eigenvalues equation:

$$k_{r_m} J'_m\left(k_{r_m}R_1\right) + j\frac{k}{Z_m} J_m\left(k_{r_m}R_1\right) = 0$$

Considering the equal energy distribution and the modal non-coupling, the total attenuation per unit length is written:

$$A_o = -10\log_{10}\left\{\sum_{mn} e^{2\mathrm{Im}(k_{z_{mn}})} / \sum_{mn} 1\right\}$$

4. RESULTS

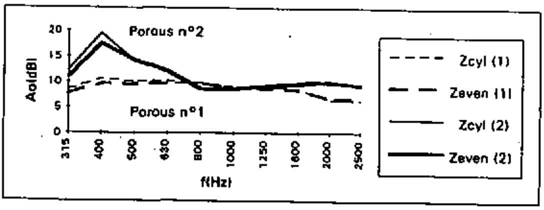
	ϕ	α_∞	σ	Λ	Λ'
Porous n°1	1	1	2300	1.0E-03	2.1E-03
Porous n°2	0.98	1.1	7000	1.7E-04	4.0E-04

Table 1:Porous materials' parameters

A first validation can be obtained by comparing two attenuation spectra in the frequency range covering the third-octave bands from 315Hz to 2.5 KHz, one computed with the above impedance model Z_{cyl} (cylindrical geometry) and the other computed with an impedance model Z_{even} of the same porous material treated as a plane layer of depth $d = R_1 - R_2$:

$$Z_{even} = -j\frac{\omega\tilde{\rho}(\omega)}{\phi\rho_0 c_0}\frac{1}{k^P_{r_m}}\cot g\left(k^P_{r_m} d\right)$$

For high enough duct radius R_1, $Z_{cyl} \rightarrow Z_{even}$. This is checked (Fig.2) for 2 porous materials, which characteristic parameters are indicated in Tab.1.



$R_1 = 0.5\text{m} \quad d = 0.1\text{m}$

Figure 2: Attenuation Spectra

5. CONCLUSION

The impedance model Z_{cyl}, and the computation tool, give correct results. The attenuation values will be improved by changing the porous material parameters, and by coupling it to another systems, like a perforated screen and/or an air-gap. Moreover, the Biot material's impedance model with cylindrical geometry has to be studied.

References

[1]P.M.Morse,'The transmission of sound inside pipes', J.Acoust.Soc.Am., **11**, 205-210 (1939)

[2]R.A.Scott,'The propagation of sound between walls of porous material', Proc.Phys.Soc., **58**, 358-368 (1946)

[3]A.Cummings,'Sound attenuation in ducts lined on two opposite walls with porous material, with some applications to splitters', J.Sound Vib., **49**, 9-35 (1976)

[4]D.L.Johnson, J.Koplik, R.Dashen,'Theory of dynamic permeability and tortuosity in fluid saturated porous media',J.Fluid Mech., **176**, 379-402 (1987)

[5]J.FAllard,'Propagation of sound in porous media'(Elsevier App.Sc., 1993)

[6]D.Lafarge, P.Lemarinier, J.F.Allard, V.Tarnow,'Dynamic Compressibility of air in porous structures at audible frequencies', Sub. to J.A.S.A. (1996)