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HOLOGRAPHIC STUDIES OF CELLO VIBRATIONS

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1. INTRODUCTION

Acoustical research on bowed string instruments has traditionally been concentrated on the violin. In spite of being a most important solo and orchestral instrument, the cello has received relatively little attention from musical acoustics researchers. In this paper, we report on studies of the vibrational motion of a cello, using TV holography.

Normal modes are associated with structural resonances. The normal modes of vibration or eigenmodes of a cello are determined by the coupled motion of the top plate, back plate, enclosed air, ribs, neck, fingerboard, etc. Thus one cannot properly label modes as "air modes," "plate modes," or "body modes," although it is tempting to do so when the motion of one part of the structure appears to dominate. We compare the vibrational motion when it is excited by a force applied to the bridge, an oscillating sound pressure applied internally, and the sound field from a loudspeaker, and when it is supported in various ways. From these observations, we attempt to describe the normal modes of vibration of the instrument.

Our designation of the modes follows that used in labeling modes of the violin [1], even though this sometimes results in a different order of mode frequencies in the cello. Perhaps when modal analyses of other cellos are reported, this labeling will change in such a way as to better describe cello modes, as compared with violin modes. The similarity between modes observed in the two instruments is striking, however.

2. SOUND SPECTRA

Room-averaged sound spectra were recorded when the cello was suspended freely by rubber bands, held in normal playing position, and lightly clamped at the center of the neck, the end pin, and at neck/corpus joint (a condition which we will hereafter refer to as "clamped"). The similarity in the peaks in these spectra, shown in Fig. 1, identifies frequencies at which the instrument radiates efficiently.

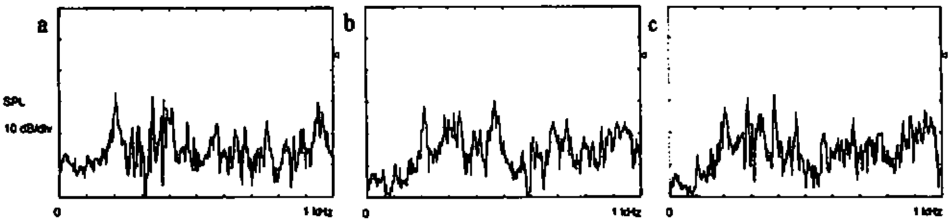

Fig. 1. Room-averaged sound spectra of a cello: a) freely supported on rubber bands; b) hand-held in playing position; c) lightly clamped, as described above.

3. CELLO CAVITY MODES

To help understand the normal modes of vibration, we studied the air cavity modes of the cello with the top, back, and ribs held nearly stationary by means of sand bags. An oscillating sound pressure was introduced internally by means of a horn compression driver and a length of rubber tubing lagged with cotton to damp acoustic resonances of the tube and driver. Two small microphones (approximately 5 mm in diameter) were used to probe the internal sound field. What we call the (m,n) cavity mode has m transverse nodal planes and n longitudinal nodal planes.

The (0,0) or Helmholtz mode was observed at 104 Hz. The modes characterized by longitudinal motion of the internal air occurred at 226 Hz (0,1), 547 Hz (0,2), 742 Hz (0,3), and 1016 (0,4). The modes characterized by transverse air motion in the lower bout were observed at 496 Hz (1,0) and 893 Hz (2,0). The (1,1) mode, characterized by transverse air motion in both bouts (but stronger in the upper bout), was observed at 609 Hz. Other modes are the (1,2) at 820 Hz the (1,3) at 980 Hz, and the (1,4) at 1129 Hz.

4. MODAL ANALYSIS USING HOLOGRAPHIC INTERFEROMETRY

Holographic interferometry offers by far the best spatial resolution of normal modes and operating deflection shapes. Whereas experimental modal testing, and various procedures for mechanical or optical scanning may look at the motion at several hundred (or even thousand) selected points, optical holography looks at almost an unlimited number of points [2]. On the other hand, it is more difficult (but certainly not impossible) to determine the phase of the motion at every point of interest. Furthermore, recording holograms on film tends to be rather time consuming. Electronic TV holography, on the other hand, offers one the opportunity to observe structural motion in real time, and a fast, convenient way to record operational deflection shapes and to determine the normal modes [3].

