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APPLICATION OF THE DUAL RECIPROCITY METHOD TO ACOUSTIC RADIATION IN AN INHOMOGENEOUS MEDIUM

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1.INTRODUCTION

This paper addresses the problem of modelling the behaviour of linear wavelike disturbances -governed in the frequency domain by a modified Helmholtz equation- as they propagate through a region of local inhomogeneity. When dealing with a domain of infinite extent, numerical models such as the wave envelope method or infinite elements [1,2] give good results. Our purpose is to propose a different method to solve this problem using the DRBEM (Dual Reciprocity Boundary Element Method) [see ref.3,4].

2.MATHEMATICAL FORMULATION

Consider a general inner problem in which a boundary surface S encloses a finite acoustic domain Ω filled with an inhomogeneous medium. The acoustical pressure $p^*(x,t)$ must satisfy the inhomogeneous wave equation.

$$\nabla.(c^2\nabla p^*) - \frac{\partial^2 p^*}{\partial t^2} = 0 \qquad (1)$$

where c is the local adiabatic sound speed. Assuming a perfect gas with a constant ratio of specific heats, we may assume that the sound speed is function of temperature only and may be written $c^2 = \gamma T$. If time harmonic solutions of the form $p^*(x,t) = p(x)e^{i\omega t}$ are then sought, the equation (1) can be written as

$$\Delta p + k_0^2 p = (k_0^2 - k^2)p - \frac{\nabla T}{T}.\nabla p \qquad (2)$$

where $k_0 = \omega / c_0$, c_0 is a reference value of sound speed and $k = \omega / c$, in which ω is the angular frequency.

By using the direct formulation, via either Green's second identity or the weighted residual formulation, equation (2) can be reformulated into a boundary integral equation as follows

$$C(P)p(P) + \int_S \left(p(Q)\frac{\partial G}{\partial n} - \frac{\partial p(Q)}{\partial n} G \right) dS(Q) = \int_\Omega \left[(k_0^2 - k^2)p - \frac{\nabla T}{T}.\nabla p \right] G d\Omega(Q) \qquad (3)$$

where G is the free-space Green's function due to a time-harmonic point source at P i.e.

$$G = \frac{i}{4} H_0^{(1)}(k_0.r) \quad \text{in 2D,} \qquad G = \frac{e^{-ik_0.r}}{4\pi r} \quad \text{in 3D,} \qquad r = \|P - Q\|$$

and C(P) is a geometric parameter, which depends on the location of the source point P with respect to S [3].

The right-hand side of (3) is now dealt with by DRBEM. Following Partridge, Brebbia and Wrobel [4], we can expand the function on the r.h.s. of (3) as

$$\left[(k_0^2 - k^2)p - \frac{\nabla T}{T}.\nabla p \right] = \sum_{j=1}^{N+L} \alpha_j F_j \qquad (4)$$

where N is the number of collocation points placed on the boundary and L is the number of internal collocation points placed inside Ω. F_j is a function associated with the point j and the α_j are initially unknown coefficients. It has been shown [4] that the best results are obtained if a comparatively simple form of F_j is used ,i.e., F_j is composed of the power series of distance functions.

In the bidimensional case, we have chosen

$$F_j = k_0.a - r - k_0^2.\frac{r^3}{9} \qquad (5)$$

where a is a characteristic length of the domain Ω. The particular solution can be obtained as follows

$$\Delta\hat{\phi}_j + k_0^2\hat{\phi}_j = F_j \qquad (6)$$

and hence $\hat{\phi}_j$ has the form

$$\hat{\phi}_j = \frac{a}{k_0} - \frac{r^3}{9} \qquad (7)$$

Applying the boundary element technique, the r.h.s. of (3) may be reformulated as follows

$$\int_{\Omega}\left[(k_0^2-k^2)p-\frac{\nabla T}{T}.\nabla p\right]G d\Omega=\sum_{j=1}^{N+L}\alpha_j\left(C(P)\hat{\phi}_j(P)+\int_S\left(\hat{\phi}_j\frac{\partial G}{\partial n}-\frac{\partial\hat{\phi}_j}{\partial n}G\right)dS\right) \quad (8)$$

