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# A WAVE APPROACH TO THE SEA OF A COUPLED RECTANGULAR PLATE SYSTEM INVOLVING A NON-UNIFORM BOUNDARY CONDITION

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## 1. INTRODUCTION

Statistical Energy Analysis (SEA) deals with the prediction of vibrational behaviour in complex mechanical systems, usually where an incomplete knowledge of structural detail is involved. Essential features of the method are the description of the system in terms of the gross properties (such as mass, surface area and loss factor) of component subsystems, and the assumption that the system at hand is one member of an ensemble of systems which are similar on the scale of gross features, but which differ in detail. The vibrational response of the system is generally expressed in terms of the ensemble-, space-, time-averaged subsystem energies.

A pivotal step in the calculation of subsystem energies is the estimation of the energy flow between subsystems. This is traditionally achieved by assuming that subsystem modal energies are equipartitioned, that modal responses are incoherent, and that the energy flow between subsystems is proportional to the difference in their modal energies [1]. The validity of these assumptions is often limited, however, and this has recently been the motivation for the development of an exact wave approach [2]. To date, this approach has been used in the SEA of a class of systems involving one or more dynamically one-dimensional wave-guides, which each support only a single wave type. These include systems of coupled beams [2] and coupled rectangular plates [3]. The present investigation extends the generality of the approach to also deal with systems of higher dynamic dimensionality and greater irregularity. General features of energy flow in these more complex systems, which do not appear in strictly uniform, dynamically one-dimensional systems, are demonstrated here for the case of a simple irregular system involving rectangular plates and a non-uniform spring/simple-support boundary condition.

## 2. WAVE COMPONENT DECOMPOSITION OF THE VIBRATION FIELD

In detail, the system comprises two uniform rectangular plates, *a* and *b*, each made of nominally the same material and having the same thickness and width. The plates are simply supported along all edges and coupled

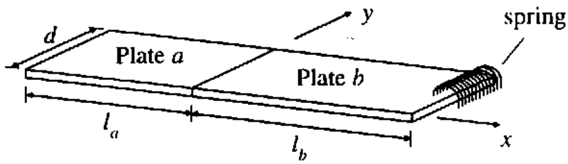


Figure 1. Simply supported plate and spring system

along a shared simply supported edge (see Figure 1). Rotation at the edge of plate *b* farthest from the coupling is partially constrained by a rotational line spring which spans some portion of the plate width. Energy is delivered to the system by time-harmonic 'rain-on-the-roof' forcing of plate *a*.

In the present analysis, the resulting vibration field is decomposed by a Fourier series expansion in the *y*-direction across the plates, into so-called 'wave components', each with a unique sinusoidal variation of amplitude across the plates and trace wavenumber along the coupling of the form $k_{yn} = n\pi / d \quad (n = 1,2,3\ldots)$. If the rotational spring is removed from the system, each wave component propagates independently in the plates, and interacts with the coupling and plate ends to produce only scattered components which have the same trace wavenumber as the incident component. The system then behaves as a number of independent dynamically one-dimensional wave-guides, and the total response is the sum of individual wave-guide responses [3]. With the rotational spring in place, however, wave component energy incident on the plate edge constrained by the spring is scattered into all other possible components, individual wave-guide responses are no longer independent, and the irregularity and the dynamic dimensionality of the system are increased.

Wave components propagate in the *x*-direction with wavenumber $k_{xn} = \sqrt{k^2 - k_{yn}^2}$, where *k* is the free bending wave wavenumber. (The small effect of near field contributions and components for which $k_{yn} > k$ are ignored here.) Dissipation of wave component energy is described in terms of the factor, $\exp(-\mu_n / 2)$, by which the amplitude of the wave component is reduced as it propagates over the length *l* of the plate. The attenuation parameter $\mu_n = \mu_0 / \sqrt{1-(k_{yn} / k)^2}$, where $\mu_0 = k\, l\eta / 2$, and $\eta$ is the small loss factor of the plate. Wave components with large trace wavenumber are thus relatively more strongly attenuated.

Overall wave component interaction with each plate is characterised by a reflection matrix $\rho$, defined so that components which enter the plate at the coupling with amplitudes $\mathbf{a}$, propagate to the opposite edge of the plate and return to the coupling with amplitudes $\rho\,\mathbf{a}$. Since the properties of plate *a* do not vary over its length, the matrix is diagonal with $|\rho_{a,nn}| = \exp(-\mu_{nn})$. The presence of the spring attached to plate *b*, on the other hand, introduces non-zero off-diagonal elements into the plate *b* matrix, with $|\rho_{b,mn}| = \exp(-(\mu_m + \mu_n)/2)\ |\rho_{spr,mn}|$, where $\rho_{spr}$ is the reflection matrix of the spring/simple-support termination. Maximum general scattering of wave component energy into other components occurs for stiff springs which span approximately half the width of the plate, and wave components with small trace wavenumber are more strongly scattered than those with large trace wavenumber.