An optical system for TV holography is shown in Fig. 2. A beam splitter BS divides the laser light to produce a reference and an object beam. The reference beam illuminates the CCD camera via a phase-stepping mirror PS and an optical fiber, while the object beam is reflected by mirror PM so that it illuminates the object to be studied. Reflected light from the object reaches the CCD camera, where it interferes with the reference beam to produce the holographic image. The speckle-averaging mechanism SAM in the object beam alters the illumination angle in small steps in order to reduce laser speckle noise in the interferograms.

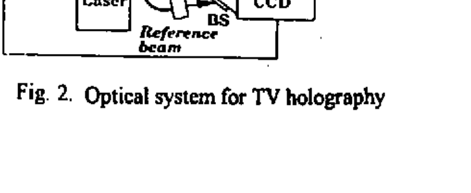

Fig. 2. Optical system for TV holography

5. NORMAL MODES OF THE CELLO

The C_1 mode at 57 Hz is a one-dimensional bending or beam mode. This mode radiates almost no sound, but it contributes to the "feel" of the instrument. The first radiating mode is the A_0 or f-hole mode at 102 Hz, which is shown in Fig. 3. This mode can be excited by a horizontal force on the bridge, an oscillating internal pressure, or by the sound field of a loudspeaker. In other cellos, the A_0 mode has been observed at frequencies as low as 82 Hz [4]. The reason for this rather large difference is not clear, since cello bodies are all rather similar in size and shape.

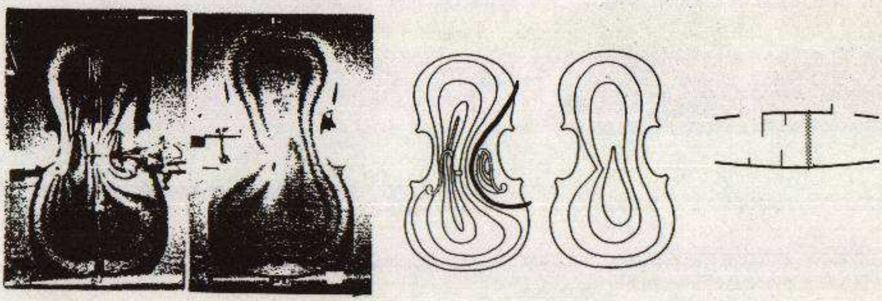

Fig. 3. A_0 mode in the top and back plates. The diagram at the right is a slice view in the bridge plane showing the amplitude and direction of motion at various points.

The T_1 mode, shown in Fig. 4, is observed at 144 Hz. The top plate motion is quite similar to that observed in the T_1 violin mode, but the motion of the back is somewhat different, and it appears to radiate less efficiently than in the violin.

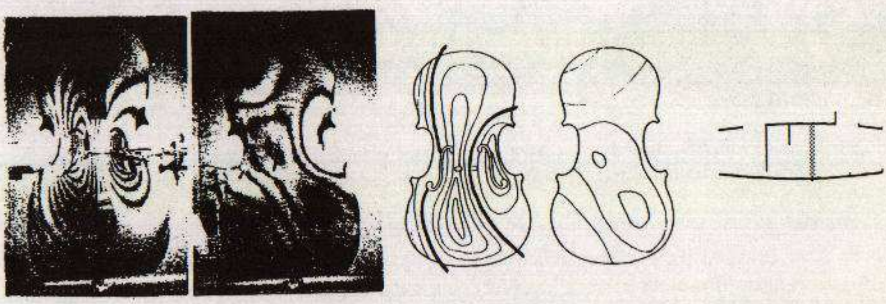

Fig. 4. T_1 mode at 144 Hz excited by a transverse bridge force. The diagram at the right is a slice view in the bridge plane showing the amplitude and direction of motion at various points.

The C_2 mode, characterized by a torsional or twisting motion, as shown in Fig. 5, is observed at 170 Hz. Unlike the violin, this mode occurs at a higher frequency than the T_1 mode.

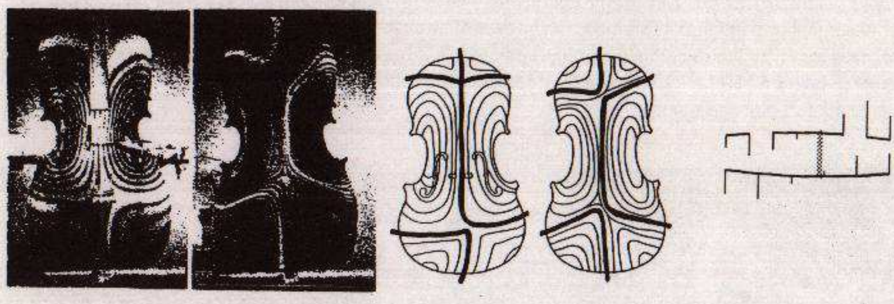

Fig. 5. C_2 mode at 170 Hz. The diagram at the right is a slice view in the bridge plane showing the amplitude and direction of motion at various points.