After discretizing the boundary S and applying the standard DRBEM procedures, a linear system of algebraic equations is obtained as

$$Hp-G\frac{\partial p}{\partial n}=\left(H\hat{\phi}_j-G\frac{\partial\hat{\phi}_j}{\partial n}\right)F^{-1}\left[K+T^{-1}\left(T_xF_x+T_yF_y\right)F^{-1}\right]p \quad (9)$$

where H and G are of the usual meaning, p is an (N+L) vector, the first N entries of which are the nodal values defined on S , and the next L entries of which are the nodal values defined on the internal collocation points. The definitions of other matrices such $F^{-1}, K, T^{-1}, T_x, T_y, F_x, F_y$ can be worked out if the standard DRBEM procedure is followed and are therefore omitted here.

3.RESULTS AND DISCUSSION

The numerical example presented here deals with the acoustic radiation in a heated 2D cavity (1 m. x 0.3 m.) opening in an infinite baffle. The rectangular cavity (see figures 1&2) is provided with a driving piston (frequency 300 Hz) at x=0 and the end x=1 (surface S_0) flush with a rigid plane wall. The lower and upper sides are rigid. The temperature field has the form $T(y)=1000y+300, (0\le y\le 0.3)$ and the external temperature is 300 K. For that particular case, we have compared the DRBEM with the FEM [5]. The boundary conditions at S_0 are

- for the DRBEM : $p=-2\int_{S_0}G\frac{\partial p}{\partial n}dS$ (10)

This result is given by the Green's theorem on the semi-infinite space(x>1).

-for the FEM, we have imposed the radiation impedance [6]:

$$Z=\pi a^2\rho c\left(1-\frac{J_1(k_0a)}{k_0a}\right)+j\frac{\pi\rho c}{2k_0^2}K_1(2k_0a) \quad (11)$$

Figures 1&2 show good agreement between the two methods.The main difference is that the radiation impedance imposes a plane wave propagation at x=1 and doesn't allow us to observe the edge fringing that appears with the DRBEM. The advantage of the DRBEM is that the boundary condition (10) is quite easy to implement, the coefficients of the impedance matrix having already been calculated.

4. CONCLUSIONS

We presented here the first results obtained using the DRBEM to acoustic problem in an inhomogeneous medium.The method is easy to use in an infinite medium and results have been obtained for a heated cavity opening in an infinite baffle and the sound radiation from an unflanged, heated pipe through a thermal jet (not presented here by lack of space). The method seems to be quite promising and is under further development.

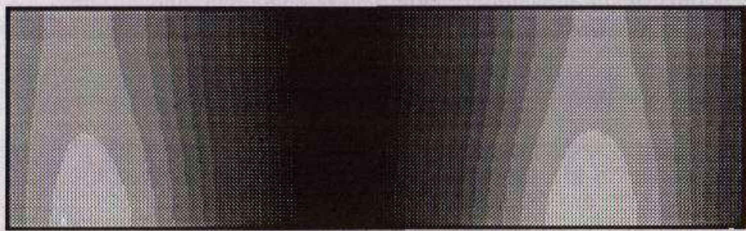

Fig. 1 FEM code

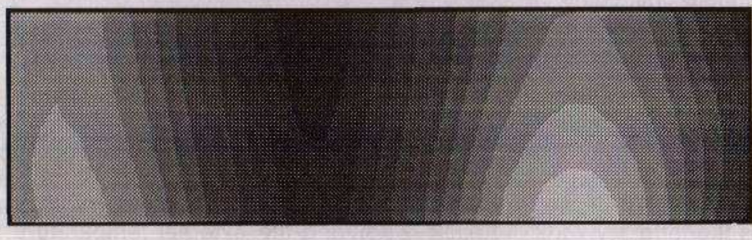

Fig. 2 DRBEM code

Normalized acoustic pressure

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