Wave component transmission at the simple-support coupling between plates is described in terms of a diagonal transmission matrix, $\mathbf{t}$, and a corresponding reflection matrix, $\mathbf{r}$. The diagonal elements of $\mathbf{t}$ have magnitude $|t_{nn}| = \sqrt{(1-(k_{yn}/k)^2/2)}$.

## 3. THE ENSEMBLE OF PLATE SYSTEMS

The coupling power between the plates may be written as

$$P_{ab} = \tfrac{1}{2}\ \mathbf{e}^H\,\mathbf{t}^H\,\mathbf{Q}^H\,[\mathbf{I} - \rho_b^H\rho_b]\,\mathbf{Q}\,\mathbf{t}\,\mathbf{e}, \tag{1}$$

where

$$\mathbf{Q} = \left[\mathbf{t}\,(\mathbf{I} - \rho_a\mathbf{r})\mathbf{t}^{-1}\,(\mathbf{I} - \mathbf{r}\,\rho_b) - \mathbf{t}\,\rho_a\mathbf{t}\,\rho_b\right]^{-1}$$

Here, $\mathbf{e}$ is the vector of 'excited' wave component amplitudes [2], defined so that the direct power incident at the coupling (i.e. $P_{ab}$ in the limit as $\mathbf{r} \to \mathbf{0}$ and $\rho_b \to \mathbf{0}$) is given by $P_{dir} = \mathbf{e}^H\mathbf{e}/2$. The symbol $^H$ denotes the Hermitian transpose. The dynamic properties of the plate system are fully represented in equation (1) by the various reflection and transmission matrices of the plates and the coupling. The sensitivity of the coupling power to the details of geometric and other properties of the system arises overwhelmingly from the sensitivity of the *phases* of the matrix elements to these details. Since only the phases taken modulus $2\pi$ are relevant in equation (1), it is assumed that a realistic ensemble of plate systems can be achieved by taking the magnitudes of the matrix elements (and $P_{dir}$) to be constant over the ensemble, while taking their phases to be random quantities, uniformly distributed in the interval $\{-\pi,\pi\}$.

An example of this ensemble averaged coupling power is shown in Figure 2, for systems comprising two *nominally* identical plates of normalised width $kd = 20.7\pi$ and a rotational spring which spans half this width. Plates in these systems share the same attenuation parameter, $\mu_0$, but may differ in detail. The coupling power is shown for springs of various non-dimensional stiffness, $K$ (the rotational stiffness divided by $kD$, where $D$ is the bending stiffness of the plate).

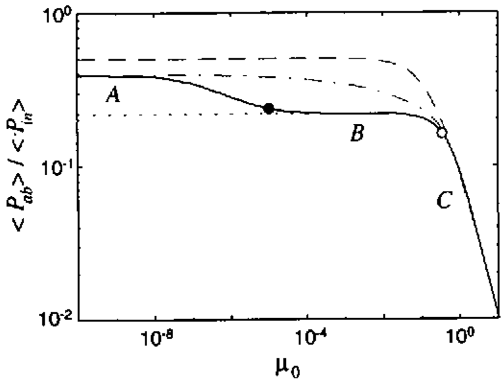


Figure 2. Ensemble averaged coupling power as a function of $\mu_0$. ———, $K=10^{-2}$; - - - - -, $K=10^{-5}$; —·—·—, $K=10^{5}$; — — —, traditional SEA estimate of coupling power; o, $\gamma_0 = 1$; •, $(1-|\rho_{spr,0}|^2)/\mu_0 = 1$.

Three distinct regimes of ensemble averaged energy flow may be identified in these systems, which are marked *A*, *B* and *C* in Figure 2 for the example of systems involving springs of non-dimensional stiffness $K=10^{-2}$. Each regime is associated with either weak or strong coupling between wave components of like trace wavenumber in the two plates, and with either weak or strong influence of scattering of energy between wave components of different trace wavenumber.

## 4. COUPLING STRENGTH

Coupling between the two plates is strong for a given wave component if the amount of energy transmitted to the other plate when incident at the coupling, is significantly greater than that dissipated by the component in a single passage over the length of the plate. The converse is true of weak coupling. An *overall* strength of coupling parameter has previously been defined [2,3] which is given, to good accuracy, by $\gamma_0 \approx |t_0| / \sqrt{\mu_{a0}\mu_{b0}}$. The condition $\gamma_0 << 1$ identifies weak coupling between the plates for all trace wavenumbers, and corresponds to region *C* in Figure 2. For systems in this region, almost all input power is dissipated in plate *a*. The coupling power and the dynamic behaviour of plate *a* are essentially independent of the properties of plate *b* (including the spring/simple-support termination), and the coupling power is inversely proportional to $\mu_{a0}$.