A mode with ring-shaped nodes in the top and back plates, similar to the C_4 mode in a violin, is found at 195 Hz. This mode is shown in Fig. 6.

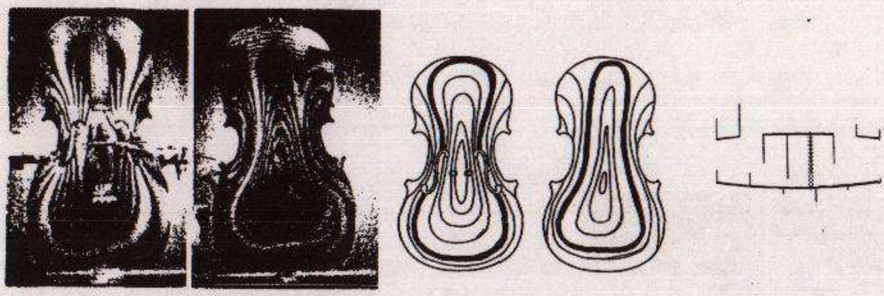

Fig. 6. C_4 mode with ring-shaped nodes in the top and back plates. The diagram at the right is a slice view in the bridge plane showing the amplitude and direction of motion at various points.

In the A_1 mode at 203 Hz, the internal air moves longitudinally between the upper and lower bouts, as in the (0,1) cavity mode. The plates more or less pivot on a nodal line near the bridge as shown in Fig. 7.

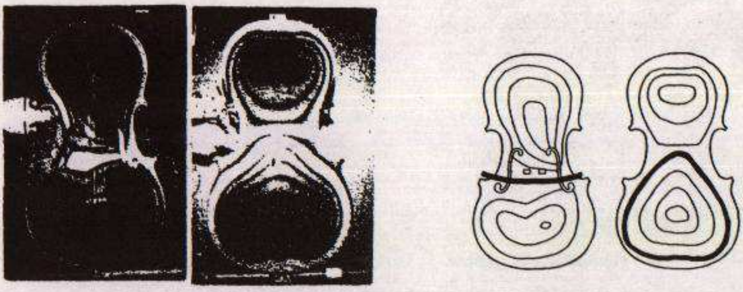

Fig. 7. A_1 mode excited with oscillating internal pressure. The internal air moves longitudinally between the upper and lower bouts, as in the (0,1) cavity mode.

The strongest radiating mode in this range, as in the violin, is the C_3 mode at 211 Hz, shown in Fig. 8, at a frequency above that of the C_4 mode. Note that the back vibrates more strongly than the top plate. In an early 19th century cello, Askenfelt [5] observed the T_1, C_3, and C_4 modes clustered closely together just below 200 Hz.

With the A-string damped, the C_3 mode occurs at 211 Hz. When the A-string is not damped, however, a pair of strong modes with identical plate motion are observed at 207 and 220 Hz. In between, at 219 Hz, we observe a sharp minimum in response, which indicates that the vibrating string opposes the horizontal force applied to the bridge.

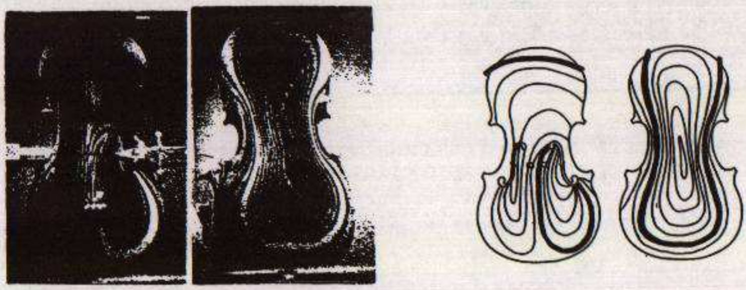

Fig. 8. C_3 mode at 211 Hz (A-string damped).

The A_2 mode at 302 Hz is characterized by longitudinal (0,1) motion of the internal air, similar to the A_1 mode but with the opposite phase. The frequencies of the A_2 and A_1 modes in the cello have the ratio 1.5, compared to a ratio of 1.75 (or 7:4) in the violin. The plate motion, shown in Fig. 9, is quite similar to that of the A_1 mode.

