

## ELASTIC CONSTANTS OF POLYURETHANE FOAM'S SKELETON FOR BIOT MODEL

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### 1. INTRODUCTION

Acoustic propagation in polyurethane foams can be predicted with phenomenological model of Biot theory [1]. For materials having elastic frame, this model needs several parameters which characterise the fluid, the matrix, the skeleton and take into account of interface effects between fluid and skeleton [2,3]. Among these parameters, mechanical properties of skeleton's foams in the case of isotropic media as Young's modulus or Poisson's ratio are studied in this paper. A quasistatic experimental method and a mechanical inversion method are presented. Measurements on polyurethane open cells foam having a very low resistivity of air flow are set out from 1 Hz to 100 Hz.

### 2. VISCOELASTIC BEHAVIOR

Polyurethane can be considered as linear solid with memory for very low strain rate [4]. So generalized standard model of viscoelasticity [5] can be used. In the Fourier domain, Young's modulus  $E$  and Poisson's ratio  $\nu$  become complex functions of frequency (noted  $f$ ). They can be written

$$\begin{cases} E^*(\omega) = E'(\omega) + jE''(\omega) & \text{where } j = (-1)^{1/2} \\ \nu^*(\omega) = \nu'(\omega) + j\nu''(\omega) & \text{and } \omega = 2\pi f \end{cases}$$

Making of use of correspondance rule between elastic problems and viscoelasticity [6], prediction of frequency response can be obtained from complex functions  $E^*(\omega)$  and  $\nu^*(\omega)$  in the theory of Biot. They are defined by measurement in the frequency domain (e.i. with harmonic excitation) presented later.

### 3. EXPERIMENTAL METHOD

Experimental method include measurement of mechanical impedance and transfert fonction. Measurements are made at low frequency below any resonance .So Inertial effects of frame and the air can be neglected.

#### Foams' samples and devices

Experimental setup is shown in Fig. 1. Cubic samples of  $50 \times 50 \times 50 \text{ mm}^3$  dimensions are commonly used. In this way, first resonance of specimen has frequency much higher than 100 Hz and precise machining can still be done. This shape lets measurements possible in three orthogonal directions in order to check isotropy and investigate anisotropic foams. Cubic sample is placed between two much more rigid plates. It is not glued but lightly prestressed. The lower plate is excited by dynamic displacement  $D_1(\omega) = d_1 e^{j\omega t}$ . On the upper fixed plate a piezoelectric sensor measure dynamic force  $F(\omega)$  transmitted by specimen. At each center cube's face are fixed targets moving with foam. Therefore, laser vibrometer can measure horizontal displacement  $D_2(\omega)$  of these points.

Dual-channel FFT analyzer is used to control source excitation of vibration exciter via a power amplifier. Displacement signal  $D_1(\omega)$  is received on first channel and source level is adjust to obtain imposed amplitude  $d_1$ . In this manner, strain rate stay independent of frequency and can be adjusted to verify linear assumption. Second channel receive either force signal  $F(\omega)$  or displacement signal  $D_2(\omega)$ . So, two frequency responses are measured : A mechanical impedance response and a transfert function of displacements respectively

$$K(\omega) = \frac{F(\omega)}{D_1(\omega)} \quad \text{and} \quad T(\omega) = \frac{D_2(\omega)}{D_1(\omega)} \quad (1) \text{ and } (2)$$

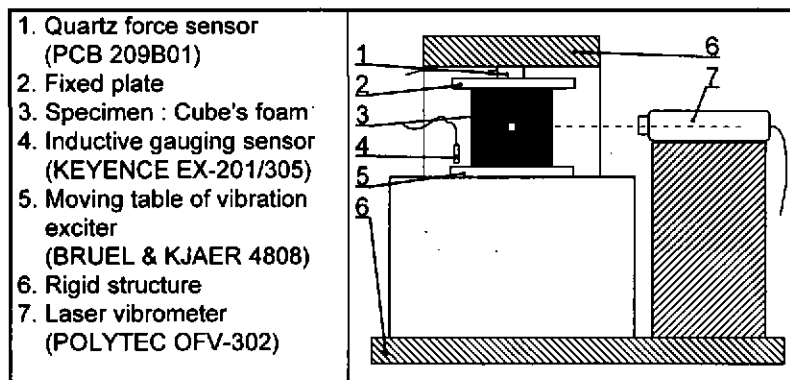


Fig. 1. : Schematic diagram of experimental configuration for measurements of viscoelastic properties of foams' skeleton.