Strong coupling arises when $\gamma_0 >> 1$, and corresponds to regions *A* and *B* in Figure 2. If the attenuation parameters, $\mu_{a0}$ and $\mu_{b0}$, are of the same order of magnitude, the powers dissipated in each of the plates are comparable. In region *B*, the coupling power is proportional to $\sqrt{\mu_{b0} / \mu_{a0}}$.

## 5. ENERGY SCATTERING BETWEEN WAVE COMPONENTS

The effects of scattering of energy between wave components of different trace wavenumber are significant in the response of the system, only when the energy transferred between components as they are reflected at the spring/simple-support termination is generally greater than that lost to damping in traversal over the length of the plate. Systems of this kind satisfy $(1-|\rho_{spr,0}|^2)/\mu_{b0} >> 1$, where $\rho_{spr,0}$ is the smallest diagonal element of $\rho_{spr}$, and correspond to region *A* in Figure 2. In the converse situation where $(1-|\rho_{spr,0}|^2)/\mu_{b0} << 1$ (regions *B* and *C*), scattering effects are negligible, and the system is effectively a uniform system of the kind described in reference [3].

The increase in coupling power indicated in Figure 2 when the effect of scattering becomes large may be attributed to the greater scattering of power from small to large trace wavenumbers than vice versa, and to the more rapid dissipation with distance of wave components with large trace wavenumber. The magnitude of the asymptotic coupling power in region *A* is independent of the stiffness or the length of the spring, with variations in the non-uniformity of the termination due to changes in these parameters affecting only the coupling power at the transition between regions *A* and *B*, and the value of $\mu_{b0}$ which marks the transition. For very non-uniform terminations (those for which the magnitudes of off-diagonal elements of $\rho_{spr}$ are large), there may be no region *B*.

## 6. COUPLING LOSS FACTOR

Expressions of coupling power in SEA traditionally involve a coupling loss factor, $\eta_{ab}$, assumed to be a property of the system and independent of the details of excitation. For the present system, it is found that

$$\frac{\eta_{ab}}{\eta_{ab}^{trad}} = \frac{\pi \mu_{a0} / \overline{T^2}}{\dfrac{\langle P_{in} \rangle}{\langle P_{ab} \rangle} - \left(1 + \dfrac{\mu_{a0}}{\mu_{b0}}\right)}, \tag{2}$$

where $\overline{T^2}$ is the diffuse field transmission coefficient of the coupling and $\eta_{ab}^{trad} = 2\overline{T^2} / \pi k l_a$ is the traditional SEA estimate of coupling loss factor [1]. The coupling loss factor is independent of the details of plate *b* when the coupling is weak (see Figure 3), and independent of the details of the spring/simple support termination when the effects of scattering are strong, i.e. $\mu_{b0} << 1 - |\rho_{spr,0}|^2$.

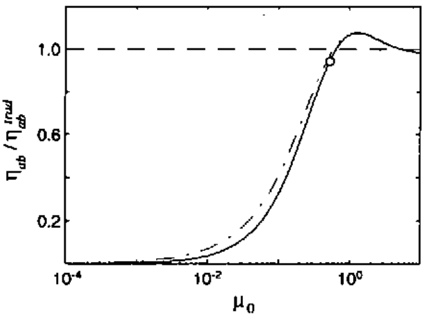


Figure 3. Coupling loss factor as a function of $\mu_0$. Key as for Figure 2.

## 6. CONCLUDING REMARKS

A wave based method, previously used in the SEA of uniform systems [2,3], has been extended for the analysis of a simple, non-uniform system. Three distinct regimes of ensemble average energy flow have been identified. One of these corresponds to weak coupling, for which there is little dependence on the boundary conditions of the system, and for which conventional SEA response estimates are accurate. Remaining regimes correspond to strong coupling, with either strong or weak effects of scattering of energy between wave components. As the damping and the lengths of the plates are reduced, the coupling strength between plates and the effects of scattering between wave components increase. Energy reflected from plate boundaries then becomes increasingly important, interference between coherent wave components arriving at the two sides of the coupling increases, and the ensemble averaged coupling power is smaller than that predicted by conventional SEA. The conventional estimate is least accurate when the effects of scattering between wave components are weak and the system is effectively uniform, but becomes more accurate as the effects of scattering increase and the coherence between components at the coupling is reduced. Stronger scattering in the receiving plate tends to increase the dissipation of power there, and to favour an equipartitioned distribution of plate energy.

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