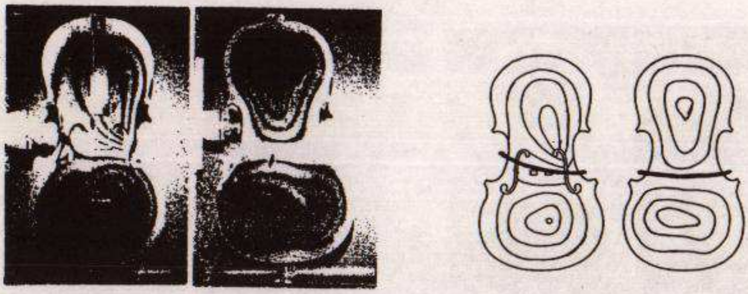

Fig. 9. A_2 mode excited with oscillating internal pressure. The internal air moves longitudinally between the upper and lower bouts, as in the A_1 mode but with the opposite phase.

In the A_3 mode at 277 Hz, the internal air appears to move transversely in the lower bout, as in the (1,0) cavity mode. The plates vibrate in accord with the changing internal pressure.

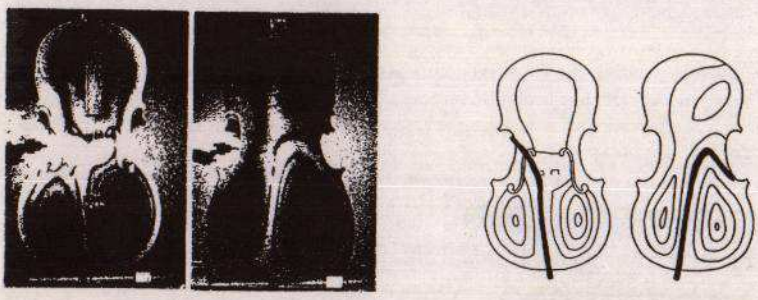

Fig. 10. A_3 mode at 277 Hz excited with oscillating internal pressure. The internal air moves transversely in the lower bout, and the plates vibrate accordingly.

6. DISCUSSION

In the table below, we have tabulated the modal frequencies observed in this cello, along with those we were able to find in the published literature. A comparison is difficult, since modal shapes are not given in most of the publications. Nevertheless, the agreement appears to be satisfactory in most cases. If one interchanges Askenfelt's C_4 and C_3 mode frequencies, the agreement is even better. In at least three of the cellos, there is an octave spacing between the A_1 and A_0 modes, which appears to be the case in most cellos

[6]. Firth [7] describes a number of modes as top-plate modes (T) or back plate modes (B). Langhoff [8] lists several modes of the neck and tailpiece but only three modes of the body.

Mode	Bynum	Ratio to violin	Comments	Askenfelt	Langhoff	Firth 1	Firth 2	Eggers
C_1	57 Hz	0.33						
A_0	102	0.36		90 Hz	85 Hz	90 Hz	104 Hz	82Hz
T_1	144	0.32	Stronger in top than back	168	146	165	185	132
C_2	170	0.42	Torsional motion		163			186?
C_4	195	0.29	Ring-shaped node	202				
A_1	203	0.43	Longitudinal air motion			188	203	
C_3	219	0.39		185				218?
A_3	277	0.25	Transverse air motion					260
A_2	302	0.37	Longitudinal air motion					312
T_2						290	292	
T_3						380	330	
T_4							440	
B_1						178	205	
B_2						295	283	
B_3						320	308	
B_4						390	390	
B_5							460	

In the table above, we give the ratios of the cello mode frequencies compared to those observed in the Hutchins violin SUS 295 [1]. The ratio of cello string frequencies to violin string frequencies is 0.33, but their body lengths and widths are more nearly in the ratio of 0.5. The cello is not meant to be a scaled up violin, of course, but the comparisons are interesting. A good discussion of simularity laws is found in Cremer [9].

7. CONCLUSIONS

Holographic interferometry is an accurate and convenient way to do modal analysis of the cello and a valuable aid in determining the normal modes of vibration. Electronic TV holography is especially convenient, since the mode shapes can be displayed essentially in real time.

The modes of a cello are found to be quite similar to the corresponding modes of a violin, although they may appear in a different order in frequency. Modal frequencies in this cello occur at about 0.29 to 0.43 times the corresponding mode frequencies in a violin. Although the cello is certainly not a scaled-up violin, it does belong to the "violin family" of instruments.

7. REFERENCES

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