### Measurements and calibrations

K and T are frequency responses functions of foam's skeleton but also of measurement apparatus. In order to reduce effects extraneous to foam's behavior, two calibrations are needed. On the one hand,  $K_s(\omega)$  measure stiffness of a spring which first frequency resonance is 400 Hz. Therefore idealized stiffness can be taken with good approximation as a pure real number noted  $K_0$ . Its value is measured on tensile machine. On the other hand,  $T_s(\omega)$  denotes transfert function T measured at the same point. Hence its idealized value is 1. Calibrating K and T with  $K_s(\omega)$ ,  $K_0$  and  $T_s(\omega)$ , new functions

$$K_c(\omega) = \frac{F(\omega)}{D_1(\omega)} \cdot \frac{K_0}{K_s(\omega)} \quad \text{and} \quad T_c(\omega) = \frac{D_2(\omega)}{D_1(\omega)} \cdot \frac{1}{T_s(\omega)} \quad (3) \text{ and } (4)$$

are obtained. Both functions are only fonction of foam's behavior with a good approximation.

### 4. NUMERICAL INVERSION

Finite element method (f.e.m.) has been chosen to obtain static approximations of  $K_c(0)$  and  $T_c(0)$  in function of E and  $\nu$ . A 3D model of experiment described in part 3 is used on PERMAS code. This model consist of a cube fixed between two rigid plates. The cube is made of elastic continuum solid caraterised by E and  $\nu$ . Plates are moved close together with static displacement  $D_1(0)$ . After simulations, static force applied on a plate by cube ( $F(0)$ ) and center displacement of side face ( $D_2(0)$ ) are given as functions of E and  $\nu$ . Hence, static functions K(0) and T(0) for different values of  $\nu$  and E are derived. If L is the edge-lenght of cubic sample, these functions are

$$K(0) = \frac{F(0)}{D_1(0)} = \frac{EL(1-\nu)}{h(\nu)} \quad \text{and} \quad T(0) = \frac{D_2(0)}{D_1(0)} = g(\nu), \quad (5) \text{ and } (6)$$

where h and g are functions fitted on f.e.m. results. It is noted that T(0) is independent of E whereas K(0) is proportional to it. For quasi-static strain, these functions are supposed independent of frequency. So, replacing (E,  $\nu$ ) with ( $E^*(\omega)$ ,  $\nu^*(\omega)$ ) in (5,6), respecting (3,4), functions  $K_c(\omega)$  and  $T_c(\omega)$  are written

$$K_c(\omega) = \frac{E^*(\omega)L(1-\nu^*(\omega))}{h(\nu^*(\omega))} \quad \text{and} \quad T_c(\omega) = g(\nu^*(\omega)). \quad (5) \text{ and } (6)$$

Hence,  $E^*(\omega)$  and  $\nu^*(\omega)$  are infered from equations (5) and (6).

## 5. RESULTS

Industrial open cell foams from Recticel compagny were tested. Fig. 2. presents an example of  $(E^*(\omega), \nu^*(\omega))$  functions whereas Table 1. gives the viscoelastic constants at 10 Hz for three foams made of identical polyurethane with different cell's size. The air did not affect measurements because of low resistivity of air flow which was lower than  $20000 \text{ Nm}^{-4}\text{s}$ .

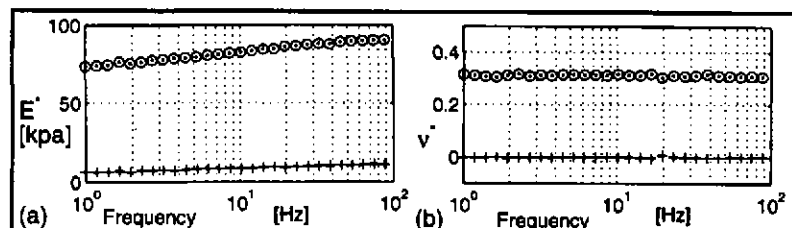


Fig. 2. : Viscoelastic properties of BulprenS60 foam : Young's modulus (a) and Poisson's ratio (b).  $\circ$  denotes real parts,  $+$  denotes imaginary parts.

Ref. of foams (Recticel Cie)	Porosity	Pore per inch	Young's modulus $E^*$ [kPa]	Poisson's ratio $\nu^*$
Bulpren S 45	0.97	45	$140.1 + 13.4j$	0.32
Bulpren S 60	0.97	60	$82.4 + 8.6j$	0.32
Bulpren S 90	0.97	90	$106.0 + 9.9j$	0.32

Table 1. : Viscoelastic properties of foams' skeleton having different cell's size. Typical results at 10 Hz.

## 6. CONCLUSION

Experimental determination of viscoelastic properties of isotropic open cell foams are obtained at low frequency. From 1 Hz to 100Hz, a weak dependence with frequency of complex functions  $E^*(\omega)$  is observed. The measured parameter  $\nu^*(\omega)$  is a frequency independent real function.

## References

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