

SOUND PROPAGATION OVER A BARRIER COMPUTED WITH THE PARABOLIC EQUATION METHOD

E M Salomons

TNO Institute of Applied Physics, PO Box 155, 2600 AD, Delft, The Netherlands

1. Introduction

Sound barriers are widely used for noise reduction in the open air. Accurate computational models for sound propagation in outdoor situations with barriers are therefore of great interest. Atmospheric refraction should be taken into account in these models, as meteorological effects on sound propagation are large.

The parabolic equation method (PE method) [1,2] is such a model. The PE method yields the sound pressure field of a monopole source in a refracting atmosphere above a finite-impedance ground plane. It is possible to include a rectangular barrier on the ground plane [3].

This paper is organized as follows. First a brief description of the PE method for situations with barriers is given. Next, numerical examples are given for situations with a non-refracting atmosphere. By comparison with analytic diffraction solutions, the accuracy of the PE method for propagation over a barrier is demonstrated. Finally, examples are given for situations with a refracting atmosphere.

2. The PE method

The PE method yields the sound pressure field of a harmonic monopole source in an inhomogeneous atmosphere above a ground surface. The inhomogeneous atmosphere is represented by a profile of the (effective) sound speed [4], including windspeed and temperature profiles. The ground surface is represented by a complex acoustical impedance.

The PE method is based on a numerical integration of a one-way wave equation, which properly accounts for waves propagating in the direction from the source to the receiver, but backscattered waves are neglected. The model is valid up to elevation angles of the order of 45°.

but this is not a serious problem as higher angles are usually not of interest in outdoor sound propagation.

The sound field computation is performed in two dimensions, in the vertical rz plane through the source and the receiver (see Fig. 1). This simplification is based on the assumption that the sound field of the monopole has rotational symmetry around the z axis. Then the three-dimensional wave equation can be replaced by a two-dimensional wave equation [5]. The assumption of rotational symmetry is equivalent with the assumption that the variation of the sound pressure with the azimuthal angle α in Fig. 1 can be neglected.

Use is made of a two-dimensional grid (see Fig. 2). At the top of the grid an absorbing layer is used, to eliminate spurious reflections from the top surface.

The sound pressure p as a function of range r and height z is denoted by the function $p(r,z)$. This function is extrapolated stepwise in positive r -direction: $p(r,z) \rightarrow p(r+\Delta r,z)$. For the first step, a starting function $p(r_0,z)$ near the source is used. It is interesting to note that the starting function may be generated by any accurate sound propagation model. For situations with complex noise barriers near the source, one may use a sound diffraction model (e.g., based on the boundary element method, or the geometrical theory of diffraction) for computing a starting function at a range r_0 behind the noise barriers. In this way, the complex barriers are excluded from the PE computation. In this paper, however, we include barriers in the PE computation.

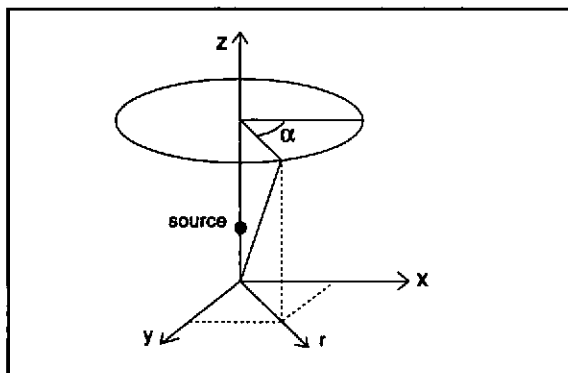


Fig. 1. Cylindrical $rz\alpha$ coordinate system, and rectangular xyz coordinate system.

The method to include a rectangular barrier in the PE method is as follows [3]. At all grid points covered by the barrier, the sound pressure is set equal to zero. In this way, the part of the sound field that falls on the screen is eliminated.

It should be noted that the reflective properties of the vertical side planes of the barrier are not well-defined [3]. For many practical situations, however, these reflective properties are of minor importance for the diffracted field. It should also be noted that the assumption of rotational symmetry implies that we model in fact a circular barrier, *i.e.*, a barrier on a horizontal circle with the source at the center [3].

For the numerical integration of the parabolic equation, an implicit Crank-Nicholson algorithm is used. The horizontal stepsize should not exceed about $\lambda/5$, where λ is the wavelength. For propagation over large distances, we use $\lambda/10$.

To give an impression of computing times, we consider our PE code in Fortran that runs on a DEC α computer. Table 1 gives the CPU times for the octave bands 16 Hz - 2 kHz. We used ten frequencies per octave band. Note that the maximum range is reduced with increasing frequency, as high-frequency are usually weak at large distances, due to molecular absorption in the air. The total CPU time required for the complete spectrum is about eight hours.

Recently, a Green's function method was developed [6] for numerical solution of the parabolic equation for atmospheric sound propagation. This Green's function PE method (GFPE) is sometimes referred to as the "fast" PE method, as computing times are smaller than computing times of the Crank-Nicholson PE method (CNPE).

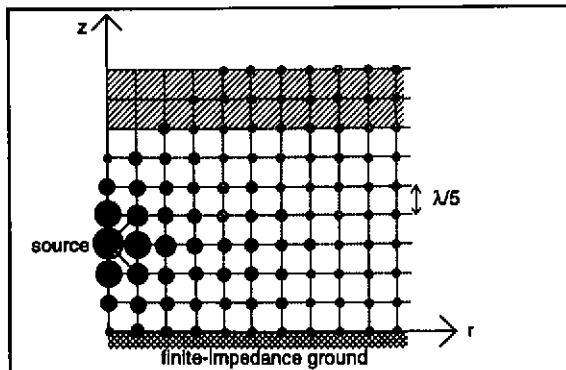


Fig. 2. Schematic representation of the grid used in the PE method. The gridspacing of typically $\lambda/5$ is indicated.

Table 1. CPU times of our Crank-Nicholson PE code in Fortran on a DEC α computer. We used ten frequencies for each octave band.

octave band frequency (Hz)	CPU time (minutes)	horizontal stepsize (meters)	number of gridpoints in vertical direction	maximum range (kilometers)
16	10	0.5	4000	10
31.5	10	0.5	4000	10
63	10	0.5	4000	10
125	20	0.25	4000	10
250	70	0.125	8000	10
500	144	0.0625	8000	10
1000	135	0.03333	8000	5
2000	112	0.02	8000	2.5

3. Numerical examples

Figure 3 shows a comparison between the PE method and an analytic diffraction theory. The computations were performed for a system with a non-refracting atmosphere, and a ground surface with an impedance computed with the empirical model of Delany and Bazley [7], assuming a flow resistivity of $300,000 \text{ Nsm}^{-4}$, a value typical for grassland. We used a height of 2 meters for the source and the receiver. A thin vertical screen with a height of 6 meters was located at 100 meters from the source.

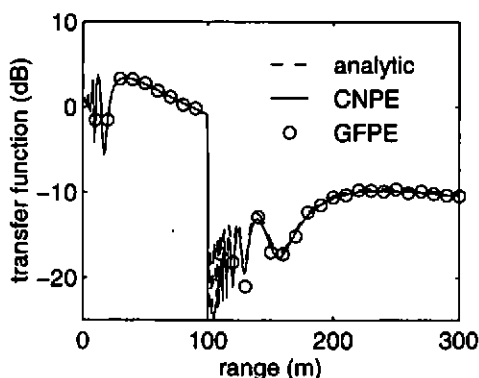


Fig. 3. Transfer function versus horizontal range, at 1000 Hz (single frequency), for a non-refracting atmosphere.